

The discovery of the topological phases of matter was a breakthrough in the condensed matter physics, establishing a new paradigm of the description of solid state systems. Moreover, the robustness of these phases against perturbations makes them potentially useful for a range of applications, including quantum computing. In this work, we study two examples of such phases, whose common feature is that they exist on a flat energy band, i.e. the interaction energy term dominates over the kinetic one.

The first phase we study is the Haldane phase, existing in spin-1 antiferromagnetic chains, characterized by four edge states behaving as effective spins-1/2. We investigate the possibility of its realization in a chain of quantum dots, being artificial spin-1 objects, embedded in a nanowire. To describe such a chain, we develop a fermionic Hubbard-Kanamori model. We solve it using the perturbation theory, exact diagonalization and density matrix renormalization group method, showing that its ground state is indeed the Haldane phase. In contrast to existing realizations of Haldane phase, the dot chain would allow to control the system parameters, to study one chain instead of an ensemble, and to investigate the Haldane phase optically. Moreover, it can be used to construct a macroscopic singlet-triplet qubit.

A second object of our investigations are the fractional Chern insulators (FCIs) -- the lattice counterparts of the fractional quantum Hall states. They exist on topologically nontrivial, nearly-flat bands of 2D crystals, playing the role of Landau levels. We study these phases in a variety of lattice models, including a new one, the Lieb lattice, proposed by us in this work. We study the influence of various factors on the stability of FCIs. First, we show the emergence of the FCI phase in the Lieb lattice and investigate its stability as a function of single-particle model parameters. This analysis shows the importance of the flatness of the Berry curvature, in agreement with the results from the literature. A second issue under consideration is the role of the interband excitations. We analyze them in the simplified case of the 1D limit of the topological flat band systems and find that the stability of the FCI-like 1D states can be linked to the dimerization of the Wannier functions. A third factor we study is the long-range interaction. We find that it causes an instability of the FCIs towards the Wigner crystallization at low filling factors. This effect is studied on small periodic plaquettes of three different lattices at a variety of filling factors. The shape of the resulting Wigner crystal is to a large extent independent from the lattice model and follows the behavior of classical point particles. While a part of our results are lattice-specific, others, e.g. the ones concerning the Wigner crystallization, allow to formulate general conclusions about the behavior of interacting particles in topological flat bands.