PhD Thesis

Excitation dynamics in a strongly disordered quantum system with long-range couplings

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Abstract

In this thesis, I study a lattice model with diagonal energy disorder and couplings diminishing with distance as a power-law, $\propto 1/r^{\mu}$. As an introduction, I present the most important properties of the model known from the literature, such as the possibility of the presence of delocalized states even in low-dimensional systems, which is at odds with well-known results employing the single parameter scaling hypothesis. I focus only on the case where the disorder given by the standard deviation of the distribution of diagonal energies is large in comparison with maximal coupling in the system.

I study the evolution of a (quasi)particle initially placed at the center of the lattice. I find an asymptotic survival occupation of the central node that is only slightly smaller than unity in the regime of large disorder for $\mu > d$, while for $\mu = d$ real transport may occur (as it was shown by P.W. Anderson for 3-dimensional system) and I determine the decay time. When the disorder is strong, only jumps from the central node to remote ones are significant. Neglecting higher order jumps in the model, it is possible to analytically solve the Schrödinger equation for the amplitudes of the probability of finding a particle on a site at a particular instant of time. The dynamics of the occupation at any node proceeds qualitatively the same way, i.e., in three consecutive phases. First, the occupation increases with the square of time, which constitutes a fundamental property of quantum systems with long-range interactions arising from the perturbation theory. Then, at a certain moment in time, the growth trend changes to linear in time. Finally, the occupation level saturates.

The triple-phase excitation dynamics is reflected in the behavior of the mean square displacement. The interpretation in transport theory terms is as follows, the first phase of the motion represents ballistic diffusion, the second — standard diffusion followed by saturation. Using numerical simulations, I calculate the mean square displacement, only for the model with coupling inversely proportional to the distance between nodes ($\mu = 1$). I find the values of the dynamic

parameters, the ballistic velocity of the first phase of motion, the diffusion coefficient for the second phase of motion, and the saturation level as a function of the disorder strength and the size of the system. The analytical solution available within the central atom model allows me to find exact formulas for the dependence of the dynamical parameters on the disorder force and on the system size for any value of μ.

The asymptotic occupation of the sites has heavy-tailed power-law distribution along the linear dimension of the system. One can therefore speak of a power-law localization of the particle in the system.

Secondly, I study the dynamics of the correlations propagating in the system defined by the two-point correlation function. During the diffusion of a (quasi)particle through the system, when the particle arrives at a particular node, a correlation is formed between it and the central node. In the single particle picture, the correlation function is proportional to the occupation of the node, which implies the triple-phase growth of the correlation at each node. Analytical derivations allow me to quantify and qualify the correlation dynamics. I also find the time needed to reach a given correlation value as a measure of the propagation speed. Because of the triple phase evolution of the correlation function, the propagation changes its time dependence. In the particular case of $\mu = 1$, the propagation starts as a ballistic motion, then, at a certain crossover time, turns into standard diffusion. When $\mu > 1$, the propagation in the first part is always sub-ballistic, while in the second part it is sub-diffusive.

Finally, I study theoretically the exciton diffusion in a realistic system of self-assembled quantum dots on a two-dimensional circular mesa. From the initially optically excited dot, transport to the remaining artificial atoms occurs. Quantum diffusion occurs via long-range couplings that are power-law functions of the distance, similarly to the model I studied earlier. The distances between the dots are large enough that their wave functions do not cross. Additionally, the size of the system may significantly exceed the emitted electromagnetic wavelength such that the dipole approximation is not valid. In doing so, I assume a finite lifetime for the exciton. Again, diffusion proceeds in three successive phases: ballistic transport, standard diffusion, and saturation. Using the results obtained earlier, I find the dynamic parameters of the evolution. I also investigate what effect the finite lifetime of the exciton has on the dynamics.