Autoreferat wraz z ankietą oceny osiągnięć naukowych

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Spis treści

Rozdział 1

Życiorys zawodowy

1.1 Dane osobowe

- **Imię i nazwisko:** Katarzyna Weron (Katarzyna Sznajd-Weron w publikacjach)
- **Data i miejsce urodzenia:** 7 kwietnia 1971, Wrocław
- **Obywatelstwo:** polskie
- **Stan cywilny:** mężatka, dwóch synów (1996, 2001)
- **Adres zamieszkania:** Dębowa 16, 51-217 Pruszowice, Polska

1.2 Aktualne miejsce zatrudnienia

Profesor nadzwyczajny, od 1.10.2013
Katedra Fizyki Teoretycznej, Wydział Podstawowych Problemów Techniki,
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1.3 Historia zatrudnienia

10.2013-teraz Profesor nadzwyczajny, Wydział Podstawowych Problemów Techniki (WPPT), Politechnika Wrocławska (PWr)

09.2011-09.2013 Profesor nadzwyczajny, Instytut Fizyki Teoretycznej (IFT), Uniwersytet Wrocławski (UWr)

07.2012-09.2013 Kierownik Katedry UNESCO Badań Interdyscyplinarnych, IFT UWr

09.2009-09.2013 Kierownik Zakładu Układów Złożonych i Dynamiki Nieliniowej, IFT UWr

01.2007-09.2011 Adiunkt z habilitacją, Instytut Fizyki Teoretycznej UWr

02.1999-12.2006 Adiunkt, Instytut Fizyki Teoretycznej UWr

09.1995-12.1998 Doktorant, Instytut Fizyki Teoretycznej UWr

1.4 Stopnie naukowe i zawodowe

16.12.2006 Doktor habilitowany nauk fizycznych w zakresie fizyki, specjalność *fizyka statystyczna* (Uniwersytet Wrocławski, Wydział Fizyki i Astronomii, Instytut Fizyki Teoretycznej) na podstawie rozprawy: *Nowa lokalna dynamika w układzie spinów isingowskich*

18.12.1998 Doktor nauk fizycznych w zakresie fizyki, specjalność *fizyka statystyczna* (Uniwersytet Wrocławski, Wydział Fizyki i Astronomii, Instytut Fizyki Teoretycznej) na podstawie rozprawy: *Modelowanie ewolucji biologicznej metodami fizyki statystycznej*; promotor: Prof. dr hab. Andrzej Pękalski

10.05.1995 Magister fizyki komputerowej (Uniwersytet Wrocławski, Wydział Fizyki i Astronomii, Instytut Fizyki Teoretycznej) na podstawie pracy: *Modelowanie dyfuzji Li na Mo(112)*; opiekun: Prof. dr hab. Andrzej Pękalski

Rozdział 2

Autoreferat

2.1 Informacja o osiągnięciach naukowych

Moje zainteresowania naukowe stopniowo ewoluowały, od zagadnień związanych z zastosowaniem metod fizyki statystycznej w modelowaniu układów biologicznych, poprzez zastosowania fizyki statystycznej w badaniu układów społecznych aż po analizę przejść fazowych w nierównowagowych układach spinowych. Mój dorobek naukowy można podzielić na trzy grupy tematyczne:

I. zastosowania fizyki statystycznej w modelowaniu układów biologicznych – w latach 1996–2003,

II. zastosowania modeli agentowych w układach społecznych, w tym w finansach, marketingu i polityce – od 2000 roku,

III. teoretyczne aspekty nierównowagowych dynamik spinowych, w tym przejścia fazowe i stacjonarne własności układów ze stanami absorpcyjnymi, ze szczególnym uwzględnieniem prawdopodobieństwa ucieczki (*exit probability*)[1] – od 2002 roku,

opisane pokrótce w rozdziałach 2.1.1–2.1.3. Mój dorobek obejmuje:

- 43 oryginalne artykuły naukowe opublikowane w czasopismach z listy JCR
- 2 recenzowane artykuły konferencyjne
- 5 artykułów popularnonaukowych

Niektóre wyniki były prezentowane również w trakcie wykładów na konferencjach i seminariach w Polsce i za granicą. W szczególności wygłosiłam:

- 15 zaproszonych i plenarnych wykładów konferencyjnych, w tym 10 zagranicznych
- 20 zaproszonych wykładów seminaryjnych, w tym 9 w placówkach zagranicznych.

[1] To jest moje autorskie tłumaczenie angielskiego terminu *exit probability*, które wydaje mi się dość dobrze oddawać znaczenie tej wielkości. Prawdopodobieństwo ucieczki oznacza bowiem prawdopodobieństwo tego, że układ *ucieknie* ze stanu początkowego z koncentracją początkową spinów 'do góry' równą x do stanu absorpcyjnego ze wszystkimi spinami 'do góry'.

Muszę tutaj dodać, że z powodów rodzinnych, zrezygnowałam z wielu zaproszeń na konferencje, seminaria i dłuższe pobyty zagraniczne. Chociaż jestem świadoma tego, że staże, wizyty naukowe i ogólnie współpraca jest niezwykle ważna dla rozwoju kariery naukowej, podjęłam świadomą decyzję, stawiając na pierwszym miejscu moją rodzinę. Wierzę, że pomimo tej kontrowersyjnej decyzji, wiele z otrzymanych przeze mnie wyników jest dostrzegana, zarówno w Polsce jak i za granicą. Liczba cytowań moich prac bez samocytowań[2] wg. bazy *Web of Science* (WoS) w latach 1996-2014 wynosi 914 . Ponadto, były one wielokrotnie cytowane w książkach i monografiach, więcej szczegółów w rozdziale 3.1.

Moje badania od zawsze koncentrowały się na interdyscyplinarnych zastosowaniach fizyki statystycznej. **Po uzyskaniu magisterium z fizyki** w maju 1995 (Uniwersytet Wrocławski, specjalność: fizyka komputerowa), zaczęłam pracę poświęconą modelowaniu ewolucji biologicznej, pod opieką Prof. Andrzeja Pękalskiego. W okresie 1996-2003, który obejmuje również studia doktoranckie w Instytucie Fizyki Teoretycznej Uniwersytetu Wrocławskiego (1995-1998), opublikowałam w tej dziedzinie 12 artykułów w czasopismach naukowych. Tej tematyki dotyczył również mój doktorat, w ramach którego zaproponowałam model ewolucji pojedynczej cechy ilościowej w metapopulacji, składającej się z wielu lokalnych populacji (tzw. demów) [8], [41], [42].[3]

Po uzyskaniu stopnia doktora w dziedzinie fizyki (w grudniu 1998 roku), kontynuowałam jeszcze przez jakiś czas badania dotyczące dynamiki układów biologicznych, chociaż od roku 2000 moje zainteresowania stopniowo ewoluowały w kierunku prostych modeli spinowych i ich zastosowań w naukach społecznych. Na pewno spory wpływ na zmianę kierunku moich badań miały przeczytane wówczas publikacje i wysłuchane wykłady prof. Serga Galama i prof. Janusza Hołysta. W roku 2000 opublikowałam, wraz z moim ojcem prof. Józefem Sznajdem, mój pierwszy artykuł na temat dynamiki opinii, w którym zaproponowaliśmy nowy prosty model, bazujący na idei spinów Isinga, znany obecnie jako *model Sznajdów* [38].[4] Ta pierwsza praca okazała się udanym mariażem wieloletniego doświadczenia mojego ojca w dziedzinie przemian fazowych w równowagowych układach magnetycznych oraz mojego zainteresowania naukami społecznymi oraz doświadczenia w dziedzinie nierównowagowych układów dynamicznych. Do dnia dzisiejszego, praca została zacytowana ponad 500 razy (zgodnie z WoS) i pozostaje najczęściej cytowanym artykułem opublikowanym w historii *International Journal of Modern Physics C*. Samemu modelowi poświęcono wiele uwagi nie tylko w publikacjach naukowych, ale również w monografiach czy książkach popularnonaukowych (np. cały rozdział w książce z 2006 roku *A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature* by Tom Siegfried). Ku mojemu zaskoczeniu, model posiada również własną stronę w anglojęzycznej wersji Wikipedii (*http://en.wikipedia.org/wiki/Sznajd_model*). Niewykluczone, że ta popularność modelu Sznajdów znacząco przyczyniła się do tego, że w roku 2001 i ponownie w roku 2002 otrzymałam prestiżowe stypendium Fundacji na Rzecz Nauki Polskiej (FNP) dla młodych naukowców.

W roku 2000 tematyka 'socjologiczna' była jeszcze stosunkowo mało popularna wśród fizyków, chociaż powoli zdobywała coraz mocniejszą pozycję w dziedzinie interdyscypli-

[2]Wykluczono te prace cytujące, w których pojawia się chociaż jeden z autorów pracy cytowanej

[3]Numeracja prac zgodna z wykazem publikacji w rozdziale 3.1.

[4]Termin *Sznajd model* został zaproponowany przez prof. Dietrich Stauffer w jego wczesnych publikacjach (2000-2002) na temat tego modelu.

narnych zastosowań fizyki statystycznej. Wkrótce zaczęły powstawać specjalne sekcje towarzystw fizycznych, cykliczne konferencje, działy czasopism czy wręcz oddzielne czasopisma poświęcone nowym, interdyscyplinarnym zastosowaniom fizyki statystycznej. W roku 2002 sekcja Fizyki Układów Socjo-ekonomicznych (*DPG Physics of Socio-Economic Systems Division*) Niemieckiego Towarzystwa Fizycznego ustanowiła ogólnoświatową nagrodę dla najlepszego młodego socjo-/ekonofizyka, wyłanianego w drodze nominacji. W 2004 roku powstała również nowa sekcja Polskiego Towarzystwa Fizycznego *Fizyka w Ekonomii i Naukach Społecznych (FENS)*. W tym samym roku odbyła się pierwsza cykliczna konferencja *European Conference on Complex Systems* gromadząca, co roku w innym europejskim mieście, uczonych z różnych dziedzin – od fizyków, matematyków, informatyków po biologów, ekonomistów czy psychologów społecznych (http://eccs13.eu/).

Generalnie jednak, w tamtym okresie zastosowania fizyki w naukach społecznych to była "egzotyka". Dlatego też, mając na uwadze zdobycie stopnia doktora habilitowanego nauk fizycznych, poza pracą dotyczącą zastosowań prostych modeli w socjologii i ekonomii, zajmowałam się teoretycznymi aspektami proponowanych modeli. W latach 2002–2005 powstał cykl 4 samodzielnych prac, opublikowanych w Physical Review E, poświęcony nowej lokalnej dynamice w układzie spinów isingowskich [3]–[6]. Te cztery prace, wraz z trzema innymi, dotyczącymi zastosowań modelu w naukach społecznych i ekonomicznych (i.e. [32], [34], [38]), stały się podstawą mojej rozprawy habilitacyjnej złożonej we wrześniu 2005 roku. W grudniu 2006 odbyło się kolokwium habilitacyjne przed Radą Instytutu Fizyki Teoretycznej Uniwersytetu Wrocławskiego, w wyniku którego otrzymałam stopień naukowy doktora habilitowanego nauk fizycznych.

Po habilitacji moje zainteresowania skoncentrowały się wokół dwóch głównych obszarów badawczych: (a) teoretycznych aspektów dynamik spinowych, w tym tych wykorzystywanych do modelowania układów społecznych; (b) zastosowania modeli mikroskopowych do analizy konkretnych zjawisk społecznych, w tym dyfuzji innowacji, skuteczności kampanii proekologicznych itp. Szczególnie owocnym w mojej karierze naukowej był rok 2007. W roku 2007 wnioskowałam i otrzymałam dwuletni ministerialny projekt badawczy własny *Nowa lokalna dynamika spinów Isinga z punktu widzenia teorii nierównowagowych układów dynamicznych i zastosowań w modelowaniu grup społecznych*. W tym samym roku ministerialny grant promotorki został również przyznany mojej doktorantce Sylwii Krupie, która w czerwcu 2009 r. obroniła pracę doktorską zatytułowaną *Analiza układów spinów isingowskich z zero-temperaturowymi lokalnymi dynamikami*. W 2007 roku miałam również zaszczyt otrzymać, wspomnianą wcześniej, ogólnoświatową nagrodę dla najlepszego młodego socjo-/ekonofizyka (*Young Scientist Award for Socio- and Econophysics*) i wygłosić zaproszone wykłady na dwóch prestiżowych imprezach - na międzynarodowej szkole *International School on Complexity. Course on Statistical Physics of Social Dynamics: Opinions, Semiotic Dynamics, and Language* (Erice, Sicily, 17.07.2007) i międzynarodowym sympozjum *Computational Philosophy: Lessons from Simple Models* (Niels Bohr Institute, Copenhagen, 13.10.2007). Wreszcie, we wrześniu 2007 roku, zorganizowałam międzynarodową konferencję na cześć siedemdziesiątej rocznicy urodzin prof. Andrzeja Pękalskiego – XXIII Max Born Symposium *Critical Phenomena in Complex Systems* (Polanica-Zdrój).

Jak wspomniałam, w okresie po habilitacji ważny obszar moich badań stanowią teoretyczne aspekty nierównowagowych dynamik spinowych. Szczególnie interesujące okazały się takie charakterystyki układów ze stanami absorpcyjnymi jak prawdopodobieństwo ucieczki ([11], [20], [25]), jak również wrażliwość charakterystyk makroskopowych – w tym przejść fazowych – na drobne zmiany wprowadzane na poziomie mikroskopowym ([1], [9], [14], [16]–[19], [23], [26]). W moim odczuciu właśnie prace z tego obszary stanowią moje największe osiągnięcie naukowe po habilitacji (patrz również rozdział 2.1.3 i 4).

Poza pracą teoretyczną, w ostatnich latach coraz intensywniej współpracuje z przedstawicielami nauk społecznych. W szczególności, w latach 2011-2014 byłam głównym wykonawcą w grancie Opus NCN HS *Modelowanie dynamiki zachowań konsumentów na rynkach oligopolistycznych za pomocą automatów komórkowych* realizowanym na Wydziale Organizacji i Zarządzania Politechniki Śląskiej w Gliwicach. Obecnie jestem głównym wykonawcą w grancie Opus NCN HS *Ekonomiczne konsekwencje kształtowania się opinii i podejmowania decyzji przez konsumentów: Modelowanie agentowe dyfuzji innowacji* realizowanym na Wydziale Informatyki i Zarządzania Politechniki Wrocławskiej, w ramach którego współpracuję głównie z ekonomistami. Byłam również współorganizatorem trzech interdyscyplinarnych spotkań (CODYM 2013, CODYM-WIOSNA 2014, CODYM 2014; szczegóły w rozdziale 3.5.7) zrzeszających badaczy z różnych dziedzin (fizyka, informatyka, nauki społeczne). W latach 2012-2013 byłam szefem *Katedry UNESCO Badań Interdyscyplinarnych* (Uniwersytet Wrocławski) i organizowałam interdyscyplinarne seminaria poświęcone układom złożonym. W ubiegłym (2014) roku zostałam zaproszona do wygłoszenia wykładu plenarnego na *VII Konferencji PsychologiI Ekonomicznej*, zorganizowanej przez Szkołę Wyższą Psychologii Społecznej (SWPS). Miałam również zaszczyt wygłosić wykład inauguracyjny na Wrocławskim Wydziale SWPS w październiku 2014. W wyniku kontaktów nawiązanych z SWPS, w 2015 roku rozpoczęłam współpracę z dr Katarzyną Byrką z SWPS, psychologiem specjalizującym się w psychologii społecznej środowiska i zdrowia. Obecnie jesteśmy w trakcie przygotowywania publikacji dotyczącej modelowania psychologicznych, społecznych i ekonomicznych barier w dyfuzji innowacji.

Obecnie jestem również opiekunem stażu po-doktorskiego dr Anny Chmiel finansowanego w ramach grantu NCN Fuga *Procesy nierównowagowe na sieciach wielopoziomowych*. W związku z tym niektóre z moich ostatnich prac dotyczą roli topologii sieci w modelach dynamiki opinii – dwa artykuły dotyczące tej tematyki zostały już opublikowane ([11], [12]), a dwa kolejne znajdują się obecnie w recenzji[5]. Ten nowy kierunek badań, zapoczątkował współpracę z Grupą Sieci Społecznych (*Social Network Group*) na Politechnice Wrocławskiej. Bez wątpienia temat sieci złożonych, w tym sieci wielopoziomowych i temporalnych, będzie jednym z moich obszarów badawczych w najbliższej przyszłości.

2.1.1 Zastosowania fizyki statystycznej w modelowaniu układów biologicznych

W latach 1996–2003 opublikowałam 12 prac dotyczących modelowania populacji biologicznych i dynamiki populacji metodami fizyki statystycznej. Ostatnia z nich, która powstała we współpracy z prof. Josephem Indekeu z Katolickiego Uniwersytetu w Leuven (Belgia), dotyczyła wpływu antybiotyków na populację bakterii [31]. Pierwsza praca z tego okresu,

[5] A. Chmiel, K. Sznajd-Weron, *Phase transitions in the q-voter model with noise on a duplex clique*, arXiv:1503.01400 [physics.soc-ph]; A. Jędrzejewski, K. Sznajd-Weron, J. Szwabiński, *Mapping the q-voter model: From a single chain to complex networks*, arXiv:1501.05091 [physics.soc-ph]

która została opublikowana w *Physical Review Letters*, była znacznie bardziej ogólna i miała na celu odpowiedzieć na pytania: *Jakie warunki są potrzebne aby populacja mogła przeżyć w danym środowisku? Na ile mogą się różnić dwa środowiska aby populacja, migrująca z jednego do drugiego środowiska, mogła przeżyć?* Aby odpowiedzieć na te pytania, zaproponowaliśmy model sieciowy, w którym każdy osobnik charakteryzowany był swoim genotypem (modelowanym sekwencją zer i jedynek) i jednoznacznie zadanym przez genotyp fenotypem (czyli zestawem cech) [43]. Założyliśmy, że reprodukcja podlega prawom Mendla, tzn. rozważaliśmy jedynie cechy jakościowe. Założyliśmy również, że środowisko opisane jest przez pewien idealny fenotyp. Na podstawie symulacji Monte Carlo, byliśmy w stanie określić warunki konieczne dla rozwoju populacji i kolonizacji nowej, początkowo pustej niszy. Model, choć niewątpliwie interesujący, był stosunkowo skomplikowany w porównaniu do innych fizycznych modeli ewolucji biologicznej, a zatem trudny do analizy teoretycznej. Z tego powodu postanowiłam nie kontynuować pracy nad tym modelem i wróciłam do niego tylko raz, w 2001 roku [37].

W trakcie studiów doktoranckich zaproponowałam nowy, sieciowy model ekosystemu, służący do badania różnicowania się w czasie i przestrzeni cech ilościowych[6] w metapopulacji złożonej z populacji lokalnych (tzw. demów). Chociaż idea modelu była zainspirowana modelem magnetyzmu Blume-Capela, bazował on również na empirycznych obserwacjach biologicznych dotyczących demów. Podobnie jak w rzeczywistych ekosystemach, zmiana cechy lokalnej populacji spowodowana była z jednej strony przepływem genów od sąsiadujących populacji, z drugiej zaś strony doborem naturalnym (GFS model od *gene flow and natural selection*). Pokazałam, że współzawodnictwo, pomiędzy tymi dwoma czynnikami, prowadzi do trzech podstawowych struktur, obserwowanych w naturze: (a) ciągły rozkład populacji, w którym obserwowany jest przestrzenny gradient danej cechy ilościowej, (b) rozróżnialne populacje graniczące strefą integracji lub (c) całkowicie izolowane populacje. Podstawową metodą badawczą były symulacje Monte Carlo, jakkolwiek część wyników udało się uzyskać drogą analityczną, dzięki zastosowaniu transformaty fourierowskiej i przybliżenia pola średniego. W ramach tej tematyki opublikowałam 5 prac ([8],[39]-[42]), z czego trzy stały się podstawą mojej rozprawy doktorskiej.

Spośród wszystkich zagadnień związanych z tematyką biologiczną, najwięcej czasu poświęciłam modelowi GFS. Jednak, moim zdaniem, najbardziej interesujące wyniki uzyskałam w serii dwóch prac dotyczących niestabilności w dynamice populacji. W [7] zadałam pytanie o istnienie krytycznej gęstości, poniżej której populacja jest skazana na zagładę (tzw. *minimalna wielkość populacji trwałej*). Aby odpowiedzieć na to pytanie, zaproponowałam znaczne uproszczenie modelu wprowadzonego w [43], co pozwoliło na podejście analityczne, przynajmniej w przybliżeniu średniego pola. Udało się pokazać analitycznie, że ten prosty model dynamiki populacji jest w stanie opisać zarówno pojemność środowiskową, która jest stabilnym stanem stałym populacji, jak i minimalną wielkość populacji trwałej. Druga praca, w tym krótkim cyklu, została przygotowana wspólnie z moim studentem, Marcinem Wolańskim [33]. W tej pracy rozważyliśmy ponownie model zaproponowany w [7] i zbadaliśmy jaka strategia może pomóc populacji przetrwać.

[6] tzn. takich, którym można przyporządkować zmienną ciągłą, np. wzrost, masa, itp.

2.1.2 Zastosowania modeli agentowych w układach społecznych

Wkrótce po uzyskaniu stopnia doktora zaczęłam się zajmować zastosowaniem idei i metod fizyki statystycznej w naukach społecznych, a konkretnie modelowaniem dynamiki opinii, która jest jednym z najintensywniej analizowanych zagadnień socjofizyki. Moim zdaniem, istnieją co najmniej dwa powody, które sprawiają, że to zagadnienie jest pociągające dla fizyków. Pierwszy określiłabym jako pokusę zbudowania pomostu między poziomami mikro i makro w opisie układów społecznych. Tradycyjnie, istnieją dwie dziedziny, które zajmują się analizą zachowań społecznych - socjologia i psychologia społeczna. Chociaż przedmiot badań w obu dziedzinach jest właściwie taki sam, to jednak podejście, związane z poziomem analizy układu, jest znacząco różne. Socjologowie zajmują się układami społecznymi z poziomu grupy społecznej, natomiast psychologowie społeczni koncentrują się na poziomie jednostki (osoby). Z punktu widzenia fizyka, relacja pomiędzy socjologią a psychologią społeczną przypomina łudzącą relację pomiędzy termodynamiką i fizyką statystyczną. Dlatego właśnie pojawia się pokusa, aby opisać i zrozumieć zachowania układów społecznych (socjologia) z poziomu oddziaływań międzyludzkich (psychologia społeczna). Ponieważ modele dynamiki opinii są często bardzo interesujące z teoretycznego punktu widzenia, drugą motywacją zajęcia się tą tematyką był rozwój nierównowagowej fizyki statystycznej. W 2000 roku zaproponowaliśmy mikroskopowy model [38], znany obecnie jako model Sznajdów, który zainspirował wielu badaczy nie tylko do zastosowań społecznych (np. kampanie marketingowe, rynki finansowe czy kampanie polityczne), ale także do analizy teoretycznych aspektów modelu.

Chciałabym dodać, że zastosowania modeli mikroskopowych w naukach społecznych są starsze niż socjofizyka, jeśli przyjąć, że socjofizyka narodziła się w 1982 roku[7]. Ponad dekadę wcześniej, Thomas Schelling zaproponował model segregacji przestrzennej, zaskakująco podobny do modelu Isinga z dynamiką Kawasakiego[8]. Ostatnio, tego typu podejście, znane obecnie jako modelowanie agentowe (*agent-based modeling* (ABM)), zdobywa coraz większą popularność w naukach społecznych, szczególnie w dziedzinie marketingu.

Moje prace, poświęcone zastosowaniom modeli agentowych w naukach społecznych, skoncentrowane są głównie na problemach związanych z marketingiem ([10], [12], [13], [15], [24], [32]), jakkolwiek pierwsza z nich, poświęcona była opisowi dynamiki ceny na rynkach finansowych [34]. Zaproponowaliśmy modyfikację modelu poprzez wprowadzenie dwóch typów graczy giełdowych – naśladowców (*followers*), którzy zachowywali się zgodnie z regułami modelu Sznajdów, oraz fundamentalistów (*fundamentalist*), posiadających swoja własną racjonalną strategię i pełną wiedzę o rynku. Już wprowadzenie tylko jednego fundamentalisty diametralnie zmieniło zachowanie modelu i pozwoliło odtworzyć empirycznie obserwowane charakterystyki rzeczywistych zwrotów cen.[9]

W kolejnej pracy [32] rozważaliśmy problem związany ze strategiami marketingowymi na rynkach duopolistycznych (tzn. opanowanych przez dwóch konkurujących producentów)[10]. W ramach zmodyfikowanego dwuwymiarowego modelu Sznajdów z zewnętrznym

[7]Wraz z publikacją S. Galam, Y. Gefen, Y. Shapir, *Sociophysics: a new approach of sociological collective behavior. I. Mean-behavior description of a strike*, J. Math. Sociol. 9, 1-13 (1982).

[8]Dietrich Stauffer and Sorin Solomon in *Ising, Schelling and self-organising segregation*, Eur. Phys. J. B 57, 473?79 (2007)

[9]W literaturze anglojęzycznej określane jako *stylized facts of financial returns.*

[10]Często podawanym przykładem duopolu jest tzw. *Cola Wars* – konkurencja pomiędzy Coca-Cola i

polem (opisujacym reklamę), próbowaliśmy odpowiedzieć na pytanie o skuteczną strategię marketingową. W oparciu o symulacje Monte Carlo, wykazaliśmy istnienie dwóch przejść fazowych – jednego związanego z początkową liczbą konsumentów danego produktu (tzw. masa krytyczna) oraz drugiego, związanego z intensywnością kampanii reklamowej. Pięć lat później wróciliśmy do tego tematu, w bardziej ogólnym przypadku rynku oligopolistycznego (kilka firm dominujących nad całym rynkiem w produkcji danego dobra) [24]. Rozpatrywaliśmy sytuację firmy, która wchodzi na rynek, opanowany już przez dwóch równorzędnych graczy, na przykładzie operatora telekomunikacyjnego *Idea* (później na *Orange*), który wchodził na rynek opanowany już przez dwie dobrze znane firmy (*Era* i *Plus*)[11]. Analiza modelu została przeprowadzona zarówno metodą Monte Carlo, jak i metodą pola średniego. Zaskakująco, najlepsze dopasowanie do danych empirycznych dotyczących rynku polskiej telefonii komórkowej, otrzymaliśmy przy założonym poziomie konformizmu (jeden z parametrów modelu) $p \in (0.3, 0.4)$, co jest zgodne z poziomem konformizmu określonym przez Solomona Ascha w serii słynnych eksperymentów społecznych.

W ubiegłych latach kontynuowałam badania w tym obszarze w ramach grantu NCN *Zastosowanie prostych modeli spinowych w marketingu społecznym i komercyjnym.* Nawiązałam również interdyscyplinarną współpracę z (1) Zespołem Modelowania Ekonomicznego na Wydziale Informatyki i Zarządzania Politechniki Wrocławskiej, (2) ekonomistką dr Agnieszką Kowalską-Styczeń z Wydziału Organizacji i Zarządzania Politechniki Śląskiej i (3) psychologiem społecznym dr Katarzyną Byrką z Szkoły Wyższej Psychologii Społecznej. W ramach tej współpracy wystąpiliśmy o dwa granty: (1) z Zespołem Modelowania Ekonomicznego realizujemy obecnie grant badawczy *Ekonomiczne konsekwencje kształtowania się opinii i podejmowania decyzji przez konsumentów: Modelowanie agentowe dyfuzji innowacji* (NCN 2013/11/B/HS4/01061) a (2) z dr Kowalską-Styczeń realizowałyśmy w latach 2011-2014 projekt *Modelowanie dynamiki zachowań konsumentów na rynkach oligopolistycznych za pomocą automatów komórkowych* (NCN 2011/01/B/HS4/02740). W efekcie tych działań w latach 2014-2014 opublikowaliśmy 4 prace dotyczące modelowania agentowego w marketingu ([10], [12], [13], [15]) a kolejne są w przygotowaniu. Moje ostatnie publikacje z tego obszaru poświęcone są dyfuzji innowacji, w szczególności związanej z produktami i usługami ekologicznymi. Jednym z poruszanych przez nas zagadnień jest rozbieżność pomiędzy zamiarem a faktycznym zachowaniem (tzw. *intention-behavior gap*), obserwowana empirycznie w przypadku niektórych innowacji takich jak dynamiczne taryfy elektryczne czy zachowania prozdrowotne.

Poza pracami związanymi z finansami i marketingiem, zajmowałam się również zastosowaniem ABM w polityce ([21], [30]). Moim zdaniem, szczególnie interesujący pomysł zaproponowaliśmy w pracy [30]. Bazując na tzw. kompasie politycznym[12], wprowadziliśmy model wykorzystujący pomysł, zaczerpnięty z modelu Ashkina-Tellera, żeby z każdą

Pepsi.

[11] Jako ciekawostkę podam, że współautorką publikacji była nasza mistrzyni olimpijska Maja Włoszczowska, która w tym okresie pisała pracę magisterską z matematyki finansowej, poświęconą zastosowaniu modelu Sznajdów w marketingu. Dzięki Maji zdobyliśmy empiryczne dane dotyczące nie tylko udziałów w rynku wszystkich trzech firm na przestrzeni lat 2000–2008, ale również kwartalnych nakładów na reklamę.

[12] Jest to model polityczny, opisujący postawy polityczne w dwóch wymiarach - stosunek do wolności ekonomicznej i stosunek do wolności osobistej (zobacz Politicalcompass.org).

jednostką związać dwa spiny Isinga – jeden reprezentujący opinię w sferze ekonomicznej, a drugi w sferze osobistej. Założyliśmy, że mechanizmy zmiany opinii, w każdym z tych dwóch obszarów są różne – w sferze ekonomicznej podlegają tzw. dynamice odpływu (model Sznajdów) a w sferze osobistej dynamice dopływu (zero-temperaturowa dynamika Glaubera).[13] Pokazaliśmy, między innymi, że osiągnięcie konsensusu pomiędzy dwiema grupami osób, różniących się tylko w sferze ekonomicznej, jest stosunkowo łatwe. Natomiast jeśli różnią się w sferze osobistej, konsensus jest niemożliwy. Poza zaskakującymi implikacjami politycznymi (szczególnie w odniesieniu do polskiej sceny politycznej w roku 2005)[14], wynik ten był interesujący również z teoretycznego punktu widzenia, wskazując na różnice pomiędzy dynamiką dopływu i odpływu. Dlatego stał się dla mnie motywacją do dalszych rozważań teoretycznych, które zaprezentuję w kolejnym rozdziale.

2.1.3 Teoretyczne aspekty nierównowagowych dynamik spinowych

Z jednej strony głównym wyzwaniem, z jakim mamy do czynienia modelując dynamikę opinii, jest opisanie niezwykle złożonego układu społecznego przy pomocy stosunkowo prostych, ale jednocześnie w miarę realistycznych reguł. To wyzwanie zainspirowało fizyków do wprowadzenia modeli, które trudno byłoby uzasadnić na bazie zjawisk fizycznych, ale są interesującą propozycją w przypadku układów społecznych. Z drugiej jednak strony okazuje się, że te "niefizyczne" modele, takie jak model Sznajdów, mogą być interesujące same w sobie i dzięki temu przyczyniać się do rozwoju nierównowagowej fizyki statystycznej.

W naszym modelu, podobnie jak w niektórych wcześniejszym modelach dynamiki społecznej[15], układ składa się z N osób, z których każda opisana jest pojedynczą dwustanową opinią $S_i = \pm 1$, podobnie jak cząstki w modelu Isinga. Chociaż może to być zaskakujące, binarne opinie są naturalne z punktu widzenia nauk społecznych. Tak zwany dychotomiczny format odpowiedzi – 1 (tak, prawda, zgadzam się) i 0 (nie, falsz, nie zgadzam się) – jest jednym z najczęściej używanych w eksperymentach społecznych[16].

Jak zauważył profesor Dietrich Stauffer, zasadnicza różnica pomiędzy modelem Sznajdów a modelem wyborcy czy Isinga polega na tym, że w modelu Sznajdów informacja przepływa w kierunku od centralnej grupy do sąsiadów, a nie w kierunku przeciwnym, jak to ma zwykle miejsce (*The crucial difference of the Sznajd model compared with voter or Ising models is that information flows outward: A site does not follow what the neigh-*

[13]Szersza dyskusja na temat dynamik odpływu i dopływu w rozdziale 2.1.3.

[14]Warto tu przypomnieć POPiS, przewidywaną koalicję dwóch zwycięskich partii wyborów parlamentarnych w 2005. Zwykle wzbraniam się od prezentowania *przewidywań* na bazie modeli ABM. Jednak w roku 2005, w trakcie Seminarium Wydziału Fizyki i Astronomii na Uniwersytecie Zielonogórskim, pokusiłam się o stwierdzenie, że taka koalicja, zgodnie z naszym modelem, jest niemożliwa.

[15]Zobacz np. S. Galam, *Majority rule, hierarchical structures and democratic totalitarianism: a statistical approach.*, J. Math. Psychol. 30, 42634 (1986); M. Lewenstein, A. Nowak, B. Latane, *Statistical mechanics of social impact.*, Phys. Rev. A 45, 76376 (1992); J. A. Hołyst, K. Kacperski, F. Schweitzer, *Social impact models of opinion dynamics.* Ann. Rev. Comput. Phys. 9, 25373 (2001).

[16]R. W. Robins, R. C. Fraley and R. F. Krueger (Eds.) *Handbook of research methods in personality psychology* New York: Guilford Press (2007)

bours tell the site, but instead the site tries to convince the neighbours).[17] Jednak dopiero po jakimś czasie zaczęto dyskutować, czy dynamika dopływu (*inflow dynamics*) faktycznie różni się od dynamiki odpływu (*outflow dynamics*). Jak na razie można spotkać w literaturze sprzeczne opinie.[18]

Inną charakterystyczną cechą tego modelu jest fakt, że wpływ społeczny zachodzi tylko w przypadku opinii jednomyślnych, a nie w przypadku bezwzględnej większości. Ta reguła stała się podstawą bardziej ogólnego modelu, tzw. modelu q-wyborcy, zaproponowanego w 2009 roku przez Castellano i innych[19]. Może nie być to zgodne z intuicją, ale właśnie taka ostra reguła znajduje uzasadnienie w eksperymentach społecznych. Zaobserwowano, że mała jednomyślna grupa jest znacznie bardziej skuteczna w przekonywaniu innych, niż większa grupa osób z niejednomyślną większością. Co więcej, w przypadku większości, może się zdarzyć, że to właśnie mniejszościowa opinia będzie bardziej przekonująca. Zanim jednak przejdę do opisu badań, związanych z tymi dwiema wymienionymi cechami modelu, czyli kierunkiem przepływu informacji i regułą jednomyślności, zacznę od chronologicznie pierwszego z problemów teoretycznych, zainspirowanych modelem Sznajdów, nad którym pracowałam.

W 2002 roku zadałam pytanie o możliwość wprowadzenia do modelu Sznajdów czegoś na kształt hamiltonianu. Ze względu na brak symetrii oddziaływań, związany z dynamiką odpływu, nie byłam w stanie wprowadzić hamiltonianu, w ścisłym tego słowa znaczeniu. Zamiast tego wprowadziłam twór, tzw. funkcję niezgody (*disagreement function*), który był odpowiedzialny za dynamikę modelu i był inspirowany hamiltonianem modelu ANNNI (*Axial Next-Nearest-Neighbor Ising*). W przeciwieństwie do hamiltonianu, funkcja niezgody była minimalizowana tylko lokalnie i w efekcie układ mógł osiągać stany stacjonarne, w których globalna funkcja niezgody nie była minimalizowana. Model był analizowany w jednym [6] i dwóch wymiarach [5] zarówno metodą symulacji Monte Carlo, jak i analitycznie w przybliżeniu średniego pola [3] i [4]. Ponadto, w [4] przy użyciu współczynnika Boltzmanna, wprowadziłam do modelu parametr (T), który grał rolę lokalnej temperatury. Model okazał się być na tyle ciekawy, ze względu na nietrywialną ewolucję czasową i złożone diagramy fazowe, że w latach 2002-2005, napisałam serię czterech autorskich prac ([3]–[6]), które zostały opublikowane w Physical Review E i stały się podstawą mojej habilitacji.

Po złożeniu rozprawy habilitacyjnej we wrześniu 2005 roku, zaczęłam analizować różnice między dynamikami dopływu i odpływu. Praca [28], opublikowana w 2006 roku w Physical Review E razem z moją doktorantką Sylwią Krupą, była pierwszą w serii prac związanych z jednym z najważniejszych problemów w dziedzinie symulacji społecznych[20].

[17] D. Stauffer, *Sociophysics: the Sznajd model and its applications.*, Computer Physics Communications 146, 93?8 (2002).

[18] Z tego co wiem, oba terminy – *inflow dynamics* i *ouflow dynamics* – zostały wprowadzone przeze mnie w [28], a obecnie są powszechnie używane w literaturze naukowej. Co więcej dyskusja dotycząca różnic pomiędzy dynamikami nadal trwa, zobacz na przykład P. Roy, S. Biswas, P. Sen, *Exit probability in inflow dynamics: nonuniversality induced by range, asymmetry and fluctuation*, Physical Review E **89**, 030103 (2014) i C. Castellano, R. Pastor-Satorras, *Irrelevance of information outflow in opinion dynamics models*, Physical Review E *83*, 016113 (2011).

[19] C. Castellano, M.A.Muňoz, R.Pastor-Satorras, *Nonlinear q-voter model*, Phys. Rev. E 80, 041129 (2009).

[20] Jak bowiem zauważyli Macy i Willer (*From factors to actors: computational sociology and agent-*

Motywacją do tej pracy była sugestia, że zero-temperaturowa dynamika Glaubera (dopływu) i Sznajdów (odpływu) są równoważne przynajmniej w jednym wymiarze. Aby systematycznie porównać obie dynamiki, wprowadziłyśmy nowy typ aktualizacji – *aktualizację częściowo synchroniczną*. W ramach takiej aktualizacji, w każdym elementarnym kroku czasowym, odwiedzamy wszystkie N węzłów i wybieramy każdego z prawdopodobieństwem c jako kandydata do obrotu. Taka aktualizacja pozwala na płynne przejście pomiędzy dwiema powszechnie stosowanymi aktualizacjami – sekwencyjną losową dla $c = 1/N$ oraz synchroniczną dla $c = 1$. Ten typ aktualizacji został wykorzystany później przez Radicchi i innych do analizy łańcucha spinów Isinga w temperaturze $T = 0$ z algorytmem Metropolisa[21] i przez nas dla łańcucha spinów Isinga z uogólnioną zero-temperaturową dynamiką Glaubera [16]. W [28] wykorzystałyśmy między innymi metodę mapowania łańcucha spinów Isinga na model dimerów (RSA) i pokazałyśmy, że nawet już taka prosta metoda ujawnia różnicę pomiędzy dynamikami dopływu i odpływu. Ponadto, analizowałyśmy obie dynamiki pod wpływem częściowo synchronicznej aktualizacji metodą symulacji Monte Carlo i wykazałyśmy jakościowe różnice pomiędzy obiema dynamikami.

Przy okazji okazało się, że aktualizacja częściowo synchroniczna wprowadza niezwykle złożone zachowanie do prostego łańcuch spinów Isinga z uogólnioną zero-temperaturową dynamika Glaubera. Ponadto okazało się, że analiza łańcuchów spinów Isinga z dynamiką Glaubera, szczególnie w przypadkach niskotemperaturowych, jest niezwykle interesująca z punktu widzenia zastosowań w dziedzinie molekularnych nanomagnesów. W roku 2001 po raz pierwszy zaobserwowano powolną magnetyzację w materiałach złożonych z pojedynczych izolowanych łańcuchów magnetycznych. Co więcej okazało się, że relaksacja takiego układu może być opisana przy pomocy, dotychczas czysto teoretycznej, dynamiki Glaubera[22]. Warto również dodać, że układ spinów Isinga oziębiany z wysokich temperatur do temperatury $T = 0$ wykazuje niezwykle interesujące zachowanie w wyższych wymiarach, nawet w przypadku aktualizacji sekwencyjnej.[23] Dlatego też, postanowiłam pozostać przy tym temacie. W [1] analizowałam relaksację łańcucha spinów Isinga z zero-temperaturową uogólnioną dynamiką Glaubera[24] pod wpływem aktualizacji synchronicznej. Wykorzystując symulacje Monte Carlo i metodę pola średniego, wykazałam istnienie nieciągłego przejścia fazowego pomiędzy fazą ferro- i anty-ferromagnetyczną. Przejście to zachodzi pod wpływem zmiany parametru W_0, który w zero-temperaturowej uogólnionej dynamice Glaubera określa prawdopodobieństwo obrotu spinu, w przypadku gdy energia pozostaje stała (tzn. algorytm Metropolisa odpowiada $W_0 = 1$ a oryginalna

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based modeling., Annu. Rev. Sociol. 28, 143166, 2002), zbyt mało wysiłku poświęca się na analizę tego, jak wyniki zależą od samej konstrukcji modelu.

[21] F. Radicchi, D. Vilone, and H. Meyer-Ortmanns, *Phase Transition between Synchronous and Asynchronous Updating Algorithms*, J. Stat. Phys. 129, 593 (2007).

[22] P. Gambardella i inni, *Ferromagnetism in one-dimensional monatomicmetal chains*, Nature 416, 301-304 (2002); A. Caneschi i inni *Glauber slow dynamics of the magnetization in a molecular Ising chain*, Europhys. Lett. 58, 771–777 (2002)

[23] A. Lipowski, *Anomalous phase-ordering kinetics in the Ising model*, Physica A 268, 6-13 (1999); V. Spirin, P. L. Krapivsky, S. Redner, *Fate of zero-temperature Ising ferromagnets*, Physical Review E 63, 036118 (2001)

[24] C . Godrèche, J. M. Luck, *Metastability in zero-temperature dynamics: statistics of attractors*, J. Phys.: Condens. Matter 17, S2573–S2590 (2005)

dynamika Glaubera $W_0 = 1/2$). Wkrótce po publikacji mojej pracy w *Physical Review E*, Yi i Kim opublikowali komentarz[25], w którym powtórzyli i potwierdzili moje wyniki dotyczące przejścia fazowego dla $W_0 = 1/2$, ale dodatkowo przeprowadzili skalowanie skończonych rozmiarów (*finite-size scaling*). Na podstawie tego skalowania, stwierdzili że obserwowane przejście jest ciągłe, a nie jak twierdziłam w [1] nieciągłe. Faktycznie, zgodnie z klasycznym twierdzeniem Landaua, przejścia pierwszego rodzaju są niemożliwe w równowagowych układach jednowymiarowych. Jednakże, zero-temperaturowa dynamika Glaubera z aktualizacją synchroniczną nie jest równowagowa, a w statystycznej fizyce nierównowagowej znanych jest już kilka modeli, które wykazują nieciągłe przejścia fazowe w jednym wymiarze, chociaż rzeczywiście nie są one tak powszechne jak przejścia ciągłe[26]. Dlatego zdecydowałam raz jeszcze przyjrzeć się temu zagadnieniu, tym razem w przypadku ogólniejszej, częściowo synchronicznej aktualizacji. Dwa lata później, razem z dwójką moich studentów opublikowaliśmy kolejną pracę w *Physical Review E* poświęcono zero-temperaturowej uogólnionej dynamice Glaubera w łańcuch spinów Isinga [16]. Potwierdziliśmy w niej moje poprzednie przewidywania, że w przypadku aktualizacji synchronicznej obserwowane przejście fazowe ferro-antyferromagnetyk jest nieciągłe. Zaobserwowane zostały wszystkie trzy sygnatury nieciągłych przejść fazowych: (1) skok parametru porządku (wykładnik krytyczny $\beta = 0$), (2) współistnienie faz i (3) histereza. Co więcej, pozostałe wykładniki krytyczne okazały się być zgodne ze skalowaniem w układach skończonych dla przejść pierwszego rodzaju, znalezionym analitycznie przez Fishera i Berkera, oraz w ramach symulacji Monte Carlo przez Bindera i Landaua.[27] Dodatkowo pokazaliśmy, że dla każdego innego typu aktualizacji częściowo synchronicznej występuje ciągłe przejście fazowe pomiędzy fazą ferromagnetyczną a tzw. aktywną.

Oprócz pracy poświęconej dynamice Glaubera (dopływu), zajęłam się też teoretyczną analizą dynamiki odpływu w ramach ogólnego modelu q-wyborcy. W skrócie, w tym modelu każda jednostka komunikuje się z panelem q sąsiadów (tzw. q-lobby). Jeśli wszystkie q węzły mają taki sam stan (czyli q-lobby jest jednomyślne), wówczas wyborca przyjmuje stan zgodny z q-lobby. W przeciwnym wypadku (czyli w przypadku braku porozumienia), jak zaproponowano oryginalnie, wyborca zmienia swój stan na przeciwny z prawdopodobieństwem ϵ. W naszych późniejszych publikacjach rozważaliśmy jednak wyłącznie przypadek $\epsilon = 0$, jako naturalne uogólnienie modelu Sznajdów[28].

Istnieją dwa szczególnie interesujące tematy związane z modelem q-wyborcy dla $\epsilon = 0$. Pierwszy, rozważany w pracach [11], [20] i [25], dotyczy niedawnej kontrowersji na temat prawdopodobieństwa ucieczki $E(x)$ w jednowymiarowym modelu q-wyborcy. Podczas gdy dla liniowego modelu wyborcy i modelu Isinga z dynamiką Glaubera, $E(x) = x$ jest do-

[25] I.G. Yi, B.J. Kim, *Comment on Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 83, 033101 (2011).

[26] See e.g. M. Henkel, H. Hinrichsen, and S. Luebeck, *Non-equilibrium Phase Transitions*, Springer, 2008.

[27] M. E. Fisher and A. N. Berker, *Scaling for first-order phase transitions in thermodynamic and finite systems*, Phys. Rev. B 26, 2507 (1982); K. Binder and D. P. Landau, *Finite-size scaling at first-order phase transitions*, Phys. Rev. B 30, 1477 (1984).

[28] W zakończeniu naszej pierwszej poświęconej modelowi Sznajdów, zaproponowaliśmy również taką regułę *jeśli nie wiesz co masz robić zrób cokolwiek*, która odpowiada $\epsilon > 0$, jakkolwiek wydaje się, że z punktu widzenia nauk społecznych bardziej uzasadniona jest reguła *jeśli nie wiesz co masz robić, nie rób nic*.

kładnym wynikiem znalezionym analitycznie, w modelu q-wyborcy z $q \geqslant 2$ prawdopodobieństwo ucieczki jest nieliniowe i jak dotąd nikomu nie udało się znaleźć analitycznej postaci $E(x)$ w sposób ścisły nawet dla $q = 2$ (co odpowiada modelowi Sznajdów).

Pierwsza próba analitycznego obliczenia $E(x)$ została podjęta w 2008 roku niezależnie przez naszą grupę [25] oraz Lambiotte i Rednera[29]. Korzystając z przybliżenia Kirkwooda, otrzymaliśmy analitycznie formułę określającą prawdopodobieństwo ucieczki w przypadku dowolnych warunków początkowych. Co zaskakujące, nasz wynik okazał się zgodny z wynikami symulacji komputerowych, nawet dla takich przypadków, w których przybliżenie teoretycznie nie powinno działać. Ponadto, identyczna postać prawdopodobieństwa ucieczki została otrzymana przez Lambiotte i Rednera. Mimo tego, trzy lata później Galam i Martins[30] zasugerowali, że nasze wyniki są poprawne tylko w przypadku układów o skończonym rozmiarze i dla układów nieskończonych prawdopodobieństwo ucieczki powinno być opisane funkcją schodkową, a nie krzywą typu S, jak znaleziono w [25].

Ponieważ nasze wcześniejsze obliczenia $E(x)$ były jednak przybliżone, potraktowałam tę sugestię poważnie i we współpracy z moimi studentami, wróciłam do tematu w 2011, tym razem jednak w szerszym kontekście modelu q-wyborcy. Zaproponowana przez nas w pracy [20] analityczna formuła dla $E(x)$ dla ogólnego modelu q-wyborcy, została potwierdzona w późniejszych publikacjach dla $q = 2$, ale zakwestionowana dla $q > 2$ w przypadku większych sieci. Pokazano jednak, że z pewnością $E(x)$ ma postać funkcji schodkowej jedynie na grafie zupełnym, a nie w przypadku łańcucha. Jeśli chodzi o dokładną analityczną postać $E(x)$ dla $q > 2$, problem nie został jeszcze definitywnie rozwiązany i nadal stanowi temat dyskusji w literaturze[31].

Drugi temat w tym obszarze badań związany jest z przejściami fazowymi w uogólnionym modelu q-wyborcy. W podstawowym modelu, podobnie jak w modelu Sznajdów, jedynym typem odpowiedzi na wpływ społeczny był tzw. konformizm, który z punktu widzenia fizyki przypomina oddziaływanie ferromagnetyczne. Jednakże, w prawdziwych układach społecznych konformizm nie jest jedynym typem odpowiedzi. Innym, powszechnie uznawanym, jest non-konformizm, który zgodnie z psychologią społeczną może oznaczać:

- Niezależność – odporność na wpływ, tzn. decyzje podejmowane są niezależnie od wpływu grupy. W tym sensie niezależność odgrywa podobną rolę co temperatura [14].

- Anty-konformizm – bunt wobec wpływu. Według psychologów, anty-konformiści są podobni do konformistów w tym sensie, że jedni i drudzy biorą pod uwagę opinie grupy (lub normę społeczną) – konformiści się z nią zgadzają, a anty-konformiści nie.

Wprowadzenie któregokolwiek rodzaju non-konformizmu do podstawowego modelu q-wyborcy powoduje pojawienie się przejścia fazowego pomiędzy fazą z magnetyzacją $m \neq 0$

[29]R. Lambiotte, S. Redner, *Dynamics of non-conservative voters*, Europhys. Lett. **82**, 18007 (2008).

[30]S. Galam, A.C.R. Martins, *Pitfalls driven by the sole use of local updates in dynamical systems*, Europhys. Lett. 95, 48005 (2011).

[31][11] i A.M. Timpanaro, C.P.C. Prado, *Exit probability of the one-dimensional q-voter model: Analytical results and simulations for large networks*, Phys. Rev. E **89**, 052808 (2014).

(interpretowaną w tego typu modelach jako *opinia publiczna*) i fazą status-quo ($m = 0$). W pracy [19] wprowadziliśmy niezależność, a w [18] anty-konformizm w przypadku modelu Sznajdów, co odpowiada modelowi q-wyborcy z $q = 2$.

W [17] rozważaliśmy uogólniony model q-wyborcy z dwoma typami non-konformizmu dla dowolnej wartości q. Motywacją do tej pracy stało się zadane przeze mnie pytanie, dotyczące różnic pomiędzy dwoma typami non-konformizmu. Chociaż różnice te są bardzo ważne z punktu widzenia psychologii społecznej i widoczne na poziomie mikroskopowym, mogą być bez znaczenia z punktu widzenia fizyka, w tym sensie, że makroskopowe zachowanie układu będzie jakościowo takie samo dla obu typów non-konformizmu. Prawdę mówiąc intuicja mojego doktoranta (współautora pracy) była taka, że rodzaj non-konformizmu nie ma znaczenia, a ja miałam nadzieje, że różnice się jednak ujawnią. Dlatego rozważyliśmy dwa modele – model q-wyborcy z niezależnością i model q-wyborcy z anty-konformizmem. Ograniczyliśmy naszą analizę do przypadku grafu zupełnego, co pozwoliło uzyskać dokładne wyniki analityczne i zastosować do opisu teorię Landaua. W rezultacie stwierdziliśmy jednoznacznie, że istnieją znaczne różnice jakościowe pomiędzy tymi dwoma modelami. Chociaż w obu modelach zaobserwowaliśmy przejście fazowe pod wpływem parametru p, określającym prawdopodobieństwo zachowania non-konformistycznego (anty-konformistycznego lub niezależnego w zależności od modelu), to charakter przejścia był różny dla obu modeli nawet na grafie zupełnym. W szczególności pokazaliśmy, że w modelu z anty-konformizmem krytyczna wartość zaburzenia p rosnie z q, podczas gdy w modelu z niezależnością krytyczna wartość zaburzenia p maleje z q. Ponadto, w przypadku modelu z anty-konformizmem przejście fazowe jest ciągłe dla dowolnej wartości q, zaś w modelu z niezależnością charakter przejścia zmienia się z ciągłego na nieciągłe dla $q = 6$. Ten ostatni wynik jest szczególnie interesujący z punktu widzenia nauk społecznych, potwierdzając fakt, że niezależność prowadzi do bardziej rewolucyjnych zmian niż anty-konformizm. Mam nadzieje, że wyniki te będą również istotne dla całego obszaru modelowania dynamiki opinii, ponieważ wcześniej problem różnic pomiędzy dwoma typami non-konformizmu był zaniedbywany a nawet niedostrzegany.

Następna praca w tej serii, [14], została napisana na zaproszenie prof. Sidneya Rednera i opublikowana w specjalnym tomie *Statistical Mechanics and Social Sciences* czasopisma Journal of Statistical Physics. W tej pracy przeprowadziliśmy dalsze badania dotyczące różnic pomiędzy dwoma typami non-konformizmu, tym razem w kontekście ogólniejszego modelu q-wyborcy z progiem. Ten uogólniony model został zaproponowany przez mojego doktoranta, Piotra Nyczkę, w celu opisania sytuacji gdy nie jest potrzebna całkowita jednomyślność aby zmienić przekonania wyborcy. Zaproponowany przez niego model przypomina, znany z nauk społecznych tzw. model progowy (*threshold model*), ale zasadnicza różnica między modelami polega na tym, że w naszej pracy rozważaliśmy (1) dynamikę odpływu, podczas gdy w modelu progowym mamy do czynienia z dynamiką dopływu, (2) grupa q osób wpływa na wyborcę, a nie wszyscy jego sąsiedzi, jak ma to zwykle miejsce w modelach progowych. Z mojego punktu widzenia, wprowadzony przez Piotra model, był znakomitym pretekstem do sprawdzenia czy oba typy niezależności różnią się jedynie w przypadku reguły jednomyślności czy może dla dowolnego progu. Ponownie więc rozważyliśmy dwie wersje modelu i okazało się, że różnice pomiędzy dwoma typami non-konformizmu są widoczne dopiero wówczas gdy, próg większościowy przekroczy wartość $3q/4$. Ta wartość zaskakująco zgadza się z wynikami eksperymentów społecznych, które wskazują na to, że o jednomyślności można już mówić w przypadku gdy 75% osób ma to samo zdanie.

W moim odczuciu praca [14] jest ważna nie tylko ze względu na nowe wyniki teoretyczne. Ponieważ publikacja miała się ukazać w tomie specjalnym, a edytorami byli zarówno fizycy jak i przedstawiciel nauk społecznych, postanowiłam poświęcić sporą część artykułu (ostatecznie pięć rozdziałów, 10 stron) na osobisty przegląd literatury dotyczącej dynamiki opinii. Moim celem było przybliżenie literatury i wiedzy, z zakresu nauk społecznych, fizykom zajmującym się dynamiką opinii, a jednocześnie opisać pewne użyteczne w tej dziedzinie koncepcje fizyczne w sposób zrozumiały dla przedstawicieli nauk społecznych. Mam nadzieję, że mi się to udało – chociaż praca jest bardzo młoda, już doczekała się 29 cytowań według Google Scholar i 9 cytowań (bez autocytowań) według Web of Science. Ponadto, krótko po publikacji tego artykułu, zostałam zaproszona do udziału w kilku prestiżowych wydarzeniach. W szczególności, zostałam zaproszona do poprowadzenia tutorialu na Zjeździe Niemieckiego Towarzystwa Fizycznego (Spring Meeting of the German Physical Society in Berlin 2015)[32], plenarnego wykładu na VII Konferencji Akademickiego Stowarzyszenia Psychologii Ekonomicznej w Szkole Wyższa Psychologii Społecznej (SWPS) oraz wykłoszenia wykładu inaugurującego rok akademicki na wrocławskim wydziale SWPS w październiku 2014.

Pytanie: *Czy szczegóły założeń, dotyczących modelowania oddziaływań społecznych na poziomie mikroskopowym, mają wpływ na zachowanie układu jako całości, czy też nie?*, które było inspiracją poprzednich prac, było również motywacją do najnowszej pracy w serii [9], opublikowanej w PLoS ONE i poświęconej słynnej psychologicznej debacie *Osobowość czy sytuacja?* Debata rozpoczęła się w późnych latach sześćdziesiątych i odnosi się do kontrowersji dotyczącej tego, czy osobowość czy może sytuacja jest bardziej istotna dla przewidzenia zachowania danej osoby. Zaproponowaliśmy, a następnie przeanalizowaliśmy, dwa warianty (*osobowość* lub *sytuacja*) tego samego modelu agentowego, konkretnie modelu q-wyborcy z niezależnością. W efekcie pokazaliśmy, że decyzja o wyborze wariantu *osobowość* (tzn. pewna frakcja p agentów to na stałe niezależni, a pozostali to konformiści) albo *sytuacja* (z prawdopodobieństwem p każdy agent zachowuje się niezależnie, a z $1-p$ jak konformista) ma kolosalne znaczenie z punktu widzenia makroskopowego zachowania układu, nawet w przypadku grafu zupełnego. Wydaje mi się, że ten wynik powinien mieć daleko idące konsekwencje, także poza modelowaniem dynamiki opinii, ponieważ w ostatnich latach modelowanie agentowe nie tylko staje się coraz popularniejszym narzędziem w naukach społecznych, ale jest nawet często traktowane jako substytut prawdziwych eksperymentów.

Moim zdaniem, rozważane w pracy [9] zagadnienie jest również interesujące z fizycznego punku widzenia, ponieważ relacja *osobowość* kontra *sytuacja* przypomina związek między podejściem typu *quenched* i *annealed*, analizowany tradycyjnie w odniesieniu do układów spinowych, a ostatnio coraz bardziej popularnym w dziedzinie sieci złożonych. Jedna z metod analitycznych stosowana w tej dziedzinie, tak zwane *heterogeniczne pole średnie*, polega na zastąpieniu prawdziwej (*quenched*) sieci, w której dane połączenie istnieje lub nie, przez graf zupełny z odpowiednimi wagami połączeń (*annealed*).[33] Pytanie,

[32] Niestety z powodu wypadku i w efekcie złamanej nogi, musiałam odwołać ten wyjazd.

[33] See e.g. S.N. Dorogovtsev, A.V. Goltsev, J.F.F. Mendes, *Critical phenomena in complex networks*, Rev. Mod. Phys. 80, 12751335 (2008).

które się natychmiast narzuca, dotyczy tego, czy faktycznie można dokonać takiej zamiany tzn. czy te oba podejścia są równoważne[34]. Odpowiedź wcale nie jest oczywista i zależy od badanego układu.

2.2 Informacje o opiece naukowej i kształceniu młodej kadry

Od początku mojej pracy akademickiej dużą wagę przywiązywałam do zajęć dydaktycznych i współpracy ze studentami. W trakcie pracy na Uniwersytecie Wrocławskim (do września 2013 roku), oprócz standardowych zajęć znajdujących się w programie studiów (*Mechanika teoretyczna*, *Fizyka statystyczna*, *Modelowanie komputerowe* czy *Teoria przejść fazowych*), prowadziłam również kursy wybieralne, które cieszyły się wśród studentów dużą popularnością (w tym kurs po angielsku *Nonlinear dynamics* i wykład *Nierównowagowe przejścia fazowe*). Szczególnie dobrze wspominam autorskie seminarium *Egzotyczna fizyka statystyczna*, prowadzone przeze mnie w latach 2000-2001 i wprowadzające studentów w nową dziedzinę interdyscyplinarnych zastosowań fizyki statystycznej. Seminarium, pomimo że nieobowiązkowe (tzn. nie znajdujące się w planie studiów) przyciągnęło kilkunastu studentów lat 3-5 fizyki doświadczalnej, teoretycznej i komputerowej. Od tamtego momentu wielokrotnie miałam przyjemność współpracy ze studentami (między innymi z Karolem Suszczyńskim, Rafałem Topolnickim, Maciejem Tabiszewskim i Marcinem Wolańskim) co zaowocowało kilkoma publikacjami naukowymi w prestiżowych czasopismach z listy JCR: ([10], [11], [16], [19], [20], [33]). **Za moje zaangażowanie w działalność dydaktyczną zostałam wyróżniona w roku 2011 Medalem Komisji Edukacji Narodowej**

W duchu rozwijania pasji i tworzenia Nauki prowadzony był również autorski kurs *Modelarnia – krytyczność i złożoność* (60 godzin w semestrze). Zajęcia te, które odbywały się na Uniwesytecie Wrocławskim w latach 2012-2013 w ramach projektu "Rozwój potencjału i oferty edukacyjnej Uniwersytetu Wrocławskiego szansą zwiększenia konkurencyjności Uczelni", miały nowoczesną formę interaktywną. Stosowane były takie metody dydaktyczne jak dyskusje, burze mózgów, prezentacje (indywidualne i grupowe), ćwiczenia numeryczne i rachunkowe. W trakcie zajęć studenci mieli okazję nie tylko zapoznania się z nowymi ideami modelowania układów złożonych i nierównowagowych przejść fazowych, ale również uczestniczenia w procesie badawczym od narodzin modelu, poprzez przegląd literaturowy, analizę modelu metodami numerycznymi i analitycznymi aż po prezentację wyników (w formie referatu i publikacji).

W latach 1999-2014 byłam opiekunem 25 prac dyplomowych (w tym 20 magisterskich) na Uniwersytecie Wrocławskim. W tym samym okresie byłam również recenzentem ponad 30 prac magisterskich na UWr i NTNU (Trondheim, Norwegia). Ponadto byłam promotorem w trzech przewodach doktorskich z nauk fizycznych (dwa zakończone, jeden otwarty), a w latach 2011-2014 recenzentem czterech rozpraw doktorskich, więcej szczegółów w rozdziale 3.5.2.

[34]A.N. Malmi-Kakkada, O.T. Valls, Ch. Dasgupta, *Ising model on a random network with annealed or quenched disorder*, Physical Review B 90, 024202 (2014).

W 2013 roku podjęłam decyzję o zmianie miejsca pracy i od października 2013 jestem zatrudniona na Wydziale Podstawowych Problemów Techniki, Politechniki Wrocławskiej. Tu, ze względu na znacznie większą liczbę studentów, mogę jeszcze intensywniej realizować się jako nauczyciel akademicki. Poza prowadzonymi kursami z fizyki, od listopada 2013 mam przyjemność być współopiekunką (wraz z prof. Antonim Mitusiem) koła naukowego fizyków Nabla. W ramach spotkań koła, zorganizowałam w roku akademickim 2013/2014 cykl wykładów poświęconych układom złożonym a w roku 2014 rozpoczęłam zajęcia z symulacji komputerowych, przerwane w 2015 ze względów zdrowotnych[35].

2.3 Informacja o działalności popularyzatorskiej i organizacyjnej

Poza pracą naukową i dydaktyczną, sporo czasu i zaangażowania poświęcałam zawsze pracy organizacyjnej i popularyzacji nauki. Brałam udział w organizacji dziewięciu międzynarodowych konferencji naukowych, będąc kierownikiem trzech z nich. Za szczególnie ważną uważam, zorganizowaną przeze mnie w 2011 roku, 47. Zimową Szkołę Fizyki Teoretycznej *Simple Models for Complex Systems*. Szkoła miała na celu nie tylko przekazanie młodym ludziom wiedzy związanej ze stosunkowo nową, intensywnie rozwijającą się i wysoce interdyscyplinarną dziedziną układów złożonych, ale również integrację środowiska akademickiego. Wykładowcami na Szkole było 12 uczonych z całego świata, którzy nie tylko są wybitnymi specjalistami w swojej dziedzinie, ale również znakomitymi dydaktykami umiejącymi wprowadzić nowicjuszy w fascynujący, interdyscyplinarny Świat Układów Złożonych. Za osobisty sukces uważam fakt, że ze względu na wysoce interdyscyplinarny charakter szkoły na udział w spotkaniu zdecydowali się nie tylko fizycy, ale również matematycy, informatycy, ekonomiści oraz przedstawiciele nauk społecznych. W sumie w Szkole wzięło udział 75 osób z całego świata.

Idea przekraczania podziałów między różnymi, często pozornie niezwykle odległymi dziedzinami, przyświeca również od samego początku katedrze UNESCO Studiów Interdyscyplinarnych na Uniwersytecie Wrocławskim (http://www.kusi.ift.uni.wroc.pl). W latach 2011-2013 miałam zaszczyt być przewodniczącą Rady Katedry. Pierwszym przewodniczącym i jej pomysłodawcą był prof. Andrzej Pękalski, którego marzeniem było stworzenie interdyscyplinarnego ośrodka badawczo-edukacyjnego o zasięgu międzynarodowym. Podstawową formą działalności Katedry były seminaria interdyscyplinarne oraz związana z nimi praca naukowa, dotycząca głównie zagadnień wymagających współdziałania ekspertów z różnych dziedzin. Ponadto Katedra była współorganizatorem kilku konferencji naukowych, w tym *Workshop on Science for Conservation & Preservation of Cultural Heritage Research & Education* (2007) z Wydziałem Chemii Uniwersytetu Wrocławskiego oraz XXIII Sympozjum Maksa Borna *Critical Phenomena in Complex Systems* (2007) z Instytutem Fizyki Teoretycznej Uniwersytetu Wrocławskiego.

W roku 2013 byłam współorganizatorem konferencji *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the Future (CODYM)*. Konferencja odbyła się

[35]W grudniu 2014 uległam wypadkowi narciarskiemu, co skutkowało operacją i kilkumiesięczną rehabilitacją.

we wrześniu 2013 roku w Barcelonie jako satelita 13. Europejskiej Konferencji Układów Złożonych (ECCS 2013). Muszę podkreślić, że ta konferencja była już prawdziwie interdyscyplinarna. Nie tylko uczestnicy pochodzili z tak odległych dziedzin jak matematyka, fizyka, informatyka, socjologia, psychologia czy lingwistyka, ale również nasz pięcioosobowy Komitet Organizacyjny składał się zarówno z przedstawicieli nauk społecznych jak i ścisłych. W ramach kontunuacji tego interdyscyplinarnego wydarzenia, w kwietniu 2014 zorganizowałam workshop CODYM-Spring'14, w którym wzięli udział naukowcy z różnych dziedzin z całego świata. W kolejnych latach, w ramach międzynarodowej interdyscyplinarnej współpracy z Timoteo Carletti (Belgia), Guillaume Deffuant (Francja), Floriana Gargiulo (Belgia), Anrea Guazzini (Włochy), Sylvie Huet (Francja) i Pawłem Sobkowiczem (Polska), zamierzamy kontynuować organizację spotkań CODYM.

Jednym z moich ulubionych zajęć związanych z życiem akademickim jest popularyzacja nauki. Od zawsze byłam aktywna w tej dziedzinie, przede wszystkim wygłaszając wykłady dla laików, między innymi na Uniwersytecie Trzeciego Wieku, na posiedzeniu klubu Lions , szkołach i przede wszystkim w ramach Dolnośląskiego Festiwalu Nauki (patrz rozdział 3.5.6). Jestem również autorem kilku artykułów popularnonaukowych, z czego trzech opublikowanych w popularnym miesięczniku *Wiedza i Życie*.

Rozdział 3

Ankieta osiągnięć naukowych

3.1 Informacja o osiągnięciach i dorobku naukowym

3.1.1 Informacja o cytowaniach

Liczba cytowań moich prac wg. bazy Web of Science (stan bazy na dzień 31.03.2015) wynosi **914 bez autocytowań**, a indeks Hirscha 12.[1]

Ponadto moje publikacje były również wielokrotnie (ok. 100 razy wg. *Google Books*) cytowane w monografiach, między innymi w:

- Parongama Sen, Bikas K. Chakrabarti, *Sociophysics: An Introduction*, Oxford University Press (2013)
- Francisek Slanina, *Essentials of Econophysics Modelling*, Oxford University Press (2013)
- Serge Galam, *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*, Springer (2012)
- Willi-Hans Steeb, *The Nonlinear Workbook: Chaos, Fractals, Cellular Automata, Neural Networks*, World Scientific (2011)
- Rodolfo Baggio, Jane Klobas, *Quantitative Methods in Tourism*, Channel View Publications (2011)
- Tom Siegfried, *A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature*, National Academies Press (2006)
- Dietrich Stauffer et al., *Biology, Sociology, Geology by Computational Physicists*, Elsevier (2006)
- Sergio Albeverio, Volker Jentsch, Holger Kantz, *Extreme Events in Nature and Society*, Springer (2006)

[1] Wykluczono te prace cytujące, w których pojawia się chociaż jeden z autorów pracy cytowanej. Indeks Hirscha podany jest według bazy *Web of Science* (liczony z autocytowaniami, wyłącznie dla prac indeksowanych w bazie WoS).

- Philip Ball, *Critical Mass: How One Thing Leads to Another*, Macmillan (2006)
- David P. Landau, Kurt Binder, *A Guide to Monte Carlo Simulations in Statistical Physics*, Cambridge University Press (2002)

3.1.2 Autorskie artykuły naukowe w czasopismach indeksowanych w JCR

Wykaz autorskich artykułów naukowych w czasopismach z listy filadelfijskiej wraz z aktualnymi wartościami 2 i 5-letnich wskaźników impact factor (odpowiednio IF_{2Y} i IF_{5Y}); z raportu JCR opublikowanego w 2014 r. oraz liczbą cytowań CYT według bazy Web of Science (stan bazy na dzień 31.03.2015) bez autocytowań (tzn. wykluczono te prace cytujące, w których pojawia się chociaż jeden z autorów pracy cytowanej):

[1] K. Sznajd-Weron, *Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 82, 031120 (2010); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 2]$

[2] K. Sznajd-Weron, *Sznajd model and its applications*, Acta Physica Polonica B 36 (2005); $[IF_{2Y} = 0.998, IF_{5Y} = 0.742, CYT = 65]$

[3] K. Sznajd-Weron, *Metastabilities in the degenerated phase of the two-component model*, Phys. Rev. E 72, 026109 (2005); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 0]$

[4] K. Sznajd-Weron, *Mean-field results for the two-component model*, Phys. Rev. E 71, 046110 (2005); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 1]$

[5] K. Sznajd-Weron, *Dynamical model of Ising spins*, Phys. Rev. E 70, 037104 (2004); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 13]$

[6] K. Sznajd-Weron, *Controlling simple dynamics by a disagreement function*, Phys. Rev. E 66, 046131 (2002); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 19]$

[7] K. Sznajd-Weron, *Instabilities in population dynamics*, Eur. Phys. J. B 16, 183 (2000); $[IF_{2Y} = 1.463, IF_{5Y} = 1.515, CYT = 7]$

[8] K. Sznajd-Weron, *Change of a continuous character caused by gene flow. An analytical approach*, Physica A 264, 432 (1999); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 0]$

3.1.3 Współautorskie artykuły naukowe w czasopismach indeksowanych w JCR

[9] K. Sznajd-Weron, J. Szwabiński, R. Weron, "Is the Person-Situation Debate Important for Agent-Based Modeling and Vice-Versa?" PLoS ONE 9(11), e112203 (2014); $[IF_{2Y} = 3.543, IF_{5Y} = 4.015, CYT = 0]$

[10] A. Kowalska-Pyzalska, K. Maciejowska, K. Suszczyński, K. Sznajd-Weron, R. Weron, *Turning green: Agent-based modeling of the adoption of dynamic electricity tariffs*, Energy Policy 72, 164-174 (2014); $[IF_{2Y} = 2.696, IF_{5Y} = 3.402, CYT = 1]$

[11] K. Sznajd-Weron, K. Suszczyński, *Nonlinear q-voter model with deadlocks on the Watts-Strogatz graph*, J. Stat. Mech. P07018 (2014); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CYT = 0]$

[12] K. Sznajd-Weron, J. Szwabiński, R. Weron, T. Weron, *Rewiring the network. What helps an innovation to diffuse?*, J. Stat. Mech. P03007 (2014); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CYT = 1]$

[13] P. Przybyła, K. Sznajd-Weron, R. Weron, *Diffusion of innovation within an agent-based model: Spinsons, independence and advertising*, Advances in Complex Systems 17, 1450004 (2014); $[IF_{2Y} = 0.786, IF_{5Y} = 0.918, CYT = 0]$

[14] P. Nyczka, K. Sznajd-Weron, *Anticonformity or Independence? – Insights from Statistical Physics*, Journal of Statistical Physics 151, 174-202 (2013); $[IF_{2Y} = 1.284, IF_{5Y} = 1.239, CYT = 9]$

[15] A. Kowalska-Styczeń, K. Sznajd-Weron, *Access to information in word of mouth marketing within a cellular automata model*, Advances in Complex Systems 15, 1250080 (2012); $[IF_{2Y} = 0.786, IF_{5Y} = 0.918, CYT = 1]$

[16] B. Skorupa, K. Sznajd-Weron, R. Topolnicki, *Phase diagram for a zero-temperature Glauber dynamics under partially synchronous update*, Phys. Rev. E 86, 051113 (2012); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 0]$

[17] P. Nyczka, K. Sznajd-Weron, J. Cislo, *Phase transitions in the q-voter model with two types of stochastic driving*, Physical Review E 86, 011105 (2012); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 8]$

[18] P. Nyczka, K. Sznajd-Weron, J. Cisło, *Opinion dynamics as a movement in a bistable potential*, Physica A 391, 317-327 (2012); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 1]$

[19] K. Sznajd-Weron, M. Tabiszewski, A. Timpanaro, *Phase transition in the Sznajd model with independence*, Europhys. Lett. 96, 48002 (2011); $[IF_{2Y} = 2.269, IF_{5Y} = 2.112, CYT = 10]$

[20] P. Przybyła, K. Sznajd-Weron and M. Tabiszewski, *Exit probability in a one-dimensional nonlinear q-voter model*, Phys. Rev. E 84, 031117 (2011); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 7]$

[21] G. Kondrat, K. Sznajd-Weron, *Spontaneous reorientations in a model of opinion dynamics with anticonformists*, Int. J. Mod. Phys. C 21, 559-566 (2010); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CYT = 4]$

[22] T. Czarnik, R. Gawda, W. Kołodziej, D. Łątka, K. Sznajd-Weron, R. Weron, *Associations between intracranial pressure, intraocular pressure and mean arterial pressure in patients with traumatic and non-traumatic brain injuries*, Injury, Int. J. Care Injured 40, 33 (2009); $[IF_{2Y} = 2.462, IF_{5Y} = 2.388, CYT = 9]$

[23] G. Kondrat, K. Sznajd-Weron, *Percolation framework in Ising-spin relaxation*, Phys. Rev. E 79, 011119 (2009); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 3]$

[24] K. Sznajd-Weron, R. Weron, M. Włoszczowska, *Outflow dynamics in modeling oligopoly markets: the case of the mobile telecommunications market in Poland*, J. Stat. Mech. P11018 (2008); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CYT = 1]$

[25] F. Slanina, K. Sznajd-Weron, P. Przybyła, *Some new results on one-dimensional outflow dynamics*, Europhys. Lett. 82, 18006 (2008); $[IF_{2Y} = 2.269, IF_{5Y} = 2.112, CYT = 18]$

[26] G. Kondrat, K. Sznajd-Weron, *Three types of outflow dynamics on square and triangular lattices and universal scaling*, Phys. Rev. E 77, 021127 (2008); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 3]$

[27] T. Czarnik, R. Gawda, D. Łątka, W. Kołodziej, K. Sznajd-Weron, R. Weron, *Noninvasive measurement of intracranial pressure: Is it possible?*,The Journal of Trauma, Injury Infection and Critical Care, 62(1), 207-211 (2007); $[IF_{2Y} = 2.961, IF_{5Y} = 3.204, CYT = 9]$

[28] K. Sznajd-Weron, S. Krupa, *Inflow versus outflow zero-temperature dynamics in one dimension*, Phys. Rev. E 74, 031109 (2006); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 8]$

[29] S. Krupa, K. Sznajd-Weron, *Relaxation under outflow dynamics with random sequential updating*, Int. J. Mod. Phys. C , Vol. 16, No. 11, 1771 (2005); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CYT = 9]$

[30] K. Sznajd-Weron, J. Sznajd, *Who is left, who is right?*, Physica A 351, 593 (2005); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 23]$

[31] J.O. Indekeu, K. Sznajd-Weron, *Hierarchical population model with a carrying capacity distribution for bacterial biofilms*, Phys. Rev. E 68, 061904 (2003); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 1]$

[32] K. Sznajd-Weron, R. Weron, *How effective is advertising in duopoly markets?*, Physica A 324, 437 (2003); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 36]$

[33] K. Sznajd-Weron, M. Wolański, *In search for the optimal strategy in population dynamics*, Eur. Phys. J. B 25 2, 253 (2002); $[IF_{2Y} = 1.463, IF_{5Y} = 1.515, CYT = 5]$

[34] K. Sznajd-Weron, R. Weron, *A simple model of price formation*, Int. J. Mod. Phys. C 13, 115 (2002); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CYT = 54]$

[35] K. Sznajd-Weron, A. Pękalski, *Model of population migration in a changing habitat*, Physica A 294, 424 (2001); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 2$]

[36] K. Sznajd-Weron, R. Weron, *A new model of mass extinctions*, Physica A 293, 559 (2001); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 3$]

[37] A. Pękalski, K. Sznajd-Weron, *Population dynamics with and without selection*, Phys. Rev. E 63, 031903 (2001); [$IF_{2Y} = 2.326, IF_{5Y} = 2.302, CYT = 5$]

[38] K. Sznajd-Weron, J. Sznajd, *Opinion evolution in closed community*, Int. J. Mod. Phys. C 11, 1157 (2000); [$IF_{2Y} = 1.125, IF_{5Y} = 0.949, CYT = 513$]

[39] K. Sznajd-Weron, A. Pękalski, *Statistical physics model of an evolving population*, Physica A 274, 91 (1999); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 3$]

[40] K. Sznajd-Weron, A.Pękalski, *Evolution of populations in a changing environment*, Physica A 269, 527 (1999); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 1$]

[41] K. Sznajd-Weron, A. Pękalski, *Change of a continuous character caused by gene flow. A Monte Carlo study*, Physica A 259, 457 (1998); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 2$]

[42] K. Sznajd-Weron, A. Pękalski, *Evolution through stabilizing selection and gene flow*, Physica A 252, 336 (1998); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CYT = 1$]

[43] I. Mróz, A. Pękalski, K. Sznajd-Weron, *Conditions for adaptation of an evolving population*, Phys. Rev. Lett. 76 3025 (1996); [$IF_{2Y} = 7.728, IF_{5Y} = 7.411, CYT = 14$]

3.1.4 Publikacje konferencyjne

1. K. Sznajd-Weron, J. Sznajd (2006) *Personal Versus Economic Freedom*, Proceedings of the Third Nikkei Econophysics Symposium Practical Fruits of Econophysics, H. Takayasu Ed., Springer-Verlag, Tokyo 355-360.

2. A. Kowalska-Pyzalska, K. Maciejowska, K. Sznajd-Weron, R.Weron (2014) *Modeling consumer opinions towards dynamic pricing: An agent-based approach*, IEEE Conference Proceedings, 11th International Conference on the European Energy Market (EEM'14), 28-30 May 2014, Kraków, Poland, DOI 10.1109/EEM.2014.6861272.

3.1.5 Członkostwo w redakcjach naukowych

1. K. Sznajd-Weron, ed. (2004) *Statistical Physics outside pure Physics* Physica A 336

2. A. Pękalski, K. Sznajd-Weron, eds. (2000) *Exotic Statistical Physics*, Physica A 285

3. R. Kutner, A. Pękalski, K. Sznajd-Weron, eds. (1999) *Anomalous Diffusion: From Basics to Applications*, Lecture Notes in Physics, Springer-Verlag, Berlin.

3.2 Informacja o aktywności naukowej

3.2.1 Informacja o udziale w konferencjach naukowych

Wykłady zaproszone i plenarne

1. *Czy modelowanie agentowe może zastąpić eksperyment społeczny?*,Wykład plenarny na VII Konferencji Akademickiego Stowarzyszenia Psychologii Ekonomicznej, Szkoła Wyższa Psychologii Społecznej, Wrocław 09-10.05.2014

2. *Agent Based Modeling in Energy Markets*, 2nd Energy Finance Christmas Workshop, Macquarie University, Sydney, 13-14.12.2012

3. *Phase transition in the Sznajd model with nonconformity*, The Unexpected Conference – SOCIOPHYSICS: Do humans behave like atoms?, CREA-Ecole Polytechnique, Paris, 13-16.11.2011

4. *Simple models for complex systems – toys or tools?*, 6-cio godzinny mini kurs na Ising Lectures 2011, 14th Annual Workshop on Phase Transitions and Critical Phenomena, Lviv, 11-15.04.2011

5. *Can we treat people like particles? - a simple model of opinion formation*, International Symposium Computational philosophy: lessons from simple models, Niels Bohr Institute, Copenhagen 11-13.10.2007

6. *Opinion dynamics in personal and economical areas do they differ?*, International School on Complexity Statistical Physics of Social Dynamics: Opinions, Semiotic Dynamics, and Language, Erice (Sicily) 14-19.07.2007

7. *From social psychology to sociology - a physicist's point of view*, AKSOE Conference Physics of socio-economic Systems, Regensburg 27.03.2007

8. *Personal versus economic freedom*, AKSOE Conference Physics of Socio-economic Systems, Dresden 26-31.03.2006

9. *Opinion evolution in sociophysics*, XI Summer School Fundamental Problems in Statistical Physics FPSPXI, Leuven 04-17.09.2005

10. *Kto jest prawicą, kto jest lewicą?*, IX Mini Sympozjum z Fizyki Statystycznej, Częstochowa 05-06.12.2004

11. *Sznajd model and its applications*, Sympozjum FENS'04, Warszawa 19-20.11.2004

12. *Personal versus economic freedom*, 3rd Nikkei Econophysics Workshop, Tokyo 09-11.11.2004

13. *Fizyka poza fizyką*, wykład plenarny na XXXVII Zjeździe Fizyków Polskich, Gdańsk 15-18.09.2003

Wybrane referaty i postery konferencyjne

1. Zestawienie referatów konferencyjnych typu *contributed* zawiera m.in.:
   (a) *Diffusion of innovation within an agent-based model*, European Conference on Complex Systems (ECCS'13), Barcelona 16-20.09.2013
   (b) *Modelowanie dyfuzji innowacji*, 42 Zjazd Fizyków Polskich, Poznań 8-13.09.2013
   (c) *Spontaneous reorientations in a model of opinion dynamics with anticonformists*, Middle European Cooperation in Statistical Physics MECO 35, Pont-a-Mousson 15-19.03.2010
   (d) *Dogadamy się czy nie? – o modelowaniu ewolucji opinii w socjofizyce*, Sympozjum FENS'06, Kraków 21-22.04.2006
2. Ponadto prezentowałam postery m.in. na następujących konferencjach/szkołach: Altenberg Summer School on Fundamental Problems in Statistical Physics (Altenberg 1997), Middle European Cooperation in Statistical Physics – MECO 22 (Szklarska Poręba 1997), MECO 24 (Lutherstadt-Wittenberg 1999), MECO 27 (Sopron 2002), MECO 28 (Saarbrücken 2003), MECO 29 (Bratislava 2004), European Conference on Complex Systems ECCS'12 (Brussels 2012).

3.2.2 Wybrane wykłady zaproszone w placówkach naukowych

Wykłady zaproszone na seminariach w zagranicznych placówkach naukowych

1. *Diffusion of innovation within an agent-based model*, FIME (Finance for Energy Market) seminar, Institut Henri Poincarré, Paryż 7.11.2014
2. *Social Physics or Sociophysics?*, Joint Seminar: Department of Physics and Centre for Computational Science and Engineering, National University of Singapore, Singapur 11.12.2012
3. *Social Physics or Sociophysics?*, Physics Friday Colloquia, Department of Physics Norwegian University of Science and Technology, Trondheim 16.09.2011
4. *Can we treat people like particles? – a simple model of opinion dynamics*, Department of Physics Norwegian University of Science and Technology Trondheim, 03.05.2009
5. *Opinion Formation, Irrational Thinking and Spreading of Minorities*, Chair of Sociology, in particular of Modelling and Simulations, ETH Zurich, Zurych 18.12.2007
6. *A simple model of opinion formation*, Institute of Industrial Science, University of Tokyo, Tokyo 12.11.2004
7. *A simple model of opinion formation*, Department of Physics, Tokyo Metropolitan University, Tokyo 10.11.2004

8. *New dynamical model of Ising spins*, Ikegami Laboratory, University of Tokyo, Tokyo 8.11.2004

9. *From social psychology to sociology - a physicist's point of view*, Division of Condensed Matter Physics of the Institute of Physics of the Academy of Sciences of the Czech Republic, Praga 6.11.2003

Wykłady zaproszone na seminariach w krajowych placówkach naukowych

1. *Jak modelować dyfuzję innowacji?*, Collegium Physicum, Wydział Fizyki Uniwersytetu im. Adama Mickiewicza w Poznaniu, Poznań 5.11.2013

2. *Czy możliwe są przejścia fazowe w układach jednowymiarowych?*, Seminarium Instytutu Fizyki, Politechnika Wrocławska, Wrocław 7.01.2013[2]

3. *Proste modele układów złożonych*, Instytut Fizyki Uniwersytetu Opolskiego, Opole 6.11.2011

4. *Phase transitions in 1D kinetic Ising model*, Seminarium Kinetyka — korelacje – złożoność, Instytut Fizyki Politechniki Wrocławskiej, Wrocław 17.11.2010

5. *Ludzi można traktować jak cząstki, tylko po co? — o sensie socjofizyki*, Centrum Informatyczne w Świerku, Świerk 19.05.2010

6. *Ludzi można traktować jak cząstki, tylko po co? — o sensie socjofizyki*, Instytut Niskich Temperatur i Badań Strukturalnych PAN, Wrocław 10.03.2010

7. *Ludzi można traktować jak cząstki, tylko po co?*, Seminarium Instytutu Fizyki, Politechnika Wrocławska, Wrocław 22.02.2010

8. *Nowy model spinów Isinga – czyli o tym jak nauki społeczne zainspirowały fizyka*, Seminarium Dynamiki Układów Złożonych, Politechnika Warszawska, Warszawa 23.10.2006

9. *Socjaliści czy Liberałowie? - socjofizyka w polityce*, Seminarium Wydziału Fizyki i Astronomii, Uniwersytet Zielonogórski, Zielona góra 25.10.2005

10. *Czy ludzi można traktować jak cząstki?*, Seminarium Fizyki Statystycznej, Uniwersytet Warszawski, Warszawa 7.05.2004

11. *Nowy dynamiczny model spinów isingowskich*, Seminarium Fizyki Statystycznej,Uniwersytet Warszawski, Warszawa 7.05.2004

[2] W tym czasie pracowałam jeszcze na Uniwersytecie Wrocławskim.

3.2.3 Członkostwo w komitetach redakcyjnych i radach naukowych czasopism

- Redaktor w sekcji Interdisciplinary Physics międzynarodowego czasopisma *Frontiers in Physics*, Frontiers, od 2013
- Redaktor czasopisma *International Journal of Statistical Mechanics*, Hindawi, od 2013
- Członek komitetu redakcyjnego w czasopismie z listy filadelfijskiej *Physica A* (IF_{5Y} = 1.684), Elsevier, od 2010

3.2.4 Wykaz realizowanych projektów naukowo-badawczych

2014-2016 Główny wykonawca, India-Polish Inter-Governmental Science & Technology Cooperation Programme, *Data Driven Approaches for Inferring Opinion Dynamics on Social Networks*, Indian Institute of Technology Kharagpur, Indie

2014-2016 Główny wykonawca, grant badawczy NCN OPUS nr 2013/11/B/HS4/01061, *Ekonomiczne konsekwencje kształtowania się opinii i podejmowania decyzji przez konsumentów: Modelowanie agentowe dyfuzji innowacji*, PWr

2014-2016 Opiekun w grancie badawczym NCN FUGA na staż po-doktorski dla dr Anny Chmiel nr 2014/12/S/ST3/00326, *Procesy nierównowagowe na sieciach wielopoziomowych*, PWr

2011-2014 Kierownik, grant badawczy NCN OPUS nr 2011/01/B/ST3/00727, *Zastosowanie prostych modeli spinowych w marketingu społecznym i komercyjnym*, UWr

2011-2014 Główny wykonawca, grant badawczy NCN OPUS nr 2011/01/B/HS4/02740, *Modelowanie dynamiki zachowań konsumentów na rynkach oligopolistycznych za pomocą automatów komórkowych*, Politechnika Śląska

2007-2009 Kierownik, ministerialny grant własny nr N N202 0194 33, *Nowa lokalna dynamika spinów Isinga z punktu widzenia teorii nierównowagowych układów dynamicznych i zastosowań w modelowaniu grup społecznych*, UWr

2007-2009 Kierownik, ministerialny grant promotorski dla Sylwii Krupy nr N N202 0907 33, *Analiza układów spinów isingowskich z zero-temperaturowymi lokalnymi dynamikami*, UWr

2000-2002 Główny wykonawca, grant KBN nr 2p03B2718

1999-2000 Kierownik, Projekt badawczy wewnętrzny 2318/W/IFT finansowany przez Uniwersytet Wrocławski

1997-1998 Kierownik, Projekt badawczy wewnętrzny 2201/W/IFT finansowany przez Uniwersytet Wrocławski

3.2.5 Informacje o kierowaniu zespołami badawczymi

2011-2014 Kierowanie zespołem badawczym składającym się z 5 pracowników i studentów, w ramach grantu NCN OPUS 2011/01/B/ST3/00727, IFT UWr

2012-2013 Kierownik Katedry *UNESCO Badań Interdyscyplinarnych* (12-14 pracowników), IFT UWr

2009-2013 Kierownik Zakładu *Układów Złożonych i Dynamiki Nieliniowej* (8-10 pracowników, 4-6 doktorantów), IFT UWr

3.3 Informacja o współpracy z otoczeniem społecznym i gospodarczym

- Recenzentka projektów badawczych ministerialnych
- *Umiejętności numeryczne*, autorski kurs we współpracy z pracodawcami (między innymi McKinsey i Google) w ramach projektu Wrocławski Absolwent *Program przygotowania kadr dla nowoczesnego sektora usług* realizowanego przez Uniwersytet Wrocławski i współfinansowanego przez Gminę Wrocław (2010-2011)
- Współpraca naukowa z *Easygreen Lejkowski Cezary* w celu promowania zachowań i działań zgodnych z Zielonymi Standardami (certyfikaty Green Brand and Global Green Consulting Center) (2010-2013)

3.4 Informacja o współpracy międzynarodowej

3.4.1 Staże i wyjazdy zagraniczne

Ze względu na sytuację rodzinną odrzuciłam zaproszenia dłuższych wyjazdów zagranicznych. Wielokrotnie natomiast przyjmowałam zaproszenia do pobytów kiludniowych połączonych zwykle z zaproszonym wykładem (patrz 3.2.2). Odbyłam tylko trzy dłuższe (powyżej 7 dni) wyjazdy zagraniczne:

1. Department of Physics, Norwegian University of Science and Technology, Trondheim (09-19.09.2011) na zaproszenie prof. Ingve Simonsena
2. Department of Physics, Norwegian University of Science and Technology, Trondheim (30.04-10.05.2009) na zaproszenie prof. Ingve Simonsena
3. Institute of Industrial Science, University of Tokyo, Tokyo (08-20.11.2004)

3.4.2 Recenzowanie prac publikowanych w czasopismach międzynarodowych ze wskaźnikiem *impact factor*

Wyjątkowo częste powoływanie na recenzenta przez:[3]

- Physical Review Letters [IF=7.411] and Physical Review E [IF=2.302] – 30 wykonanych recenzji
- Physica A [IF=1.684] – 33 wykonanych recenzji

Regularne powoływanie na recenzenta przez:

- Advances in Complex Systems [IF=0.918]
- European Physical Journal B [IF=1.515]
- Europhysics Letters [IF=2.112]
- Journal of Statitical Physics [IF=1.239]
- International Journal of Modern Physics C [IF=0.949]
- Physics Letters A [IF=1.706]

Sporadyczne powoływanie na recenzenta przez:

- Behavioural Processes [IF=1.760]
- Complexity [IF=1.290]
- Journal of the Royal Society – Interface [IF=4.875]

3.4.3 Członkostwo w międzynarodowych organizacjach i towarzystwach naukowych

1. Członek Towarzystwa Układów Złożonych (*Complex Systems Society*), od 2012

3.4.4 Udział w międzynarodowych zespołach eksperckich

1. Członek jury w prestiżowej międzynarodowej nagrodzie Young-Scientist Award for Socio- and Econophysics 2014 (grudzień 2014)
2. Członek komisji habilitacyjnej Laury Hernandez, Laboratoire de Physique Théorique et Modelisation (LPTM), joint laboratory of CNRS and Université de Cergy Pontoise (listopad 2014 - luty 2015)
3. Współorganizatorka *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2014 Satellite, Lucca, 24.09.2013

[3]Podane są 5-letnie wskaźniki IF.

4. Współorganizatorka *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2013 Satellite, Barcelona, 18-19.09.2013

5. Członek komitetu naukowego *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2012 Satellite, Brussels, 5-6.09.2012

6. Członek komitetu naukowego *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2011 Satellite, Wien, 14-15.09.2011

7. Recenzentka prac dyplomowych w Department of Physics, NTNU, Trondheim, Norwegia

3.4.5 Udział w międzynarodowych zespołach badawczych

1. Udział w projekcie *Data Driven Approaches for Inferring Opinion Dynamics on Social Networks* sponsorowanym w latach 2014-2016 przez India-Polish Inter-Governmental Science & Technology Co; współpraca z grupą z Indian Institute of Technology Kharagpur (science24.com/events/1602/boa/boa.pdf)

2. Współpraca z prof. Frantiskiem Slaniną, Instytut Fizyki, Czeska Akademia Nauk, Praga, Czechy (od 2003 roku)

3. Współpraca z prof. Josephem Indekeu, Sekcja Fizyki Teoretycznej, Uniwersytet Katolicki w Leuven, Belgia (od 1997 roku)

3.5 Informacja o dorobku dydaktycznym, popularyzatorskim i organizacyjnym

3.5.1 Prowadzone wykłady i seminaria naukowe

1. Prowadzenie seminarium Zakładu Dynamiki Nieliniowej i Układów Złożonych, Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski (2009-2013)

2. Prowadzenie seminarium Katedry UNESCO Studiów Interdyscyplinarnych, Uniwersytet Wrocławski (2009-2013)

3. *Statistical Physics Modern Theory of Phase Transitions* (wykład i ćwiczenia po angielsku), Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (2013/2014)

4. *Phase Transitions in Complex Systems* (wykład monograficzny), Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (2014)

5. *Mechanika i Termodynamika* (wykład i ćwiczenia), Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (2013-2015)

6. *Fale i elektromagnetyzm* (wykład, ćwiczenia i laboratorium), Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (2014)

7. *Modelarnia – krytyczność i złożoność* (wykład, seminarium, konwersatorium oraz laboratorium komputerowe) , Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2012-2013)

8. *Klasyczna fizyka teoretyczna 2* (wykład), Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2011-2013)

9. *Nierównowagowe przejścia fazowe* (wykład monograficzny) , Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2011)

10. *Fizyka statystyczna* (wykład), Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2006-2012)

11. *Teoria przejść fazowych i zjawisk krytycznych* (wykład) Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2006-2012)

12. *Modelowanie komputerowe* (wykład, laboratorium komputerowe) Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2000-2013)

13. Seminaria magisterskie i licencjackie, Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2008-2010)

14. *Egzotyczna fizyka statystyczna* (seminarium), Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2000-2001)

3.5.2 Opieka naukowa nad doktorantami i osobami ubiegającymi się o nadanie stopnia doktora

1. Promotor: Piotr Przybyła *Nierównowagowa dynamika spinów isingowskich z punktu widzenia teorii układów złożonych i zastosowań interdyscyplinarnych*, Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska, otwarcie przewodu 3.12.2013, przewidywany termin obrony 2015/2016

2. Promotor: Piotr Nyczka *Przejścia Fazowe w uogólnionym modelu q-wyborcy na grafie zupełnym*, Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, rozprawa doktorska obroniona 24.02.2015

3. Promotor: Sylwia Krupa *Analiza układów spinów isingowskich z zero-temperaturowymi lokalnymi dynamikami*, Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, rozprawa doktorska obroniona 19.06.2009

3.5.3 Recenzje prac doktorskich i dyplomowych

1. dr Agnieszka Czaplicka, *Procesy transportu i ewolucja topologii hierarchicznych sieci złożonych*, Wydział Fizyki, Politechnika Warszawska (obrona 20.10.2014)

2. dr Maciej Jagielski, *Zastosowanie nieliniowego równania Langevina, równania Fokkera-Plancka oraz modeli błądzeń losowych do opisu dochodów gospodarstw domowych Polski i Unii Europejskiej*, Wydział Fizyki, Uniwersytet Warszawski (obrona 09.06.2014)

3. dr Jacek Wendykier, *Sieciowe modele typu drapieżniki i ofiary – zastosowanie w modelowaniu nowotworów*, Instytut Fizyki, Uniwersytet Opolski (obrona 10.10.2013)

4. dr Tomasz Gubiec, *Modele błądzenia losowego w czasie ciągłym z pamięcią. Zastosowanie do opisu dynamiki rynków finansowych*,Wydział Fizyki, Uniwersytet Warszawski (obrona 12.12.2011)

5. recenzentka ponad 30 prac magisterskich i dyplomowych

3.5.4 Artykuły i prace o charakterze popularnonaukowym

1. G. Kontrym-Sznajd, K. Sznajd-Weron *Jak zainteresować uczniów fizyką?*, Problemy dydaktyki fizyki, Wrocławskie Wydawnictwo Oświatowe ATUT, Centrum Edukacji Nauczycielskiej Uniw. Wrocł., Wrocław-Czeszów 2013, ISBN 978-83-7432-992-7str. 57-66

2. A. Pękalski, K. Sznajd-Weron, *Układy złożone na Uniwersytecie Wrocławskim*, Przegląd Uniwersytecki, grudzień 2004

3. K. Sznajd-Weron, *W sieci małego świata*, Wiedza i Życie, luty/04, 68-71 (2004)

4. K. Sznajd-Weron, *Seks według wzoru*, Wiedza i Życie, kwiecień/02, 46-49 (2002)

5. K. Sznajd-Weron, *Opowieść o fizyce egzotycznej*, Wiedza i Życie, październik/01, 46-49 (2001)

3.5.5 Przygotowane materiały do e-learningu

1. Skrypt do wykładu *Fizyka statystyczna* dostępny na serwisie studenckim http://panoramix.ift.uni.wroc.pl oraz na mojej stronie domowej

2. Skrypt do wykładu *Teoria przejść fazowych i zjawisk krytycznych* dostępny na serwisie studenckim http://panoramix.ift.uni.wroc.pl/ oraz na mojej stronie domowej

3. Skrypt w formie prezentacji multimedialnych do wykładu *Modelowanie komputerowe* dostępny na serwisie studenckim http://panoramix.ift.uni.wroc.pl/

4. Skrypty do kursu *Modelarnia – krytyczność i złożoność* dostępny na serwisie studenckim http://panoramix.ift.uni.wroc.pl/, na stronie Wydziału Fizyki i Astronomii Uniwersytetu Wrocławskiego w zakładce projekty POKL oraz na mojej stronie domowej

5. Opracowanie slajdów (pdf) do niemal wszystkich prowadzonych kursów. Materiały do zajęć prowadzonych w danym semestrze są dostępne dla studentów na mojej stronie domowej

3.5.6 Aktywny udział w imprezach popularyzujących naukę, kulturę oraz sztukę

1. *Czy psychologia może się spotkać z fizyką?* Wykład Inauguracyjny w Szkole Wyższej Psychologii Społecznej, Wrocław 7.10.2014

2. *Co w praktyce oznacza nieskończoność?* Fizyczno Astronomiczna Konferencja Studencka, FAK 2014, Politechnika Wrocławska, Wrocław 16.05.2014

3. Seria wykładów poświęcona układom złożonym w ramach koła naukowego fizyków Nabla, Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (2013-2014)

4. *W sieci jeszcze mniejszego świata - życie na facebooku i nie tylko*, Fizyczno Astronomiczna Konferencja Studencka, FAK 2013, Wrocław 19.05.2013

5. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka*, Uniwersytet Trzeciego Wieku, Wrocław 19.11.2012

6. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka*, XIV Dolnośląski Festiwal Nauki, Uniwersytet Wrocławski, Wrocław 21.09.2011

7. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka*, Szczecin humanistyczny, Rektorat Uniwersytetu Szczecińskiego, Szczecin 4.04.2011

8. *Dogadamy się czy nie? – czyli co ma fizyka do socjologii*, XII Dolnośląski Festiwal Nauki, Uniwersytet Wrocławski, Wrocław 19.09.2009

9. *Czy ludzi można traktować jak cząstki - spojrzenie fizyka*, Oddział Wrocławskiego Polskiego Towarzystwa Socjologicznego, Instytut Socjologii, Uniwersytet Wrocławski, Wrocław 15.12.2004

10. *Czy Bóg ma przepis? - od chaosu deterministycznego po fraktale*, VI Dolnośląski Festiwal Nauki, Uniwersytet Wrocławski, Wrocław, 09.2004

11. *Czy ludzi można traktować jak cząstki?*, Szkoła Główna Gospodarstwa Wiejskiego, Warszawa, 6.05.2004

12. *Jak przekonywać innych? - socjofizyka: model Sznajdów*, Obóz Naukowym Krajowego Funduszu na Rzecz Dzieci, Świdr-Otwock, 8.05.2003

13. *Jak przekonywać innych? czyli socjofizyka*, VI Dolnośląski Festiwal Nauki, Uniwersytet Wrocławski, Wrocław wrzesień 2003 oraz Wałbrzych październik 2003

14. *Katastrofy oczami fizyków - od lawin piasku po wielkie wymierania*, V Dolnośląski Festiwal Nauki, Uniwersytet Wrocławski, Wrocław 20.09.2002 oraz Wałbrzych 4.10.2002

3.5.7 Organizacja międzynarodowych konferencji naukowych

1. Współorganizatorka *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2014 Satellite, Lucca 24.09.2014
2. Dyrektor CODYM Spring Workshop (CODYM-Spring'14), Politechnika Wrocławska, Wrocław 7-8.04.2014
3. Współorganizatorka *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2013 Satellite, Barcelona 18.09.2013
4. Dyrektor, 47 Zimowa Szkoła Fizyki Teoretycznej *Simple Models for Complex Systems*, Lądek-Zdrój 7-12.02.2011
5. Dyrektor, XXIII Sympozjum Maksa Borna *Critical Phenomena in Complex Systems*, Polanica-Zdrój 3-6.09.2007
6. Współorganizatorka, *Workshop on Science for Conservation & Preservation of Cultural Heritage Research & Education*, Wydział Chemii Uniwersytetu Wrocławskiego, Wrocław 4-5.06.2007
7. Sekretarz, XVIII Sympozjum Maxa Borna *Statistical Physics Outside Pure Physics*, Lądek-Zdrój 22-25.09.2003
8. Sekretarz, 36 Zimowa Szkoła Fizyki Teoretycznej *Exotic Statistical Physics*, Lądek-Zdrój 11-19.02.2000
9. Sekretarz, XI Sympozjum Maxa Borna *Anomalous Diffusion: from Basis to Applications*, Lądek-Zdrój 20-27.05.1998

3.5.8 Pozostała działalność organizacyjna i dydaktyczna

1. Sekretarz w zarządzie sekcji FENS (Fizyka w ekonomii i naukach społecznych) Polskiego Towarzystwa Fizycznego (2014-2016)
2. Sekretarz komisji habilitacyjnej dr hab. Grzegorza Pawlika, Instytut Fizyki, Politechnika Wrocławska (2014)
3. Sekretarz komisji habilitacyjnej dr hab. Dariusza Grecha, Wydział Fizyki i Astronomii, Uniwersytet Wrocławski (2013)
4. Opiekunka koła naukowego Fizyków Nabla, Wydział Podstawowych Problemów Techniki, Politechnika Wrocławska (od 2013)
5. Lider do spraw nauczania fizyki na Wydziale Elektrycznym Politechniki Wrocławskiej (od 2013)
6. Przewodnicząca komisji dyscyplinarnej dla nauczycieli akademickich na Wydziale Fizyki i Astronomii Uniwersytetu Wrocławskiego (2012-2013)

7. Członek wydziałowego zespołu ds. jakości kształcenia na Wydziale Fizyki i Astronomii Uniwersytetu Wrocławskiego (2011-2013)

3.6 Informacja o otrzymanych nagrodach oraz wyróżnieniach naukowych i dydaktycznych

2011 Medal Komisji Edukacji Narodowej

2010 Nagroda Rektora Uniwersytetu Wrocławskiego za osiągnięcia dydaktyczne i organizacyjne

2007 *Young-Scientist Award for Socio- and Econophysics* – prestiżowa nagroda przyznawana przez Niemieckie Towarzystwo Fizyczne corocznie tylko jednej osobie z całego świata (nominowanej przez środowisko naukowe)

2006 Nagroda Rektora Uniwersytetu Wrocławskiego za osiągnięcia dydaktyczne i organizacyjne

2005 Nagroda Rektora Uniwersytetu Wrocławskiego za osiągnięcia naukowe

2003 Stypendium krajowe Fundacji na rzecz Nauki Polskiej dla młodych naukowców

2002 Stypendium krajowe Fundacji na rzecz Nauki Polskiej dla młodych naukowców

2002 Nagroda Zespołowa Ministra Edukacji Narodowej i Sportu za współautorstwo cyklu prac dotyczących zastosowania metod symulacji komputerowej, metody Monte Carlo, do zagadnień ewolucji biologicznej i adsorbcji sztywnych prętów na powierzchni

2000 Nagroda Rektora Uniwersytetu Wrocławskiego za osiągnięcia dydaktyczne

1999 Nagroda Zespołowa Ministra Edukacji Narodowej za współautorstwo cyklu prac dotyczących zastosowania metod fizyki statystycznej do biologii i socjologii

1997 Nagroda Zespołowa Ministra Edukacji Narodowej za współautorstwo cyklu prac dotyczących różnych modeli zjawisk zachodzących w materii skondensowanej i układach biologicznych

1995 Nagroda II stopnia im. Arkadiusza Piekary przyznawana przez Polskie Towarzystwo Fizyczne za pracę magisterską

Rozdział 4

Informacja o najważniejszym osiągnięciu naukowym.

W moim odczuciu, najważniejszym osiągnięciem naukowym po habilitacji jest poniższa seria dziesięciu prac, zainicjowanych własnymi pomysłami a przygotowanymi w znacznej większości z moimi studentami (M. Tabiszewski, R. Topolnicki, K. Suszczyński) lub doktorantami (S. Krupa, P. Przybyła, B. Skorupa, P. Nyczka). Wszystkie prace w serii koncentrują się na tym samym zagadnieniu – wrażliwości wielkości makroskopowych nierównowagowych modeli spinowych na detale wprowadzane na poziomie mikroskopowym:

[P0] K. Sznajd-Weron, S. Krupa, *Inflow versus outflow zero-temperature dynamics in one dimension*, Phys. Rev. E 74, 031109 (2006).[1]

[P1] F. Slanina, K. Sznajd-Weron, P. Przybyła, *Some new results on one-dimensional outflow dynamics*, Europhys. Lett. 82, 18006 (2008).

[P2] K. Sznajd-Weron, *Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 82, 031120 (2010).

[P3] K. Sznajd-Weron, M. Tabiszewski, A. Timpanaro, *Phase transition in the Sznajd model with independence*, Europhys. Lett. 96, 48002 (2011).

[P4] P. Przybyła, K. Sznajd-Weron and M. Tabiszewski, *Exit probability in a one-dimensional nonlinear q-voter model*, Phys. Rev. E 84, 031117 (2011).

[P5] B. Skorupa, K. Sznajd-Weron, R. Topolnicki, *Phase diagram for a zero-temperature Glauber dynamics under partially synchronous update*, Phys. Rev. E 86, 051113 (2012).

[P6] P. Nyczka, K. Sznajd-Weron, J. Cislo, *Phase transitions in the q-voter model with two types of stochastic driving*, Phys. Rev. E 86, 011105 (2012).

[1] Chociaż formalnie ten artykuł został opublikowany po nadaniu mi stopnia doktora habilitowanego, zamieszczam go na tej liście z dwóch powodów. Po pierwsze, złożyłam rozprawę habilitacyjną już we wrześniu 2005, a pracę nad tym artykułem rozpoczęłam dopiero na początku roku 2006. Po drugie, ta praca zainicjowała kolejne prace znajdujące się w tym cyklu.

[P7] P. Nyczka, K. Sznajd-Weron, *Anticonformity or Independence? – Insights from Statistical Physics*, Journal of Statistical Physics 151, 174-202 (2013).

[P8] K. Sznajd-Weron, K. Suszczyński, *Nonlinear q-voter model with deadlocks on the Watts-Strogatz graph*, J. Stat. Mech. P07018 (2014).

[P9] K. Sznajd-Weron, J. Szwabiński, R. Weron, *Is the Person-Situation Debate Important for Agent-Based Modeling and Vice-Versa?* PLoS ONE 9(11), e112203 (2014).

Jak już wspomniałam w rozdziale 2.1.3, jednym z głównych problemów w dziedzinie symulacji społecznych jest, jak zauważył Macy i Willer[2], zbyt mała troska o analizę tego, w jakim stopniu konstrukcja modelu wpływa na wyniki. Jest to szczególnie istotny problem, gdyż symulacje społeczne są często traktowane jako substytut eksperymentu społecznego. Ponieważ podobny problem zauważam równiez w socjofizyce, uznałam, że warto przyjrzeć się bliżej niektórym spornym zagadnieniom. W powyższej serii prac starałam się odpowiedzieć na kilka pytań, które można zawrzeć w jednym ogólnym pytaniu: *Jak, czasami pozornie drobne, różnice wprowadzone na poziomie mikroskopowym modelu, ujawniają się w skali makroskopowej?* W szczególności skupiłam się na następujących pytaniach:

- Czy dynamiki dopływu i wypływu są równoważne [P0, P1]?
- Jaka jest rola aktualizacji w dynamikach spinowych [P0, P2, P5]?
- Jak stacjonarne własności modelu (w tym stany absorpcyjne i diagramy fazowe) zależą od warunków początkowych i wielkości grupy wpływu [P1, P3, P4, P6, P8]?
- Jakie są różnice, z punktu widzenia makroskopowego zachowania układu (grupy społecznej), pomiędzy różnymi rodzajami nonkonformizmu (antykonformizm lub niezależność) wprowadzanymi na poziomie mikroskopowym (jednostki) skoro oba mogą być traktowane jako pewien szum zaburzający konsensus [P3, P6, P7] ?
- Czy założenia, dotyczące modelowania reakcji agenta (cechy osobiste albo sytuacja) na wpływ społeczny, mają znaczący wpływ na makroskopowe zachowanie układu [P9]?

Szukając odpowiedzi na powyższe pytania, przy okazji udało mi się również uzyskać kilka ciekawych wyników teoretycznych dotyczących prawdopodobieństwa ucieczki (*exit probability*) i przejść fazowych dla zero-temperaturowej dynamiki Glaubera oraz modelu q-votera (szczegóły w rozdziale 2.1.3). Dlatego mam nadzieję, że powyższy cykl prac będzie miał wpływ nie tylko na rozwój interdyscyplinarnej dziedziny układów złożonych, w szczególności na obszar modelowania agentowego, ale również przyczyni się do rozwoju nierównowagowej fizyki statystycznej.

[2] M. W. Macy, R. Willer, *From factors to actors: computational sociology and agent-based modeling.*, Annu. Rev. Sociol. 28, 143–166 (2002).

Summary of professional achievements

Katarzyna Weron
(Katarzyna Sznajd-Weron in publications)

April 19, 2015

Contents

Chapter 1

Personal data

1.1 Bio data

- **Name and surname:** Katarzyna Weron (Katarzyna Sznajd-Weron in publications)
- **Date and place of birth:** 7th April 1971, Wrocław
- **Nationality:** Polish
- **Marital status:** Married, two sons (1996, 2001)
- **Home address:** Dębowa 16, 51-217 Pruszowice, Poland

1.2 Present position

Professor (Extraordinary professor) since 1.10.2013
Department of Theoretical Physics, Faculty of Fundamental Problems of Technology,
Wrocław University of Technology (PWr),
Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland
Tel. +48-71-320-2159, fax +48-71-328-3696, mob. +48-605-459671
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1.3 Academic positions held

10.2013-today Professor (Extraordinary professor), Department of Theoretical Physics, Wrocław University of Technology (PWr)

09.2011-09.2013 Professor (Extraordinary professor), Institute of Theoretical Physics (IFT), University of Wrocław (UWr)

07.2012-09.2013 Head of UNESCO Chair of Interdisciplinary Studies, IFT UWr

09.2009-09.2013 Head of Complex Systems and Nonlinear Dynamics (CoSyNoDy) Division, IFT UWr

01.2007-09.2011 Associate Professor, IFT UWr

02.1999-12.2006 Assistant Professor, IFT UWr

09.1995-12.1998 Ph.D. student, IFT UWr

1.4 Academic degrees

16.12.2006 Habilitation in Physics (Statistical Physics), IFT UWr; Dissertation title: *A new local dynamics in the Ising spins system.*

18.12.1998 Ph.D. in Physics (Statistical Physics), IFT UWr; Dissertation title: *Modeling of biological evolution via methods of statical physics*; Supervisor: Prof. dr hab. Andrzej Pękalski

10.05.1995 M.Sc. in Physics (Computational Physics), IFT UWr; Dissertation title: *Modeling diffusion Li on Mo(112)*; Supervisor: Prof. dr hab. Andrzej Pękalski

Chapter 2

Summary of professional achievements

2.1 Scientific achievements

My research interest has gradually evolved from applications of statistical physics in modeling population dynamics and biological evolution, through sociophysics, in particular, modeling opinion dynamics, to theoretical aspects of non-equilibrium spin systems. My academic achievements may be divided into three groups:

I. applications of statistical physics in modeling biological systems (1996–2003),

II. applications of agent-based models in social systems, including financial, marketing and political oriented problems (2000–today),

III. theoretical aspects of non-equilibrium spin dynamics, including phase transitions and first passage properties (2002–today),

which are comprehensively discussed in Sections 2.1.1–2.1.3. They include:

- 43 peer-reviewed articles in JCR-listed journals,
- 2 conference papers,
- 5 popular science and related articles.

Some of the results have been presented also at conferences and seminars in Poland and abroad, in particular, I had:

- 13 invited and plenary conference talks,
- 20 invited seminar talks, including 9 abroad.

I must add here that – for family reasons – I rejected a number of invitations to conferences, seminars and long-term visits abroad. Although, I am aware that internships, research visits and networking are very important for the development of a scientific career, it was a fully thought through decision to give priority to my family. I believe, that in spite of this many of my results have been already recognized in Poland and abroad, which is confirmed by the fact that according to *Web of Science* (WoS) in the years 1996-2014 my publications have been cited 914 times[1]. Moreover, my papers have been also repeatedly cited in monographs, see Section 3.1.

My research has always been focused on interdisciplinary applications of statistical physics. **After obtaining an M.Sc. degree** in Physics in May 1995 (University of Wrocław, specialization: computational physics), I started to work on modeling biological evolution under the supervision of Prof. Andrzej Pękalski. In the period 1996-2003, which includes Ph.D. studies in the Institute of Theoretical Physics, University of Wrocław (1995-1998), I published 12 journal articles and my doctoral dissertation in this research area. In the latter I proposed a model to describe the evolution of a single, quantitative (continuous) trait in a metapopulation that consists of a number of local populations (demes) [8], [41], [42].[2]

After obtaining a doctoral degree in Physical Sciences in December 1998, I continued my research on biological systems for some time. However, since 1999, I have gradually shifted my interest to spin dynamics and their applications in social sciences, initially motivated by the articles and lectures of prof. Serge Galam and prof. Janusz Hołyst. In 2000 I published together with my father, prof. Józef Sznajd, my first article on opinion dynamics, in which we proposed a new simple model with binary opinions and outflow dynamics[3], presently known as the *Sznajd model*[4] [38]. This paper proved to be a successful marriage of my father's experience in dealing with equilibrium phase transitions in magnetic systems and my long-term interest in social sciences, as well as my experience in non-equilibrium dynamical systems. To this day, the paper has been cited over 500 times (according to WoS) and remains the most cited article published in the *International Journal of Modern Physics C*. The model itself received a lot of attention, including dedicated chapters in review articles and books (for example, in the 2006 book *A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature* by Tom Siegfried) and its own wiki-page (*http://en.wikipedia.org/wiki/Sznajd_model*). Quite likely this popularity of the Sznajd model contributed significantly to the fact that in 2001 and 2002 I received the prestigious scholarship of the Foundation for Polish Science (FNP) for young researchers.

Back in 2000, sociophysics was still a relatively unpopular topic among physicists. However, in the years that followed it gained considerable recognition as one of the major interdisciplinary applications of statistical physics. Special sections of physical societies, conferences and journals started to appear, even separate interdisciplinary journals devoted to new applications of statistical physics were set up. In particular, in 2002 the *Physics of Socio-Economic Systems* Division of the German Physical Society (DPG) established a prestigious worldwide award – the *Young Scientist Award for Socio- and Econophysics*. Two years later, in 2004 a new section of Polish Physical Society – *Physics in Economy and Social Science Section(FENS)* – was established. In the same year the

[1]Excluding self-citations of all authors.

[2]Article numbering is consistent with the list of publications in Section 3.1.

[3]See Section 2.1.3 for a discussion on outflow and inflow dynamics.

[4]The term *Sznajd model* was coined by prof. Dietrich Stauffer in his early publications (2000-2002) on the model.

annual conference – the *European Conference on Complex Systems*, gathering every year scholars from different disciplines including physicists, mathematicians, computer scientists, biologists, economists and social psychologists, was organized for the first time.

Despite this growing interest, in the early 2000s applications of physics in the social sciences were still treated as 'exotic', also in Poland. Therefore, having in mind obtaining a habilitation (i.e. a 'higher doctorate') in physical sciences, I started to work on the theoretical aspects of the proposed models of opinion dynamics. In the years 2002-2005 I published in *Physical Review E* a series of four papers devoted to the new zero-temperature outflow spin dynamics inspired by the Sznajd model [3]–[6]. In September 2005, I submitted these four articles, together with three other papers on applications of spin dynamics in social sciences (i.e. [32], [34], [38]), as my habilitation dissertation. In December 2006, the habilitation colloquium was held at the Institute of Theoretical Physics, University of Wrocław, after which I have been awarded a habilitation in physical sciences.[5]

After habilitation I have continued research in the field of sociophysics. I have focused on two main areas: (A) theoretical aspects of spin dynamics, including those inspired by physical and social systems and (B) practical applications of microscopic models in social systems (opinion dynamics, diffusion of innovation, environmental marketing campaigns, etc.). The year 2007 was particularly fruitful in my scientific carrier. That year I applied for and received funding from the Polish Ministry of Science and Higher Education (MNiSW) for a two-year research project *A new local Ising spin dynamics from the perspective of non-equilibrium theory of dynamical systems and applications in modeling social systems* (grant no. N N202 0194 33). In the same year my PhD student, Sylwia Krupa, was awarded a MNiSW grant for Ph.D. candidates. She defended her thesis in June 2009. In 2007 I also had the honor to receive the *Young Scientist Award for Socio- and Econophysics* of the German Physical Society and gave invited lectures at two prestigious events – the *International School on Complexity. Course on Statistical Physics of Social Dynamics: Opinions, Semiotic Dynamics, and Language* (Erice, Sicily, 17.07.2007) and the international symposium *Computational Philosophy: Lessons from Simple Models* (Niels Bohr Institute, Copenhagen, 13.10.2007). Finally, in September 2007 I organized the XXIII Max Born Symposium *Critical Phenomena in Complex Systems* (Polanica-Zdrój).

In the post-habilitation period I have also focused on theoretical aspects of non-equilibrium spin dynamics. I have been interested in the first passage properties of these dynamics (see [11], [20], [25]), as well as sensitivity of the macroscopic properties – including phase transitions – to details introduced on the microscopic level (see [1], [9], [14], [16]–[19], [23], [26]). In my opinion, the most important scientific achievement after habilitation is devoted to this subject (see Section 2.1.3 and Chapter 4).

In the recent years I have also intensified collaboration with social scientists. In particular, I was a senior investigator in the grant *Modeling of consumer behavior dynamics in oligopoly markets by cellular automata* (Faculty of Organization and Management, Silesian University of Technology, 2011-2014), co-organizer of three interdisciplinary meetings (CODYM 2013, CODYM-SPRING 2014, CODYM 2014; for details see Section 3.5.7) and in the years 2012-2013 I was the head of the *UNESCO Chair of Interdisciplinary Studies*

[5] Habilitation is an academic degree sometimes referred to as 'higher doctorate'. It is equivalent to obtaining a tenured professorship at a North American university.

(University of Wrocław). In 2014 I was invited, as one of two plenary lecturers, to the *7th Conference on Behavioral Economics*, organized by the University of Social Sciences and Humanities (SWPS) in May 2014. I was also honored to give the inaugural lecture in October 2014 at the Wrocław Campus of SWPS. Presently I am a senior investigator in the grant *Economic consequences of consumer opinion formation and decision making: Agent-based modeling of innovation diffusion* (Faculty of Computer Science and Management, Wrocław University of Technology, 2014-2016), within which I work mainly with economists. Moreover, in 2015 I have started a collaboration with Dr. Katarzyna Byrka from SWPS, a social psychologist specializing in environmental and health psychology. Presently we are working on the model of psychological, social and economical barriers in diffusion of innovation.

Currently I am also the supervisor in the postdoc research grant *Fuga3* lead by Dr. Anna Chmiel (grant no. 2014/12/S/ST3/00326; 2014-2016) devoted to dynamical models on multi-level networks. Therefore some of my recent papers concern the role of topology in opinion dynamics models, including two published articles ([11], [12]) and two more under review[6]. As a consequence of this new line of research, I have started a collaboration with the Social Network Group at Wrocław University of Technology. Without doubt the subject of complex networks, including multiplex and temporal networks, will be one of my research areas in the near future.

2.1.1 Applications of statistical physics in modeling biological systems

In total, in the years 1996-2003 I published 12 articles in the area of modeling biological evolution and population dynamics by methods of statistical physics. My first paper in this series was published in *Physical Review Letters* and was aimed at answering the following questions: *Under what conditions can a population survive in a given environment? If the population may also migrate to another, initially empty, space – what are the necessary similarities between the two environments, in order that the population can develop in both regions?* To answer these questions we proposed a lattice model, in which each individual was characterized by its genome (modeled by a sequence of zeros and ones) and the phenotype of the individual (i.e. a set of features), which simply resulted from its genotype [43]. In this paper the reproduction was based on Mendel's laws, i.e. we considered only qualitative traits. We also assumed that the environment was characterized by a certain ideal phenotype. Based on Monte Carlo simulations we were able to determine the conditions necessary for a population to grow in a given environment and colonize a new, empty niche. The model, although interesting, was relatively complicated compared to other physical models of biological evolution and therefore difficult to analyze theoretically. For this reason I have not continued to work on this model in the subsequent years and I have come back to it only once, in 2001 [37].

During my Ph.D. studies I have proposed an agent-based model to describe the evolution of a single quantitative (continuous) trait in a meta-population that consists of many

[6] A. Chmiel, K. Sznajd-Weron, *Phase transitions in the q-voter model with noise on a duplex clique*, arXiv:1503.01400 [physics.soc-ph]; A. Jędrzejewski, K. Sznajd-Weron, J. Szwabiński, *Mapping the q-voter model: From a single chain to complex networks*, arXiv:1501.05091 [physics.soc-ph]

local populations, so called demes. In the model, the evolution was caused by gene flow and natural selection (hence the model name – GFS) and each agent represented a single deme, characterized by a continuous variable describing the mean value of the selected quantitative trait. Interactions between neighboring demes were described by a nonlinear equation, inspired by the Hamiltonian of the Blume-Capel model. Natural selection was realized by a non-linear term and gene flow by a linear (diffusive) term of the equation. I showed that the competition between these two terms was responsible for three basic types of population structures observed in nature: (a) a continuously distributed population, where spatial arrangement of a quantitative character (like body size or skin color) showed gradient forms, (b) integration zones between two different populations and (c) totally isolated populations. Due to the simplicity of the model I was able to analyze it not only through Monte Carlo simulations but also analytically, using the mean field approximation (MFA) and later using Fourier analysis. In total I have published 5 papers on the GFS model ([8],[39]-[42]) and three of them constituted my Ph.D. thesis.

Among all issues related to biological evolution I have been working on, I devoted the most time to the GSF model. However, in my opinion, the most interesting results have been obtained in a two-paper series on instabilities in population dynamics. In [7] I asked the question about the existence of the critical density, below which the population was doomed to extinction, the so-called *minimum viable population*. To answer this question I significantly simplified the model proposed in [43] and thus allowed for analytical treatment, at least within MFA. I was able to show that this simple model of population dynamics was capable of describing both the carrying capacity, which is a stable steady state for the population, and the minimum viable population. The second paper in the series ([33]) was written jointly with my student, Marcin Wolański. In this paper we considered the model introduced in [7] and examined what strategy could help the population to survive.

2.1.2 Applications of agent-based models in social systems

Soon after obtaining the doctoral degree I have started to work on applications of statistical physics in social sciences, in particular on opinion dynamics – one of the most investigated subfields of sociophysics. In my opinion there are at least two important reasons why physicists study this topic. The first motivation comes from social sciences and can be described as a temptation to build a bridge between the micro and macro levels in describing social systems. Traditionally, there are two main disciplines that study social behavior – sociology and social psychology. Although the subject of the study is the same for both disciplines, the usually taken approach is very different. Sociologists study social systems from the level of the social group, whereas social psychologists concentrate on the level of the individual. From the physicist's point of view this is similar to the relationship between thermodynamics and statistical physics. This analogy raises the challenge to describe and understand the collective behavior of social systems (sociology) from the level of interpersonal interactions (social psychology). The second motivation to deal with opinion dynamics is related to the development of non-equilibrium statistical physics, because models of opinion dynamics are often very interesting from the theoretical point of view. In 2000 we have proposed such a microscopic model [38], presently known as the

Sznajd model, that inspired many researchers not only to apply it to investigate social phenomena (e.g. marketing campaigns, financial markets or political campaigns) but also to study theoretical aspects of the model.

It should be stressed here, that the application of microscopic models in social systems is an older idea than sociophysics itself, if we assume that sociophysics was 'born' in 1982.[7] Over a decade earlier, Thomas Schelling has proposed a model of spacial segregation, which is strikingly similar to the Ising model with Kawasaki dynamics.[8] Nowadays, this type of approach is known in social sciences as agent-based modeling (ABM) and in recent years has become increasingly popular, in particular in the field of marketing.

If one looks at my works devoted to applications of agent-based models in social systems, one can see that I have also focused on problems related to marketing. However, my first real-life application of the Sznajd model was in finance [34]. The aim was to describe the mechanism of price formation in financial markets in terms of a modified Sznajd model. Introducing two types of traders – *followers* and a *fundamentalist* – we were able to reproduce major stylized facts of financial returns. The next paper in this collection – [32] – concerned marketing strategies in duopoly markets (i.e. markets with two competing suppliers). We tried to answer the question *When is advertising effective and when is it not?* within a two dimensional modified Sznajd model with an external field. Using Monte Carlo simulations we showed the existence of two phase transitions – one related to the initial number of customers of a given product (so-called critical mass) and second related to the level (or intensity) of advertising. Five years later, this study was continued in a more general setting of oligopoly markets [24]. We studied the problem of a new market entrant challenging two incumbents of roughly the same size, like the mobile telecom *Idea* (later rebranded to *Orange*) challenged in the year 2000 two well established telecoms at the time (*Era* and *Plus*). The general behavior of two related ABM models was studied using MFA and Monte Carlo simulations. Interestingly, the best fits to real data from the Polish telecom market were obtained for conformity level $p \in (0.3, 0.4)$, which agreed very well with the conformity level found by Solomon Asch in his famous social experiment.

In the recent years, I have continued research in this field under the grant *The use of simple spin models in social and commercial marketing* funded by the National Science Centre (NCN). I have also begun several interdisciplinary collaborations: (1) with the Economic Modeling Group from the Department of Operations Research, Wrocław University of Technology, (2) with economist Dr. Agnieszka Kowalska-Styczeń from the Department of Organization and Management, Silesian University of Technology, and very recently (3) with social psychologist Dr. Katarzyna Byrka from the University of Social Sciences and Humanities. As a result of these actions, I have published in the years 2012-2014 four articles on agent-based models in marketing ([10],[12],[13],[15]) and further papers are under preparation. These publications focus mainly on diffusion of innovation, in particular of ecological products and services. One of the problems that we have tackled is the so-called intention-behavior gap, observed empirically for some

[7]With the article S. Galam, Y. Gefen, Y. Shapir, *Sociophysics: a new approach of sociological collective behavior. I. Mean-behavior description of a strike*, J. Math. Sociol. 9, 1-13 (1982).

[8]This was probably noted for the first time by Dietrich Stauffer and Sorin Solomon in *Ising, Schelling and self-organising segregation*, Eur. Phys. J. B 57, 473-479 (2007)

innovations, e.g. dynamic electricity tariffs, green energy or health-promoting behaviors.

Besides of the 6 above mentioned papers devoted to marketing and one to financial markets, I have been also working on politically motivated problems ([21], [30]). In my opinion, a particularly interesting idea has been proposed in [30]. In this paper we introduced a model motivated by the political compass and based on the Ashkin-Teller idea to assign two Ising spins to each site agent – one representing opinion in the economic area and one in the personal area. We assumed that the mechanisms of opinion formation in the economic area were given by the outflow dynamics, whereas in the personal area by the inflow dynamics.[9] We found, among others, that the formation of consensus between groups of agents which differed only in the economic area was quite easy, whereas if they differed in the personal area no consensus was possible. This result was also interesting from the theoretical point of view and pointed to a qualitative difference between outflow and inflow dynamics. This has motivated me to conduct theoretical studies discussed in the following section.

2.1.3 Theoretical aspects of non-equilibrium spin dynamics

On one hand, the main challenge that we face when dealing with opinion dynamics models is the requirement to describe a complex social system by a relatively simple model. This has inspired many physicists to build models, like the Sznajd model, that cannot be easily justified by physical phenomena. On the other hand, such models may be interesting in themselves and may be treated as small building blocks that contribute to the construction and better understanding of the still emerging non-equilibrium statistical physics.

In the Sznajd model, similarly as in a few earlier models of social dynamics[10], the system consists of N individuals, each holding one of two opposite opinions $S_i = \pm 1$, like particles in the Ising model. Strange as it may sound, binary opinions are natural from the social point of view. A dichotomous response format with 1 (yes, true, agree) and 0 (no, false, disagree) as response options is one of the most common in social experiments.

As noted by Dietrich Stauffer: *The crucial difference of the Sznajd model compared with voter or Ising models is that information flows outward: A site does not follow what the neighbours tell the site, but instead the site tries to convince the neighbours.*[11] Of late, a debate on whether inflow dynamics is different from outflow dynamics has emerged.[12] Another characteristic feature of the model is that unanimity, instead of majority, is needed to convince others. This feature underlies the more general q-voter model proposed

[9] See Section 2.1.3 for a discussion on outflow and inflow dynamics.

[10] See e.g. S. Galam, *Majority rule, hierarchical structures and democratic totalitarianism: a statistical approach.*, J. Math. Psychol. 30, 426-434 (1986); M. Lewenstein, A. Nowak, B. Latane, *Statistical mechanics of social impact.*, Phys. Rev. A 45, 763-776 (1992); J. A. Hołyst, K. Kacperski, F. Schweitzer, *Social impact models of opinion dynamics.* Ann. Rev. Comput. Phys. 9, 253-273 (2001).

[11] D. Stauffer, *Sociophysics: the Sznajd model and its applications.*, Computer Physics Communications 146, 93-98 (2002).

[12] To the best of my knowledge, both terms – *inflow dynamics* and *ouflow dynamics* – have been introduced by me in [28]. Now they are widely used in the literature and the debate on the differences between these two dynamics is not over yet, see e.g. P. Roy, S. Biswas, P. Sen, *Exit probability in inflow dynamics: nonuniversality induced by range, asymmetry and fluctuation*, Physical Review E **89**, 030103 (2014) and C. Castellano, R. Pastor-Satorras, *Irrelevance of information outflow in opinion dynamics models*, Physical Review E *83*, 016113 (2011).

in 2009 by Castellano et al.[13] and can be justified based on social experiments – it has been observed that a small unanimous group may be more efficient at persuading others than a much larger group with a non-unanimous majority. However, before I proceed to problems related to the two mentioned features of the Sznajd model, namely *outflow dynamics* and the *unanimity rule*, I will start with the first theoretical aspect that I was working on.

In 2002 I have asked a question about the possibility of introducing something like a Hamiltonian for the Sznajd model. Due to the lack of symmetry related to outflow dynamics, I was not able to introduce a Hamiltonian in the strict sense. Instead I have introduced an object called the disagreement function, which controlled the dynamics and had the form of the axial next-nearest-neighbor Ising (ANNNI) model Hamiltonian. However, in contrast to the Hamiltonian, the disagreement function is minimized only locally. I have considered the model in one [6] and two [5] dimensions using Monte Carlo simulations, as well as the mean-field approach in [3] and [4]. Moreover, in [4], using the Boltzmann factor I have introduced a parameter (T) that played the role of local temperature. Besides determining phase diagrams, I have also considered the time evolution of the system. The model occurred to be very interesting exhibiting rich phase diagrams as well as nontrivial dynamics. As a result, in the years 2002-2005, I have written a series of four single-author papers ([3]–[6]), which were published in Physical Review E and became the basis of my habilitation.

After submitting the habilitation dissertation in September 2005, I have started to study the differences between inflow and outflow dynamics. Article [28], published in 2006 with my Ph.D. student Sylwia Krupa, was the first paper in a series related one of the major problems in the field of social simulations[14]. The motivation for this work came from the observation that Glauber (inflow) and Sznajd (outflow) zero-temperature dynamics are equivalent for the one dimensional Ising ferromagnet with nearest-neighbor interactions. To systematically compare the dynamics we have introduced *partially synchronous updating*. Within such updating in each elementary time step we visit all N sites of the system and select each of them with probability c as a candidate to get flipped. This allows to tune the type of updating from random sequential for $c = 1/N$ to synchronous for $c = 1$. This type of updating has been used later by Radicchi et al. to investigate the Ising spin chain at zero temperature for the Metropolis algorithm[15] and by us to study generalized zero-temperature Glauber dynamics [16]. In [28] we have used the method of mapping an Ising spin system onto the dimer RSA model and we have shown that already this simple mapping allows us to see the differences between inflow and outflow zero-temperature dynamics. Moreover, we have investigated both dynamics under partially synchronous updating using Monte Carlo simulations and have shown qualitative differences between inflow and outflow dynamics.

[13] C. Castellano, M.A.Muňoz, R.Pastor-Satorras, *Nonlinear q-voter model*, Phys. Rev. E 80, 041129 (2009).

[14] Namely, Macy and Willer (*From factors to actors: computational sociology and agent-based modeling.*, Annu. Rev. Sociol. 28, 143166, 2002) observed that there was a little effort to provide analysis of how results differ depending on the model designs.

[15] F. Radicchi, D. Vilone, and H. Meyer-Ortmanns, *Phase Transition between Synchronous and Asynchronous Updating Algorithms*, J. Stat. Phys. 129, 593 (2007).

Interestingly, it turned out that synchronous and partially synchronous updating introduces a very complex behavior to the generalized zero-temperature Glauber dynamics. Given that in the past decade low-temperature Glauber dynamics for the one-dimensional Ising system has been observed experimentally and is now regarded as one of the most important theoretical approaches in the field of molecular nanomagnets[16], my results may have far reaching consequences. Moreover, in general the relaxation of Ising ferromagnets with zero-temperature spin-flip dynamics exhibits very complex behavior[17].

Therefore I have continued the work on this topic. In [1] I have investigated the relaxation to the ground state, after a quench from high temperature, for one-dimensional zero-temperature Glauber dynamics[18] under synchronous updating. Using Monte Carlo simulations and MFA, I have shown the existence of a discontinuous phase transition between ferromagnetic and antiferromagnetic phases induced by parameter W_0, which describes the probability of flipping the spin in the case in which the energy of the system does not change after the flip (for the Metropolis algorithm $W_0 = 1$ and for the original Glauber dynamics $W_0 = \frac{1}{2}$). Soon after my paper appeared in *Physical Review E*, Yi and Kim published a comment[19], in which they repeated and confirmed my results but additionally provided finite-size scaling. On the basis of the obtained scaling they claimed that the observed phase transition was continuous, in contrast to my conjecture of a discontinuous phase transition. Indeed, according to the classical theorem of Landau, first-order phase transitions are impossible in one-dimensional equilibrium systems. However, zero-temperature Glauber dynamics under synchronous updating is definitely a non-equilibrium model and in non-equilibrium statistical mechanics, several models have been shown to exhibit a first-order transition even in one spatial dimension.[20] Therefore, I decided to look at this issue again more carefully and, as a result, I have published together with my students another paper on one-dimensional Glauber dynamics [16]. This time we have considered the more general case of partially synchronous updating. We have been able to confirm that for the synchronous updating mode there is a discontinuous phase transition between two ordered phases (ferromagnetic and antiferromagnetic). Three signatures of a discontinuous phase transition have been found in this case: (1) a jump of the ordering parameter (critical exponent $\beta = 0$), (2) phase coexistence, and (3) hysteresis cycles. Moreover, we have shown that for any other type of updating there is a continuous order-disorder transition (between the ferromagnetic and so-called active phases).

Besides working on Glauber (inflow) dynamics, I have been working on outflow dy-

[16] P. Gambardella et al., *Ferromagnetism in one-dimensional monatomicmetal chains*, Nature 416, 301-304 (2002); A. Caneschi et al. *Glauber slow dynamics of the magnetization in a molecular Ising chain*, Europhys. Lett. 58, 771-777 (2002)

[17] A. Lipowski, *Anomalous phase-ordering kinetics in the Ising model*, Physica A 268, 6-13 (1999); V. Spirin, P. L. Krapivsky, S. Redner, *Fate of zero-temperature Ising ferromagnets*, Physical Review E 63, 036118 (2001)

[18] C.Godrèche, J. M. Luck, *Metastability in zero-temperature dynamics: statistics of attractors*, J. Phys.: Condens. Matter 17, S2573-S2590 (2005)

[19] I.G. Yi, B.J. Kim, *Comment on Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 83, 033101 (2011).

[20] See e.g. M. Henkel, H. Hinrichsen, and S. Luebeck, *Non-equilibrium Phase Transitions*, Springer, 2008.

namics within the q-voter model. In short, in the q-voter model each individual interacts with a set of q neighbors (a q-lobby). If all q neighbors share the same state (i.e. the q-lobby is unanimous) the individual conforms to this state and – as originally proposed – otherwise (i.e. in case of disagreement) the individual changes its state with probability ϵ. However, in some later publications the model with $\epsilon = 0$ and the outflow dynamics was studied, as a natural generalization of the Sznajd model.

There are two particularly interesting topics related to the q-voter model. The first, considered in papers [11], [20] and [25] concerns the recent controversy on the exit probability[21], $E(x)$, for the one-dimensional q-voter model. While for the linear voter model and the Ising model with Glauber dynamics, $E(x) = x$ is an exact result, in the q-voter model with $q \geq 2$ the exit probability is nonlinear and so far no one has come up with an exact analytical result, even for $q = 2$ (i.e. for the Sznajd model). The difficulties to find the exact solution arise from the fact that the average magnetization in the q-voter model is not conserved. The first attempt to calculate $E(x)$ analytically, was proposed independently by our group [25] and by Lambiotte and Redner[22]. Using the Kirkwood approximation, we have obtained an analytical formula for the exit probability and have shown that it agrees very well with computer simulations, even for such initial conditions for which the Kirkwood approximation cannot be easily justified.

However, three years later, it has been suggested by Galam and Martins[23] that our results are valid only for finite-size systems and the solution should approach a step-like function for infinite systems. It should be stressed, that this suggestion was taken seriously due to the approximate nature of the solution presented in [25] and considered again in [20], where an analytical formula for the exit probability of the general q-voter model has been provided. This result has been confirmed by later publications.[24] Moreover, it has been shown analytically that the step-like exit probability is an exact result for the q-voter on the complete graph, but not for a one-dimensional lattice.

The second line of research is related to phase transitions in the generalized q-voter model. In the basic model, the interactions between individuals were limited to the so-called conformity. This type of social influence reminds us physicists of ferromagnetic interactions. However, in real societies conformity, although the most common, is not the only type of social influence. The second widely recognized social response is non-conformity, which can take one of two forms:

- Independence – resisting influence. In this case the situation is evaluated independently of the group norm. Based on this definition we have argued that independence plays a role similar to temperature.

- Anticonformity – rebelling against influence. Anticonformists are similar to conformists in the sense that both take into account the group norm – conformists agree

[21] I.e. the probability that the system ends up with all spins up starting with a fraction x of up-spins.

[22] R. Lambiotte, S. Redner, *Dynamics of non-conservative voters*, Europhys. Lett. **82**, 18007 (2008).

[23] S. Galam, A.C.R. Martins, *Pitfalls driven by the sole use of local updates in dynamical systems*, Europhys. Lett. 95, 48005 (2011).

[24] See [11] and A.M. Timpanaro, C.P.C. Prado, *Exit probability of the one-dimensional q-voter model: Analytical results and simulations for large networks*, Phys. Rev. E **89**, 052808 (2014).

with the norm, anticonformists disagree. This reminds us of anti-ferromagnetic interactions.

Introduction of any of the two types of non-conformity to the basic q-voter model results in a phase transition between the phase with magnetization ($m \neq 0$; interpreted in such models as 'public opinion') and status-quo ($m = 0$). In [18] we have introduced anti-conformity to the Sznajd model (which corresponds to the q-voter model with $q = 2$). We have considered the model on the complete graph which has allowed for an analytical treatment for infinite and finite systems. We have shown that opinion dynamics can be understood as a movement of public opinion in a symmetric bistable effective potential and we have found the full phase diagram for the considered model. We have also shown that Monte Carlo simulations coincide well with the analytical results already for a system of size $N = 100$, which is an important result because for other topologies Monte Carlo simulations are still the main research tool.

In [17] we have considered the q-voter model with non-conformity for an arbitrary value of q. In this paper we have asked an important question related to the differences between two types of non-conformity. Although these differences are very important for social scientists and visible on the microscopic level, they may be irrelevant from the physical point of view, in a sense that macroscopic behavior of the system is qualitatively the same under both types non-conformity. Therefore, we have considered two models – the q-voter model with independence and the q-voter model with anti-conformity. Again, we have limited our study to the topology of the complete graph, to get exact analytical results. Surprisingly, we have observed that there are significant qualitative differences between the two models. In particular, we have shown that in the model with anti-conformity the critical value of noise increases with parameter q, whereas in the model with independence the critical value of noise decreases with q. Moreover, in the model with anti-conformity the phase transition is continuous for any value of q, whereas in the model with independence the transition switches from continuous to discontinuous at $q = 6$. I believe that our results are particularly important for modeling opinion dynamics, because previously the problem was neglected or even unnoticed in the field.

The next paper in the series, [14], was written at the invitation of prof. Sidney Redner and published in the special issue *Statistical Mechanics and Social Sciences* of the Journal of Statistical Physics. In this paper we have further studied the *independence vs. anti-conformity* issue, originally initiated in [17]. In addition to the introduction of the generalized q-voter model with threshold and finding the phase diagram for this model, I devoted five chapters (10 pages) to the review of the opinion dynamics literature. My aim was to bring the social knowledge and literature to sociophysicists and to describe physical concepts useful for opinion dynamics modeling in a form understandable to social scientists. I believe I have succeeded. Although the paper was published only very recently, it has already received 29 citations according to Google Scholar and 9 citations (without self-citations) according to Web of Science. Moreover, shortly after publication of this article I was invited to several prestigious scientific events. In particular, to give a tutorial lecture on opinion dynamics at the Spring Meeting of the German Physical Society in Berlin (March 2015), a plenary lecture at the 7th Conference on Behavioral Economics (May 2014) and the inaugural lecture at the University of Social Sciences and Humanities (Wrocław Campus; October 2014).

The question: *Do the modeling assumptions we make regarding social interactions have substantial impact on the simulated behavior of the system as a whole or not?* that inspired me to work on the previously discussed papers was also the motivation for the most recent article [9], published in PLoS ONE and devoted to the problem known is psychology as the *person-situation debate.* The debate, started in the late 1960s, refers to the controversy concerning whether the personal traits or the situation is more important in determining a person's behavior. Studying two variants (person vs. situation) of the same agent-based model of opinion formation, we have shown that the decision to choose either personal traits (a fraction p of agents in the society are permanently conformists and $1-p$ permanently non-conformists) or the situation (each agent behaves as non-conformist with probability p and as conformist with probability $1-p$) as the primary factor driving social interactions is of critical importance even in the case of a complete graph. I strongly believe that this sensitivity to modeling assumptions has far reaching consequences also beyond opinion dynamics, since agent-based models are becoming a popular tool among economists and policy makers and are often used as substitutes of real social experiments.

In my opinion, the problem is also interesting from the physical point of view because it is similar in spirit to the discussion regarding *quenched vs. annealed disorder.* The latter has recently regained popularity – this time in the context of complex networks. One of the analytical methods used in this field, the so-called *heterogeneous mean field* approach, i.e. an improved mean-field theory that accounts for heterogeneity of a complex network, relies on replacing a real (or *quenched*) network by a weighted, fully connected graph (or *annealed*).[25] The question that arises is whether this substitution is justified, or rather – when is it appropriate.[26] The answer may be nontrivial. My initial guess regarding the person vs. situation modeling setup was that there would be no significant differences between the two approaches on the macroscopic scale, at least in the case of the complete graph. And this indeed was the reply I was giving when asked on different occasions. Only recently have I realized that this problem – while very difficult, if possible at all, to solve via social experiments – could be easily addressed within a microscopic agent-based model. And the obtained results were stunning – assuming a person-specific response to social influence at the microscopic level generally leads to a completely different and a less realistic aggregate or macroscopic behavior than an assumption of a situation-specific response [9]; a result that has been reported by social psychologists for a range of experimental setups, but has been downplayed or ignored in the opinion dynamics literature.

[25] See e.g. S.N. Dorogovtsev, A.V. Goltsev, J.F.F. Mendes, *Critical phenomena in complex networks*, Rev. Mod. Phys. 80, 12751335 (2008).

[26] A.N. Malmi-Kakkada, O.T. Valls, Ch. Dasgupta, *Ising model on a random network with annealed or quenched disorder, Physical Review B 90, 024202 (2014).*

2.2 Achievements in the area of teaching and advising

I have always spent a lot of time preparing for and felt enthusiastic about teaching classes (lectures, computer labs and exercises) to undergraduate and graduate students, for details see Section 3.5.1. While working at the University of Wrocław (until September 2013), in addition to standard classes such as *Theoretical Mechanics*, *Statistical Physics*, *Computer Modeling* or *Theory of Phase Transitions*, I have prepared several elective courses that turned out to be very popular among the students (including English-language lectures *Nonlinear Dynamics* and *Nonequilibrium Phase Transitions*). However, the most successful courses were completely original and unusual, going far beyond the standard lecture-discussion class form. In 2000-2001 I offered a course called *Exotic Statistical Physics*, which introduced undergraduate students to the new field of interdisciplinary applications of statistical physics. Being a mix of lectures, student seminars and problem solving, this course attracted students from experimental, theoretical and computational physics curricula. In the same spirit of developing passion and creativity, in the years 2012-2013 I offered a course called *Modeling – Criticality and Complexity.* This course had a modern, interactive form which included discussions, brainstorming, student presentations, problem solving and a mini-workshop at the end. Students had the opportunity not only to learn about new trends in modeling complex systems and non-equilibrium phase transitions, but also to participate in the research process itself – from the birth of the model, through a review of the literature and model analysis, to the presentation of the results. During these activities I had the pleasure to work with the best undergraduate and graduate students (including Karol Suszczyński, Rafał Topolnicki, Maciej Tabiszewski and Marcin Wolański), which resulted in articles published in prestigious scientific journals ([10], [11], [16], [19], [20], [33]). For my involvement in teaching, in 2011 I was awarded the Medal of the Commission of National Education.

In the years 1999-2014 I was the supervisor in 25 M.Sc. dissertations at the University of Wrocław (UWr). During this period I was also a reviewer of more than 30 M.Sc./B.Sc. theses at UWr and NTNU (Trondheim, Norway). Moreover, I have been the supervisor in three Ph.D.'s (two completed, one ongoing) in physics, see Section 3.5.2, and in the years 2011-2014 I was a reviewer of 4 doctoral dissertations, see Section 3.5.3.

In 2013, I have moved to the Wrocław University of Technology (PWr), where I am currently employed on the position of (Extraordinary) Professor[27] in the Department of Theoretical Physics. In addition to teaching undergraduate and graduate courses in the Physics and Applied Math curricula at PWr, since November 2013 I have been the co-advisor (with prof. Antoni Mituś) of the Society of Physics Students *Nabla*. In the years 2013-2014 I have organized a series of lectures devoted to modeling and analysis of complex systems for the *Nabla* students.

[27] In Polish: *profesor nadzwyczajny.*

2.3 Information about activities popularizing science and organizational work

I have always spent a lot of time and committed myself to organizational work and popularizing science. I took part in the organization of nine international scientific conferences (including the Max Born Symposia and the Winter Schools in Theoretical Physics, for details see Section 3.5.7) and I was the Director of three of them. I am the most proud of organizing the 47th Winter School in Theoretical Physics *Simple Models for Complex Systems*, in the year 2011. The school was intended not only to provide young people with the knowledge related to a relatively new, intensely growing and a highly interdisciplinary field of complex systems, but also to integrate the academic community. Our 12 lecturers from around the world were not only recognized experts in the field, but also excellent teachers who knew how to introduce newcomers to the fascinating interdisciplinary 'world of complex systems'. Due to the highly interdisciplinary nature of the School, not only physicists, but also mathematicians, computer scientists, economists and social scientists decided to participate in the meeting. In total, 75 people from around the world attended the School.

The idea of crossing borders between different, often seemingly distant areas, was also the motivation for establishing the UNESCO Chair of Interdisciplinary Studies at the University of Wrocław (http://www.kusi.ift.uni.wroc.pl). The first chairman and founder was prof. Andrzej Pękalski, whose dream was to create an international research center of interdisciplinary education. From the very beginning I was actively involved in the process of creating and then running the Chair. In the years 2011-2013, I was the Head of the Chair.

One of my favorite activities related to the academia is popularizing science. I have been always active in the field, offering lectures to non-specialists, among other at the University of the Third Age, charities, schools and the Lower-Silezia Science Festival (see Section 3.5.6). I am also the author of several popular-science articles, three of them published in *Wiedza i Życie*, a popular Polish journal serving the community in a similar way as *Scientific American* does worldwide.

Chapter 3

A detailed list of professional achievements

3.1 Information about scientific publications

3.1.1 Information about citations

The number of citations indexed in Web of Science (data for 31.03.2015) is 914 without self-citations and the Hirsch index is equal to 12.[1]

My publications have been also cited about 100 times[2] in books, including such monographs as:

- Parongama Sen, Bikas K. Chakrabarti, *Sociophysics: An Introduction*, Oxford University Press (2013)
- Francisek Slanina, *Essentials of Econophysics Modelling*, Oxford University Press (2013)
- Serge Galam, *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*, Springer (2012)
- Willi-Hans Steeb, *The Nonlinear Workbook: Chaos, Fractals, Cellular Automata, Neural Networks*, World Scientific (2011)
- Rodolfo Baggio, Jane Klobas, *Quantitative Methods in Tourism*, Channel View Publications (2011)
- Tom Siegfried, *A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature*, National Academies Press (2006)
- Dietrich Stauffer et al., *Biology, Sociology, Geology by Computational Physicists*, Elsevier (2006)

[1] Excluding self-citations of all authors. Hirsch index is calculated automatically by Web of Science.

[2] Without self-citations. Due to the lack of international databases of citations in books the provided numer of citations is only approximate. It was obtained by searching Google Books.

- Sergio Albeverio, Volker Jentsch, Holger Kantz, *Extreme Events in Nature and Society*, Springer (2006)
- Philip Ball, *Critical Mass: How One Thing Leads to Another*, Macmillan (2006)
- David P. Landau, Kurt Binder, *A Guide to Monte Carlo Simulations in Statistical Physics*, Cambridge University Press (2002)

3.1.2 List of single-author articles in JCR-listed journals

List of single-author publications with actual values of the 2 and 5-year impact factors (IF_{2Y} i IF_{5Y})[3] and the number of citations (CIT) indexed in Web of Science (excluding self-citations of all authors)[4]:

[1] K. Sznajd-Weron, *Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 82, 031120 (2010); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 2]$

[2] K. Sznajd-Weron, *Sznajd model and its applications*, Acta Physica Polonica B 36 (2005); $[IF_{2Y} = 0.998, IF_{5Y} = 0.742, CIT = 65]$

[3] K. Sznajd-Weron, *Metastabilities in the degenerated phase of the two-component model*, Phys. Rev. E 72, 026109 (2005); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 0]$

[4] K. Sznajd-Weron, *Mean-field results for the two-component model*, Phys. Rev. E 71, 046110 (2005); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 1]$

[5] K. Sznajd-Weron, *Dynamical model of Ising spins*, Phys. Rev. E 70, 037104 (2004); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 13]$

[6] K. Sznajd-Weron, *Controlling simple dynamics by a disagreement function*, Phys. Rev. E 66, 046131 (2002); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 19]$

[7] K. Sznajd-Weron, *Instabilities in population dynamics*, Eur. Phys. J. B 16, 183 (2000); $[IF_{2Y} = 1.463, IF_{5Y} = 1.515, CIT = 7]$

[8] K. Sznajd-Weron, *Change of a continuous character caused by gene flow. An analytical approach*, Physica A 264, 432 (1999); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 0]$

3.1.3 List of multi-author articles in JCR-listed journals

List of single-author publications with actual values of the 2 and 5-year impact factors (IF_{2Y} i IF_{5Y}) and the number of citations (CIT) indexed in Web of Science (excluding self-citations of all authors):

[3] Impact Factors are taken from the report published in 2014.
[4] Data for 31.03.2015.

[9] K. Sznajd-Weron, J. Szwabiński, R. Weron, "Is the Person-Situation Debate Important for Agent-Based Modeling and Vice-Versa?" PLoS ONE 9(11), e112203 (2014); $[IF_{2Y} = 3.543, IF_{5Y} = 4.015, CIT = 0]$

[10] A. Kowalska-Pyzalska, K. Maciejowska, K. Suszczyński, K. Sznajd-Weron, R. Weron, *Turning green: Agent-based modeling of the adoption of dynamic electricity tariffs*, Energy Policy 72, 164-174 (2014); $[IF_{2Y} = 2.696, IF_{5Y} = 3.402, CIT = 1]$

[11] K. Sznajd-Weron, K. Suszczyński, *Nonlinear q-voter model with deadlocks on the Watts-Strogatz graph*, J. Stat. Mech. P07018 (2014); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CIT = 0]$

[12] K. Sznajd-Weron, J. Szwabiński, R. Weron, T. Weron, *Rewiring the network. What helps an innovation to diffuse?*, J. Stat. Mech. P03007 (2014); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CIT = 1]$

[13] P. Przybyła, K. Sznajd-Weron, R. Weron, *Diffusion of innovation within an agent-based model: Spinsons, independence and advertising*, Advances in Complex Systems 17, 1450004 (2014); $[IF_{2Y} = 0.786, IF_{5Y} = 0.918, CIT = 0]$

[14] P. Nyczka, K. Sznajd-Weron, *Anticonformity or Independence? – Insights from Statistical Physics*, Journal of Statistical Physics 151, 174-202 (2013); $[IF_{2Y} = 1.284, IF_{5Y} = 1.239, CIT = 9]$

[15] A. Kowalska-Styczeń, K. Sznajd-Weron, *Access to information in word of mouth marketing within a cellular automata model*, Advances in Complex Systems 15, 1250080 (2012); $[IF_{2Y} = 0.786, IF_{5Y} = 0.918, CIT = 1]$

[16] B. Skorupa, K. Sznajd-Weron, R. Topolnicki, *Phase diagram for a zero-temperature Glauber dynamics under partially synchronous update*, Phys. Rev. E 86, 051113 (2012); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 0]$

[17] P. Nyczka, K. Sznajd-Weron, J. Cisło, *Phase transitions in the q-voter model with two types of stochastic driving*, Physical Review E 86, 011105 (2012); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 8]$

[18] P. Nyczka, K. Sznajd-Weron, J. Cisło, *Opinion dynamics as a movement in a bistable potential*, Physica A 391, 317-327 (2012); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 1]$

[19] K. Sznajd-Weron, M. Tabiszewski, A. Timpanaro, *Phase transition in the Sznajd model with independence*, Europhys. Lett. 96, 48002 (2011); $[IF_{2Y} = 2.269, IF_{5Y} = 2.112, CIT = 10]$

[20] P. Przybyła, K. Sznajd-Weron and M. Tabiszewski, *Exit probability in a one-dimensional nonlinear q-voter model*, Phys. Rev. E 84, 031117 (2011); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 7]$

[21] G. Kondrat, K. Sznajd-Weron, *Spontaneous reorientations in a model of opinion dynamics with anticonformists*, Int. J. Mod. Phys. C 21, 559-566 (2010); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CIT = 4]$

[22] T. Czarnik, R. Gawda, W. Kołodziej, D. Łątka, K. Sznajd-Weron, R. Weron, *Associations between intracranial pressure, intraocular pressure and mean arterial pressure in patients with traumatic and non-traumatic brain injuries*, Injury, Int. J. Care Injured 40, 33 (2009); $[IF_{2Y} = 2.462, IF_{5Y} = 2.388, CIT = 9]$

[23] G. Kondrat, K. Sznajd-Weron, *Percolation framework in Ising-spin relaxation*, Phys. Rev. E 79, 011119 (2009); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 3]$

[24] K. Sznajd-Weron, R. Weron, M. Włoszczowska, *Outflow dynamics in modeling oligopoly markets: the case of the mobile telecommunications market in Poland*, J. Stat. Mech. P11018 (2008); $[IF_{2Y} = 2.056, IF_{5Y} = 1.914, CIT = 1]$

[25] F. Slanina, K. Sznajd-Weron, P. Przybyła, *Some new results on one-dimensional outflow dynamics*, Europhys. Lett. 82, 18006 (2008); $[IF_{2Y} = 2.269, IF_{5Y} = 2.112, CIT = 18]$

[26] G. Kondrat, K. Sznajd-Weron, *Three types of outflow dynamics on square and triangular lattices and universal scaling*, Phys. Rev. E 77, 021127 (2008); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 3]$

[27] T. Czarnik, R. Gawda, D. Łątka, W. Kołodziej, K. Sznajd-Weron, R. Weron, *Noninvasive measurement of intracranial pressure: Is it possible?*,The Journal of Trauma, Injury Infection and Critical Care, 62(1), 207-211 (2007); $[IF_{2Y} = 2.961, IF_{5Y} = 3.204, CIT = 9]$

[28] K. Sznajd-Weron, S. Krupa, *Inflow versus outflow zero-temperature dynamics in one dimension*, Phys. Rev. E 74, 031109 (2006); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 8]$

[29] S. Krupa, K. Sznajd-Weron, *Relaxation under outflow dynamics with random sequential updating*, Int. J. Mod. Phys. C , Vol. 16, No. 11, 1771 (2005); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CIT = 9]$

[30] K. Sznajd-Weron, J. Sznajd, *Who is left, who is right?*, Physica A 351, 593 (2005); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 23]$

[31] J.O. Indekeu, K. Sznajd-Weron, *Hierarchical population model with a carrying capacity distribution for bacterial biofilms*, Phys. Rev. E 68, 061904 (2003); $[IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 1]$

[32] K. Sznajd-Weron, R. Weron, *How effective is advertising in duopoly markets?*, Physica A 324, 437 (2003); $[IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 36]$

[33] K. Sznajd-Weron, M. Wolański, *In search for the optimal strategy in population dynamics*, Eur. Phys. J. B 25 2, 253 (2002); $[IF_{2Y} = 1.463, IF_{5Y} = 1.515, CIT = 5]$

[34] K. Sznajd-Weron, R. Weron, *A simple model of price formation*, Int. J. Mod. Phys. C 13, 115 (2002); $[IF_{2Y} = 1.125, IF_{5Y} = 0.949, CIT = 54]$

[35] K. Sznajd-Weron, A. Pękalski, *Model of population migration in a changing habitat*, Physica A 294, 424 (2001); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 2$]

[36] K. Sznajd-Weron, R. Weron, *A new model of mass extinctions*, Physica A 293, 559 (2001); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 3$]

[37] A. Pękalski, K. Sznajd-Weron, *Population dynamics with and without selection*, Phys. Rev. E 63, 031903 (2001); [$IF_{2Y} = 2.326, IF_{5Y} = 2.302, CIT = 5$]

[38] K. Sznajd-Weron, J. Sznajd, *Opinion evolution in closed community*, Int. J. Mod. Phys. C 11, 1157 (2000); [$IF_{2Y} = 1.125, IF_{5Y} = 0.949, CIT = 513$]

[39] K. Sznajd-Weron, A. Pękalski, *Statistical physics model of an evolving population*, Physica A 274, 91 (1999); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 3$]

[40] K. Sznajd-Weron, A. Pękalski, *Evolution of populations in a changing environment*, Physica A 269, 527 (1999); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 1$]

[41] K. Sznajd-Weron, A. Pękalski, *Change of a continuous character caused by gene flow. A Monte Carlo study*, Physica A 259, 457 (1998); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 2$]

[42] K. Sznajd-Weron, A. Pękalski, *Evolution through stabilizing selection and gene flow*, Physica A 252, 336 (1998); [$IF_{2Y} = 1.772, IF_{5Y} = 1.684, CIT = 1$]

[43] I. Mróz, A. Pękalski, K. Sznajd-Weron, *Conditions for adaptation of an evolving population*, Phys. Rev. Lett. 76 3025 (1996); [$IF_{2Y} = 7.728, IF_{5Y} = 7.411, CIT = 14$]

3.1.4 Conference papers

1. K. Sznajd-Weron, J. Sznajd (2006) *Personal Versus Economic Freedom*, Proceedings of the Third Nikkei Econophysics Symposium Practical Fruits of Econophysics, H. Takayasu Ed., Springer-Verlag, Tokyo 355-360.

2. A. Kowalska-Pyzalska, K. Maciejowska, K. Sznajd-Weron, R.Weron (2014) *Modeling consumer opinions towards dynamic pricing: An agent-based approach*, IEEE Conference Proceedings, 11th International Conference on the European Energy Market (EEM'14), 28-30 May 2014, Kraków, Poland, DOI 10.1109/EEM.2014.6861272.

3.1.5 Edited volumes

1. K. Sznajd-Weron, ed. (2004) *Statistical Physics outside pure Physics* Physica A 336

2. A. Pękalski, K. Sznajd-Weron, eds. (2000) *Exotic Statistical Physics*, Physica A 285

3. R. Kutner, A. Pękalski, K. Sznajd-Weron, eds. (1999) *Anomalous Diffusion: From Basics to Applications*, Lecture Notes in Physics, Springer-Verlag, Berlin.

3.2 Information about scholarly activities

3.2.1 Participation in conferences

Invited and plenary talks

1. *Can agent-based modeling replace a social experiment?*, plenary lecture, 7th Conference of the Academic Economic Psychology Society, University of Social Sciences and Humanities, Wrocław 09-10.05.2014

2. *Agent Based Modeling in Energy Markets*, 2nd Energy Finance Christmas Workshop, Macquarie University, Sydney, 13-14.12.2012

3. *Phase transition in the Sznajd model with nonconformity*, The Unexpected Conference – SOCIOPHYSICS: Do humans behave like atoms?, CREA-Ecole Polytechnique, Paris, 13-16.11.2011

4. *Simple models for complex systems – toys or tools?*, Ising Lectures 2011, 14th Annual Workshop on Phase Transitions and Critical Phenomena, Lviv, 11-15.04.2011

5. *Can we treat people like particles? - a simple model of opinion formation*, International Symposium Computational philosophy: lessons from simple models, Niels Bohr Institute, Copenhagen 11-13.10.2007

6. *Opinion dynamics in personal and economical areas do they differ?*, International School on Complexity Statistical Physics of Social Dynamics: Opinions, Semiotic Dynamics, and Language, Erice (Sicily) 14-19.07.2007

7. *From social psychology to sociology - a physicist's point of view*, AKSOE Conference Physics of Socio-economic Systems, Regensburg 27.03.2007

8. *Personal versus economic freedom*, AKSOE Conference Physics of Socio-economic Systems, Dresden 26-31.03.2006

9. *Opinion evolution in sociophysics*, XI Summer School Fundamental Problems in Statistical Physics FPSPXI, Leuven 04-17.09.2005

10. *Kto jest prawicą, kto jest lewicą? (Who is right, who is left?)*, IX Mini Symposium in Statistical Physics, Częstochowa 05-06.12.2004

11. *Sznajd model and its applications*, 1st Polish Econo- and Sociophysics Symposium, Warszawa 19-20.11.2004

12. *Personal versus economic freedom*, 3rd Nikkei Econophysics Workshop, Tokyo 09-11.11.2004

13. *Physics beyond the Physics*, plenary lecture on XXXVII Congress of Polish Physicists, Gdańsk 15-18.09.2003

Selected contributed talks and posters

1. The list of contributed talks includes in particular:

    (a) *Diffusion of innovation within an agent-based model*, European Conference on Complex Systems (ECCS'13), Barcelona 16-20.09.2013

    (b) *Modelowanie dyfuzji innowacji* (*Modeling innovation diffusion*), 42nd General Meeting of Polish Physicists, Poznań 8-13.09.2013

    (c) *Spontaneous reorientations in a model of opinion dynamics with anticonformists*, Middle European Cooperation in Statistical Physics MECO 35, Pont-a-Mousson 15-19.03.2010

    (d) *Dogadamy się czy nie? – o modelowaniu ewolucji opinii w socjofizyce* (*Will we reach a consensus or not? – on modeling opinion evolution in sociophysics*), 2nd Polish Econo- and Sociophysics Symposium, Krakow 21-22.04.2006

2. The list of posters includes in particular (only conference/school names are provided): Altenberg Summer School on Fundamental Problems in Statistical Physics (Altenberg 1997), Middle European Cooperation in Statistical Physics – MECO 22 (Szklarska Poreba 1997), MECO 24 (Lutherstadt-Wittenberg 1999), MECO 27 (Sopron 2002), MECO 28 (Saarbrücken 2003), MECO 29 (Bratislava 2004), European Conference on Complex Systems ECCS'12 (Brussels 2012).

3.2.2 Selected invited seminar talks

Invited seminar talks abroad:

1. *Diffusion of innovation within an agent-based model*, FIME (Finance for Energy Market) seminar, Institut Henri Poincarré, Paris 7.11.2014

2. *Social Physics or Sociophysics?*, Joint Seminar: Department of Physics and Centre for Computational Science and Engineering, National University of Singapore, Singapore 11.12.2012

3. *Social Physics or Sociophysics?*, Physics Friday Colloquia, Department of Physics Norwegian University of Science and Technology, Trondheim 16.09.2011

4. *Can we treat people like particles? – a simple model of opinion dynamics*, Department of Physics Norwegian University of Science and Technology, Trondheim 03.05.2009

5. *Opinion Formation, Irrational Thinking and Spreading of Minorities*, Chair of Sociology, in particular of Modelling and Simulations, ETH Zurich, Zurich 18.12.2007

6. *A simple model of opinion formation*, Institute of Industrial Science, University of Tokyo, Tokyo 12.11.2004

7. *A simple model of opinion formation*, Department of Physics, Tokyo Metropolitan University, Tokyo 10.11.2004

8. *New dynamical model of Ising spins*, Ikegami Laboratory, University of Tokyo, Tokyo 8.11.2004

9. *From social psychology to sociology - a physicist's point of view*, Division of Condensed Matter Physics of the Institute of Physics of the Academy of Sciences of the Czech Republic, Prague 6.11.2003

Invited seminar talks in Poland:

1. *Jak modelować dyfuzję innowacji?* (*How to model diffusion of innovation?*), Collegium Physicum, Faculty of Physics, Adam Mickiewicz University in Poznań, Poznań 5.11.2013

2. *Czy możliwe są przejścia fazowe w układach jednowymiarowych?* (*Are phase transitions possible in one-dimensional systems?*), Seminar of the Institute of Physics, Wrocław University of Technology, Wrocław 7.01.2013[5]

3. *Proste modele układów złożonych* (*Simple models of complex systems*), Institute of Physics, Opole University, Opole 6.11.2011

4. *Phase transitions in 1D kinetic Ising model*, Coherence-Correlations-Complexity Seminar, Wrocław University of Technology, Wrocław 17.11.2010

5. *Ludzi można traktować jak cząstki, tylko po co? – o sensie socjofizyki* (*People can be treated as particles, but what for? – about the sense of sociophysics*), Świerk Computing Centre, Świerk 19.05.2010

6. *Ludzi można traktować jak cząstki, tylko po co? – o sensie socjofizyki* (*People can be treated as particles, but what for? – about the sense of sociophysics*), Institute of Low Temperature and Structure Research, Polish Academy of Sciences, Wrocław 10.03.2010

7. *Ludzi można traktować jak cząstki, tylko po co?* (*People can be treated as particles, but what for?*), Seminar of the Institute of Physics, Wrocław University of Technology, Wrocław 22.02.2010

8. *Nowy model spinów Isinga – czyli o tym jak nauki społeczne zainspirowały fizyka* (*A new model of Ising spins, or how social science inspired physicists.*), Dynamics of Complex Systems Seminar, Warsaw University of Technology, Warsaw 23.10.2006

9. *Socjaliści czy Liberałowie? – socjofizyka w polityce* (*Socialists or Liberals? – sociophysics in politics*), Faculty of Physics and Astronomy, University of Zielona Góra, Zielona Góra 25.10.2005

10. *Czy ludzi można traktować jak cząstki?* (*Can people be treated as particles?*), Seminar on Statistical Physics, University of Warsaw, Warsaw 7.05.2004

11. *Nowy dynamiczny model spinów isingowskich* (*A new dynamic model of Ising spins.*), Seminar on Statistical Physics, University of Warsaw, Warszawa 7.05.2004

[5] At that time I was still working at the University of Wrocław.

3.2.3 Editorial board membership

- Associate Editor, *Frontiers in Physics*, section Interdisciplinary Physics, Frontiers, since 2013
- Editorial Board, *International Journal of Statistical Mechanics*, Hindawi, since 2013
- Advisory Editor, *Physica A: Statistical Mechanics and its Applications* (IF_{5Y} = 1.684), Elsevier, since 2010

3.2.4 List of research projects

2014-2016 Senior Investigator, India-Polish Inter-Governmental Science & Technology Cooperation Programme, *Data Driven Approaches for Inferring Opinion Dynamics on Social Networks*, Indian Institute of Technology Kharagpur, India

2014-2016 Senior Investigator, Polish Science Foundation (NCN) grant no. 2013/11/B/HS4/01061, *Ekonomiczne konsekwencje kształtowania się opinii i podejmowania decyzji przez konsumentów: Modelowanie agentowe dyfuzji innowacji* (*Economic consequences of consumer opinion formation and decision making: Agent-based modeling of innovation diffusion*), PWr

2014-2016 Supervisor in the postdoc research grant *FUGA3* lead by Dr. Anna Chmiel, NCN grant no. 2014/12/S/ST3/00326, *Procesy nierównowagowe na sieciach wielopoziomowych* (*Non-equilibrium processes on multilevel networks*), PWr

2011-2014 Principal Investigator, NCN grant no. 2011/01/B/ST3/00727, *Zastosowanie prostych modeli spinowych w marketingu społecznym i komercyjnym* (*Simple spin models in applications to social and commercial marketing*), UWr

2011-2014 Senior Investigator, NCN grant no. 2011/01/B/HS4/02740, *Modelowanie dynamiki zachowań konsumentów na rynkach oligopolistycznych za pomocą automatów komórkowych* (*Modeling of consumer behavior dynamics in oligopoly markets by cellular automata*), Silesian University of Technology, Poland

2007-2009 Principal Investigator, Polish Ministry grant N N202 0194 33, *Nowa lokalna dynamika spinów Isinga z punktu widzenia teorii nierównowagowych układów dynamicznych i zastosowań w modelowaniu grup społecznych* (*A new local dynamics of Ising spins from the point of view of non-equilibrium dynamical systems and applications in modeling social groups*), UWr

2007-2009 Principal Investigator, 'Supervisory grant' for Sylwia Krupa, Polish Ministry grant N N202 0907 33, *Analiza układów spinów isingowskich z zero-temperaturowymi lokalnymi dynamikami* (*An analysis of Ising spin systems with zero-temperature local dynamics*), UWr

2000-2002 Senior Investigator, KBN grant no. 2p03B2718, UWr

1999-2000 Principal Investigator, Project for young researchers, University of Wroclaw 2318/W/IFT, UWr

1997-1998 Principal Investigator, Project for young researchers, University of Wroclaw 2201/W/IFT, UWr

3.2.5 Information about research group leadership

2011-2014 Head of a team of 5 researchers and students, NCN OPUS grant no. 2011/01/B/ST3/00727, IFT UWr

2012-2013 Head of *UNESCO Chair of Interdisciplinary Studies* (12-14 researchers), IFT UWr

2009-2013 Head of *Complex Systems and Nonlinear Dynamics* Division (8-10 researchers, 4-6 Ph.D. students), IFT UWr

3.3 Information about cooperation with the social and economic environment

- Reviewer of Polish Ministry grants
- Course leader and lecturer *Numerical skills*, in collaboration with local government and business (including McKinsey and Google) under the project *Wroclaw graduate, Program to prepare staff for the modern service sector*, University of Wroclaw and Wroclaw city (2010-2011)
- Scientific collaboration with *Easygreen Lejkowski Cezary* in promoting behaviors and actions consistent with the Green Standards (Green Brand and Global Green Consulting Center) (2010-2013)

3.4 Information about international cooperation

3.4.1 Internships and scientific visits

Due to family reasons I have rejected several invitations for longer visits or internships. However, I have accepted invitations for short-terms visits related usually to invited lectures (see Sec. 3.2.2). Only three times I have decided to go for longer visits (above 7 days):

1. Department of Physics, Norwegian University of Science and Technology, Trondheim (9 - 19.09.2011)
2. Department of Physics, Norwegian University of Science and Technology, Trondheim (30.04 - 10.05.2009)
3. Institute of Industrial Science, University of Tokyo, Tokyo (8-20.11.2004)

3.4.2 Refereeing for JCR-listed journals

Extensive peer-review service for:[6]

- Physical Review Letters [IF=7.411] and Physical Review E [IF=2.302] – 30 reports
- Physica A [IF=1.684] – 33 reports

Regular peer-review service for:

- Advances in Complex Systems [IF=0.918]
- European Physical Journal B [IF=1.515]
- Europhysics Letters [IF=2.112]
- Journal of Statitical Physics [IF=1.239]
- International Journal of Modern Physics C [IF=0.949]
- Physics Letters A [IF=1.706]

Ad-hoc peer-review service for:

- Behavioural Processes [IF=1.760]
- Complexity [IF=1.290]
- Journal of the Royal Society – Interface [IF=4.875]

3.4.3 Membership in international organizations and societies

1. Member of *Complex Systems Society*, since 2012

3.4.4 Participation in international expert committees

1. Member of the international jury for Young Scientist Award (YSA) in Socio- and Econophysics 2015

2. Member of habilitation jury for Laura Hernandez, Laboratoire de Physique Théorique et Modelisation (LPTM), joint laboratory of CNRS and Université de Cergy Pontoise (December 2014)

3. Member of international organizing committee *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2014 Satellite, Lucca, 24.09.2014

4. Member of international organizing committee *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2013 Satellite, Barcelona, 18-19.09.2013

[6] 5-year impact factors provided.

5. Member of scientific committee *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2012 Satellite, Brussels, 5-6.09.2012

6. Member of scientific committee *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2011 Satellite, Wien, 14-15.09.2011

7. Referee of M.Sc thesis, Department of Physics, NTNU, Trondheim, Norwegia

3.4.5 Participation in international research groups

1. Investigator in international project *Data Driven Approaches for Inferring Opinion Dynamics on Social Networks* sponsored by India-Polish Inter-Governmental Science & Technology Co

2. Collaboration with prof. Frantisek Slanina, Institute of Physics, Czech Academy of Sciences, Praha, Czech (from 2003)

3. Collaboration with prof. Joseph Indekeu, Theoretical Physics Section, Katholieke Universiteit Leuven, Belgium (from 1997)

3.5 Information about teaching/advising, popularizing and organizational activities

3.5.1 Courses taught

1. *Modern theory of phase transitions* (lecture + discussion class in English), Faculty of Fundamental Problems of Technology, PWr (2013-2014)

2. *Phase transitions in complex systems* (lecture in English), Faculty of Fundamental Problems of Technology, PWr (2014)

3. *Mechanics and thermodynamics* (lecture + discussion class), Faculty of Fundamental Problems of Technology, PWr (2013-2015)

4. *Waves and Electromagnetism* (lecture + discussion class + laboratory), Faculty of Fundamental Problems of Technology, PWr (2014)

5. *Modeling – criticality and complexity* (lecture + discussion class + computer laboratory), Faculty of Physics and Astronomy, UWr (2012-2013)

6. *Classical theoretical physics 2* (lecture), Faculty of Physics and Astronomy, UWr (2011-2013)

7. *Non-equilibrium phase transitions* (lecture), Faculty of Physics and Astronomy, UWr (2011)

8. *Statistical physics 1* (lecture), Faculty of Physics and Astronomy, UWr (2006-2012)

9. *Theory of phase transitions and critical phenomena* (lecture), Faculty of Physics and Astronomy, UWr (2006-2012)

10. *Computer modeling* (lecture + computer laboratory), Faculty of Physics and Astronomy, UWr (2000-2013)

11. *Diploma seminar*, Faculty of Physics and Astronomy, UWr (2008-2010)

12. *Exotic statistical physics* (seminar), Faculty of Physics and Astronomy, UWr (2000-2001)

3.5.2 Advising doctoral students

1. Supervisor: Piotr Przybyła *Nierównowagowa dynamika spinów isingowskich z punktu widzenia teorii układów złożonych i zastosowań interdyscyplinarnych* (*Non-equilibrium spin dynamics from theoretical and applicative point of view*), Faculty of Fundamental Problems of Technology, PWr, ongoing, Ph.D. opening procedure: 3.12.2013, expected defence date: 2015/2016

2. Supervisor: Piotr Nyczka *Przejścia Fazowe w uogólnionym modelu q-wyborcy na grafie zupełnym* (*Phase transitions in the generalized q-voter model on the complete graph*), Institute of Theoretical Physics, UWr. Ph.D. defence: 24.02.2015

3. Supervisor: Sylwia Krupa *Analiza układów spinów isingowskich z zero-temperaturowymi lokalnymi dynamikami* (*Analysis of Ising spin systems with zero-temperature local dynamics*), Institute of Theoretical Physics, UWr. Ph.D. defence: 19.06.2009

3.5.3 Refereeing doctoral and other dissertations

1. dr Agnieszka Czaplicka, *Procesy transportu i ewolucja topologii hierarchicznych sieci złożonych* (*Transport processes and topology evolution of hierarchical networks*), Faculty of Physics, Warsaw Technical University (Ph.D. defence: 20.10.2014)

2. dr Maciej Jagielski, *Zastosowanie nieliniowego równania Langevina, równania Fokkera-Plancka oraz modeli błądzeń losowych do opisu dochodów gospodarstw domowych Polski i Unii Europejskiej* (*Application of nonlinear Langevin equation, Fokker-Planck equation and random-walk models to modeling of household incomes in Poland and EU*), Faculty of Physics, University of Warsaw (Ph.D. defence: 09.06.2014)

3. dr Jacek Wendykier, *Sieciowe modele typu drapieżniki i ofiary – zastosowanie w modelowaniu nowotworów* (*Predator-prey lattice models – applications to cancer modeling*), Institute of Physics, Opole University (Ph.D. defence: 10.10.2013)

4. dr Tomasz Gubiec, *Modele błądzenia losowego w czasie ciągłym z pamięcią. Zastosowanie do opisu dynamiki rynków finansowych* (*Continuous-time random walk models with memory. Applications to financial market dynamics*), Faculty of Physics, University of Warsaw (Ph.D. defence: 12.12.2011)

5. reviewer of over 30 M.Sc. and B.Sc. dissertations

3.5.4 Popular science and related articles (in Polish)

1. G. Kontrym-Sznajd, K. Sznajd-Weron *Jak zainteresować uczniów fizyką?* (*How to get students interested in physics?*), Problemy dydaktyki fizyki, Wrocławskie Wydawnictwo Oświatowe ATUT, Centrum Edukacji Nauczycielskiej Uniw. Wrocł., Wrocław-Czeszów 2013, ISBN 978-83-7432-992-7str. 57-66

2. A. Pękalski, K. Sznajd-Weron, *Układy złożone na Uniwersytecie Wrocławskim* (*Complex systems at the University of Wrocław*), Przegląd Uniwersytecki, grudzień 2004

3. K. Sznajd-Weron, *W sieci małego świata* (*In the small world network*), Wiedza i Życie luty/04, 68-71 (2004)

4. K. Sznajd-Weron, *Seks według wzoru* (*Sex according to a formula*), Wiedza i Życie kwiecień/02, 46-49 (2002)

5. K. Sznajd-Weron, *Opowieść o fizyce egzotycznej* (*A tale of exotic physics*), Wiedza i Życie październik/01, 46-49 (2001)

3.5.5 Development of e-learning materials

1. Preparation of slides (pdf) and problems (pdf, xls) for nearly all credit courses taught. Materials for courses taught in the current semester are available for students from my webpage http://www.if.pwr.wroc.pl/˜katarzynaweron

2. Lecture notes *Fizyka statystyczna* (*Statistical physics*) on http://panoramix.ift.uni.wroc.pl and on my webpage

3. Lecture notes *Teoria przejść fazowych i zjawisk krytycznych* (*Theory of phase transitions and critical phenomena*) on http://panoramix.ift.uni.wroc.pl/ and on my webpage

4. Slides for the course *Modelowanie komputerowe* (*Computer modeling*) on http://panoramix.ift.uni.wroc.pl/

5. Lecture notes and slides for the course *Modelarnia – krytyczność i złożoność* (*Modeling – criticality and complexity*) on http://panoramix.ift.uni.wroc.pl/, homepage of the Faculty of Physics and Astrononomy UWr (POKL projects) and on my webpage

3.5.6 Active participation in events popularizing science

1. *Czy psychologia może się spotkać z fizyką?* (*Can psychology meet physics?*), Inaugural lecture at University of Social Sciences and Humanities, Wrocław Faculty 23.10.2014

2. *Co w praktyce oznacza nieskończoność?* (*What does infinity mean in practice?*) Student conference FAK 2014, Wrocław 16.05.2014

3. A series of lectures on complex systems for Society of Physics Students *Nabla*, PWr Wrocław (2013-2014)

4. *W sieci jeszcze mniejszego świata – życie na Facebooku i nie tylko* (*In the network of an even smaller world – life on Facebook and not only*), Student conference FAK 2013, Wrocław 19.05.2013

5. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka* (*What can be the reason for a revolution? A physicist's view on social systems*), University of the Third Age, Wrocław 19.11.2012

6. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka* (*What can be the reason for a revolution? A physicist's view on social systems*), XIV Lower Silesian Science Festival, Wrocław 21.09.2011

7. *Jaki może być powód rewolucji? Czyli o układach społecznych oczami fizyka* (*What can be the reason for a revolution? A physicist's view on social systems*), Szczecin Humanistyczny, University of Szczecin, Szczecin 4.04.2011

8. *Dogadamy się czy nie? – czyli co ma fizyka do socjologii* (*Will we reach a consensus or not? – or what has physics to do with sociology*), XII Lower Silesian Science Festival, Wrocław 19.09.2009

9. *Czy ludzi można traktować jak cząstki - spojrzenie fizyka* (*Can we treat people like particles – A physicist's view*), Polish Sociological Association, Institute of Sociology, UWr 15.12.2004

10. *Czy Bóg ma przepis? - od chaosu deterministycznego po fraktale* (*Does God have a recipe? – from deterministic chaos to fractals*), VI Lower Silesian Science Festival, Wrocław 09.2004

11. *Czy ludzi można traktować jak cząstki?* (*Can we treat people like particles?*), Warsaw Agricultural University, Warsaw 6.05.2004

12. *Jak przekonywać innych? – socjofizyka: model Sznajdów* (*How to convince others? – sociophysics: the Sznajd model*), Science Camp of the Polish Children's Fund, Świdr-Otwock, 8.05.2003

13. *Jak przekonywać innych? czyli socjofizyka* (*How to convince others? or sociophysics*), VI Lower Silesian Science Festival, Wrocław 09.2003 and Wałbrzych 10.2003

14. *Katastrofy oczami fizyków – od lawin piasku po wielkie wymierania* (*Physicists' view on catastrophes – from sand avalanches to mass extinctions*), V Lower Silesian Science Festival, Wrocław 20.09.2002 and Wałbrzych 4.10.2002

3.5.7 Membership in organizing committees

1. Organizing committee, *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2014 Satellite, Lucca 24.09.2014

2. Director, CODYM Spring Workshop (CODYM-Spring'14), PWr, Wrocław 7-8.04.2014

3. Organizing committee, *Cultural and Opinion Dynamics: Modeling, Experiments and Challenges for the future*, ECCS 2013 Satellite, Barcelona 18.09.2013

4. Director, 47th Winter School in Theoretical Physics *Simple Models for Complex Systems*, Lądek-Zdrój 7-12.02.2011

5. Director, XXIII Max Born Symposium *Critical Phenomena in Complex Systems*, Polanica-Zdrój 3-6.09.2007

6. Organizing committee, *Workshop on Science for Conservation & Preservation of Cultural Heritage Research & Education*, Wydział Chemii Uniwersytetu Wrocławskiego, Wrocław 4-5.06.2007

7. Secretary, XVIII Max Born Symposium *Statistical Physics Outside Pure Physics*, Lądek-Zdrój 22-25.09.2003

8. Secretary, 36th Winter School in Theoretical Physics *Exotic Statistical Physics*, Lądek-Zdrój 11-19.02.2000

9. Secretary, XI Max Born Symposium *Anomalous Diffusion: from Basis to Applications*, Lądek-Zdrój 20-27.05.1998

3.5.8 Other organizational activities

1. Secretary of the Polish Physical Society, Section of Physics in Economy and Social Sciences (2014-2016)

2. Secretary of habilitation commitee for dr hab. Grzegorz Pawlik, PWr (2014)

3. Secretary of habilitation commitee for dr hab. Dariusz Grech, UWr (2013)

4. Co-advisor (with prof. A. Mituś) of the Society of Physics Students *Nabla* (since 2013)

5. Leader for the teaching of physics at the Faculty of Electrical Engineering PWr (since 2013)

6. Head of the Disciplinary Board for academic teachers at the Department of Physics and Astronomy UWr (2012-2013)

7. Member of the faculty team for the quality of education at the Department of Physics and Astronomy, UWr (2011-2013)

3.6 Information about awards

2011 Medal of the Commission of National Education, Poland

2010 Rector Award for teaching and organization achievements, UWr

2007 International Young-Scientist Award for Socio- and Econophysics, German Physical Society

2006 Rector Award for teaching and organization achievements, UWr

2005 Rector Award for scientific achievements, UWr

2003 Scholarship of the Foundation for Polish Science (FNP) for young researchers

2002 Scholarship of the Foundation for Polish Science (FNP) for young researchers

2002 Award of the Ministry of National Education (MEN) for a series of papers on *applications of computer simulation methods, Monte Carlo methods, in biological evolution and surface adsorption of stiff bars*

2000 Rector Award for teaching, UWr

1999 Award of MEN for a series of papers on *applications of statistical physics in biology and sociology*

1997 Award of MEN for a series of papers on *modeling phenomena in condensed matter and biological systems*

1995 2nd award for M.Sc thesis in physics, Polish Physical Society

Chapter 4

Information about the most important scientific achievement

In my opinion, the most important scientific achievement after habilitation is the following series of ten papers, generally initiated exclusively by my ideas and published with my students (M. Tabiszewski, R. Topolnicki, K. Suszczyński) or my Ph.D. students (S. Krupa, P. Przybyła, B. Skorupa, P. Nyczka). They all concern and study the same problem – the sensitivity of macroscopic properties of non-equilibrium spin models to details introduced at the microscopic level:

[P0] K. Sznajd-Weron, S. Krupa, *Inflow versus outflow zero-temperature dynamics in one dimension*, Phys. Rev. E 74, 031109 (2006).[1]

[P1] F. Slanina, K. Sznajd-Weron, P. Przybyła, *Some new results on one-dimensional outflow dynamics*, Europhys. Lett. 82, 18006 (2008).

[P2] K. Sznajd-Weron, *Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode*, Phys. Rev. E 82, 031120 (2010).

[P3] K. Sznajd-Weron, M. Tabiszewski, A. Timpanaro, *Phase transition in the Sznajd model with independence*, Europhys. Lett. 96, 48002 (2011).

[P4] P. Przybyła, K. Sznajd-Weron and M. Tabiszewski, *Exit probability in a one-dimensional nonlinear q-voter model*, Phys. Rev. E 84, 031117 (2011).

[P5] B. Skorupa, K. Sznajd-Weron, R. Topolnicki, *Phase diagram for a zero-temperature Glauber dynamics under partially synchronous update*, Phys. Rev. E 86, 051113 (2012).

[P6] P. Nyczka, K. Sznajd-Weron, J. Cislo, *Phase transitions in the q-voter model with two types of stochastic driving*, Phys. Rev. E 86, 011105 (2012).

[1]Although this paper was published before I formally obtained my habilitation, I include it here for two reasons. Firstly, I submitted my habilitation dissertation in September 2005 and started working on this paper only in the beginning of 2006. Secondly, it initiated a whole series of papers.

[P7] P. Nyczka, K. Sznajd-Weron, *Anticonformity or Independence? – Insights from Statistical Physics*, Journal of Statistical Physics 151, 174-202 (2013).

[P8] K. Sznajd-Weron, K. Suszczyński, *Nonlinear q-voter model with deadlocks on the Watts-Strogatz graph*, J. Stat. Mech. P07018 (2014).

[P9] K. Sznajd-Weron, J. Szwabiński, R. Weron, *Is the Person-Situation Debate Important for Agent-Based Modeling and Vice-Versa?* PLoS ONE 9(11), e112203 (2014).

As I have already written in Section 2.1.3, one of the main problems in the field of social simulations is, as noted by Macy and Willer[2], little effort to provide analysis of how results differ depending on the model designs. This is a particularly important problem, since social simulations are often treated as a substitute of the social experiment. Honestly speaking, a similar problem is present also in sociophysics. Therefore, I have found it important to clarify at least some of the issues. In this series of papers, I have asked and tried to answer several questions that can be combined into a single general question: *How do, sometimes seemingly minor, differences introduced at the microscopic level of the model, manifest at the macroscopic scale?* In particular I have focused on the following:

- Is the outflow dynamics equivalent to the inflow dynamics [P0,P1]?
- What is the role of updating in spin dynamics [P0,P2,P5]?
- How do the stationary properties of the model (e.g. the absorbing state, the phase diagram) depend on the initial conditions and the size of the influence group [P1,P3,P4,P6,P8]?
- How do different types of non-conformity (anti-conformity or independence; which can be treated as noise), introduced at the microscopic level (individuals), manifest at the macroscopic level (the society) [P3,P6,P7]?
- Do the modeling assumptions regarding the type of agent's response to the influence (personal traits vs. situation) have substantial impact on the macroscopic behavior of the system or not [P9]?

Addressing these questions, I have been able to obtain several interesting theoretical results regarding first passage properties and phase transitions for the zero-temperature generalized Glauber dynamics and the q-voter model (for details see Section 2.1.3). Therefore, I believe that this series of papers will have impact not only on the interdisciplinary field of complex systems, and in particular on agent-based modeling, but will also contribute to the developing field of non-equilibrium statistical physics.

[2] M. W. Macy, R. Willer, *From factors to actors: computational sociology and agent-based modeling.*, Annu. Rev. Sociol. 28, 143–166 (2002).

Inflow versus outflow zero-temperature dynamics in one dimension

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It has been suggested that Glauber (inflow) and Sznajd (outflow) zero-temperature dynamics for the one-dimensional Ising ferromagnet with nearest-neighbor interactions are equivalent. Here we compare the two dynamics from the analytical and computational points of view. We use the method of mapping an Ising spin system onto the dimer RSA model and show that already this simple mapping allows us to see the differences between inflow and outflow zero-temperature dynamics. Then we investigate both dynamics with synchronous, partially synchronous, and random sequential updating using the Monte Carlo technique and compare both dynamics in the number of persistent spins, clusters, mean relaxation time, and relaxation time distribution.

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I. INTRODUCTION

The majority of natural phenomena observed in physics, biology, geology, social sciences, etc., are nonequilibrium processes. Unfortunately, the theory of nonequilibrium statistical mechanics is far less developed than its equilibrium counterpart. As a result, these ubiquitous phenomena are poorly understood [1]. The zero-temperature dynamics of simple models such as Ising ferromagnets provides interesting examples of nonequilibrium dynamical systems with many attractors (absorbing configurations, blocked configurations, zero-temperature metastable states) [2]. In this paper we focus on so-called single-spin-flip dynamics for the one-dimensional Ising ferromagnet. The best-known example of such dynamics for the Ising model is Glauber dynamics [3]. It can be viewed as "inflow" dynamics, since the center spin is influenced by its nearest neighbors [4]. Another type of dynamics, which can be called "outflow" dynamics, since the information flows from the center spin (or spins) to the neighborhood, has been introduced to describe opinion formation in social systems [5]. It has been suggested [6,7] that both dynamics for an Ising ferromagnet with nearest-neighbor interactions are equivalent, at least in one dimension. However, this seems to be true only in some particular cases. The aim of this paper is to compare generalized outflow and inflow dynamics for a chain of Ising spins and show in which cases these are equivalent and in which they differ.

It should be noticed here that the models studied in this paper are closely related to the majority-rule (MR) model introduced by Krapivsky and Redner [8]. In the MR model a selected group of G spins adapts to the state of the local majority and eventually the system reaches consensus (all spins up or all spins down). Interestingly, a system described by the MR model and an Ising spin system with outflow dynamics and random sequential updating (known as the Sznajd model) behave similarly in some aspects. For instance, the exit probability has almost the same, nontrivial dependence on the initial magnetization [4,8], in contrast to the linear voter model [8,9,11].

However, both inflow and outflow dynamics with random sequential updating and the MR model belong to a very general class of voter models (VMs). Following Liggett [9,10], VM models are continuous-time Markov processes, which are described by specifying the rates at which the system changes from one configuration to another. Changes are generally local, in that only several sites change state at any given time, and the rates for such transitions depend on the configurations near those sites. The inflow dynamics has already been reformulated in terms of a linear voter model, which is exactly soluble [9,13]. Probably the outflow dynamics could be reformulated in terms of a nonlinear voter model. Unfortunately, except for the linear voter model case and some very special cases of nonlinear voter models, the exact symmetry of voter models places them beyond the reach of currently available techniques for rigorous mathematical analysis, but at least some Monte Carlo simulation results are known [12]. Thus, it would be interesting to reformulate the outflow dynamics in terms of a nonlinear voter model and check if this is one of the few fortunate solvable cases or at least if there are some Monte Carlo results for related voter models. Although this is an interesting and important task we leave it for future work and concentrate here rather on comparisons of both dynamics under various updating schemes.

In Secs. II and III we recall ideas of inflow and outflow dynamics and formulate the generalized versions of both dynamics. We take both dynamics under a common roof, reformulating them without using the concept of energy. In Sec. IV we use the illuminating method of mapping the Ising spin system onto the dimer RSA model and make simple mean-field-like calculations to show the difference between the dynamics. In Sec. V we present Monte Carlo results for both dynamics with several kinds of updating including synchronous, partially synchronous, and random sequential updating. The summary and conclusions are the subjects of Sec. VI of the paper.

II. INFLOW DYNAMICS

The best-known example of such dynamics for the Ising model is Glauber dynamics. Within Glauber dynamics, in a broad sense, each spin is flipped $S_i(\tau)\rightarrow -S_i(\tau+1)$ with a rate

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$W(\delta E)$ per unit time and this rate is assumed to depend only on the energy difference implied in the flip [2]. The two most usual choices of flipping rates in the case of discrete updates are the heat-bath and Metropolis algorithms; both obey the detailed balance condition

$$\frac{W(\delta E)}{W(-\delta E)} = \exp(-\beta\delta E). \tag{1}$$

Recently it was shown [2] that there is a vast family of dynamical rates, besides these two choices, which obeys the condition (1). Among them the class of zero-temperature dynamics defined as

$$W(\delta E) = \begin{cases} 1 & \text{if } \delta E < 0, \\ W_0 & \text{if } \delta E = 0, \\ 0 & \text{if } \delta E > 0, \end{cases} \tag{2}$$

is very interesting. The zero-temperature limits of the heat-bath and Metropolis rates are, respectively, $W_0^{HB}=1/2$ and $W_0^{M}=1$. For any nonzero value of the rate W_0 corresponding to free spins, the dynamics belongs to the universality class of the zero-temperature Glauber model. This is a prototypical example of phase ordering by domain growth (coarsening). The typical size of ordered domains of consecutive $\uparrow$ and $\downarrow$ spins grows as $L(t)\sim t^{1/2}$. The particular value $W_0=0$ corresponds to the constrained zero-temperature Glauber dynamics ([2] and references therein). In the constrained zero-temperature Glauber dynamics, the only possible moves are flips of isolated spins and the system therefore eventually reaches a blocked configuration, where there is no isolated spin [2]. Very interesting results for the zero-temperature Glauber dynamics have also been obtained using computer simulations [14–17].

In out-of-equilibrium systems, there is usually no energy function and the system is only defined by its dynamical rules [18]. This is also the case of the sociophysics Sznajd model. For this reason we reformulate the definition of the zero-temperature Glauber dynamics for the Ising ferromagnet without using the concept of energy in the following way:

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } \sum_{NN} S_{NN} > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } \sum_{NN} S_{NN} = 0, \\ -1 & \text{if } \sum_{NN} S_{NN} < 0, \end{cases} \tag{3}$$

where $\Sigma_{NN}S_{NN}$ denotes the sum over nearest neighbors.

In one dimension, which is the case of this paper, the above definition can be obviously written as

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} = 0, \\ -1 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} < 0. \end{cases} \tag{4}$$

III. OUTFLOW DYNAMICS

Outflow dynamics was introduced to describe opinion change in a society. The idea is based on the fundamental social phenomenon called "social validation." However, in this paper we do not focus on social applications of the model (for interested readers, reviews can be found in [19–22]). On the contrary, here we investigate the dynamics from the theoretical point of view.

In the original model a pair of neighboring spins S_i and S_{i+1} was chosen and if $S_iS_{i+1}=1$ the two neighbors of the pair followed its direction, i.e., $S_{i-1}\rightarrow S_i$ $(=S_{i+1})$ and $S_{i+2}\rightarrow S_i$ $(=S_{i+1})$. Such a rule was also used in all later papers dealing with the one-dimensional case of the model. However, the case in which $S_iS_{i+1}=-1$ was noted as far less obvious. For example, in the original paper in the case of $S_iS_{i+1}=-1$, $S_{i-1}\rightarrow S_{i+1}$ and $S_{i+2}\rightarrow S_i$. However, it was noticed in several papers that such a rule is unrealistic in a model trying to represent the behavior of a community. Moreover, it can be seen from the following two rules that the original Sznajd model with both ferromagnetic and antiferomagnetic rules is equivalent to the single simple rule that every spin takes the direction of its next-nearest neighbor independently of the product S_iS_{i+1}.

Ferromagnetic rule. If $S_i(\tau)S_{i+1}(\tau)=1$ then $S_{i-1}(\tau+1)\rightarrow S_i(\tau)$ and $S_{i+2}(\tau+1)\rightarrow S_{i+1}(\tau)$ is equivalent to the rule: that if $S_i(\tau)S_{i+1}(\tau)=1$ then $S_{i-1}(\tau+1)\rightarrow S_{i+1}(\tau)$ if $S_{i+2}(\tau+1)\rightarrow S_i(\tau)$.

Antiferromagnetic rule. If $S_i(\tau)S_{i+1}(\tau)=-1$ then $S_{i-1}(\tau+1)\rightarrow S_{i+1}(\tau)$ and $S_{i+2}(\tau+1)\rightarrow S_i(\tau)$.

Thus, the two rules above can be rewritten as a simple single rule: $S_{i-1}(\tau+1)\rightarrow S_{i+1}(\tau)$ and $S_{i+2}(\tau+1)\rightarrow S_i(\tau)$.

In later papers we proposed two modifications of the model in which the antiferromagnetic rule was replaced by one of rules described below.

Modification 1. If $S_i(\tau)S_{i+1}(\tau)=-1$, then $S_{i-1}(\tau+1)\rightarrow S_{i-1}(\tau)$ and $S_{i+2}(\tau+1)\rightarrow S_{i+2}(\tau)$.

Modification 2. If $S_i(\tau)S_{i+1}(\tau)=-1$, then $S_{i-1}(\tau+1)\rightarrow -S_{i-1}(\tau)$ and $S_{i+2}(\tau+1)\rightarrow -S_{i+2}(\tau)$ with probability 1/2.

The generalized dynamics including the two modifications above, can be written as

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } S_{i+1}(\tau)+S_{i+2}(\tau) > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } S_{i+1}(\tau)+S_{i+2}(\tau) = 0, \\ -1 & \text{if } S_{i+1}(\tau)+S_{i+2}(\tau) < 0. \end{cases} \tag{5}$$

It is easy to see that modification 1 corresponds to $W_0^1 = 0$ and modification 2 to $W_0^2 = 1/2$.

IV. MAPPING ONTO THE DIMER MODEL

As was mentioned in previous sections, for $W_0=0$ the system under inflow (Glauber) dynamics described by the formula (4) eventually reaches a blocked configuration, where there is no isolated spin. On the other hand the system under outflow dynamics described by (5) always reaches a ferromagnetic steady state. Thus, for $W_0=0$ the difference between outflow and inflow dynamics is obvious. Nevertheless, within the mean-field approach [23] and Galam's unifying frame [6] both dynamics are equivalent, i.e., there is no difference between outflow and inflow dynamics, even for $W_0=0$.

Here we use the illuminating method of mapping the Ising spin system onto the dimer RSA model, which has already been done for the inflow dynamics [2]:

$$X_i = S_i S_{i+1} = 1 \Rightarrow \circ,$$

$$X_i = S_i S_{i+1} = -1 \Rightarrow \bullet. \tag{6}$$

In the case of inflow dynamics the following transitions, which change the state of the system, are possible:

spins	particles
$\downarrow\uparrow\downarrow \to \downarrow\downarrow\downarrow$	$\bullet\bullet \to \circ\circ$
$\uparrow\downarrow\uparrow \to \uparrow\uparrow\uparrow$	$\bullet\bullet \to \circ\circ$
$\downarrow\uparrow\uparrow \xrightarrow{W_0} \downarrow\downarrow\uparrow$	$\bullet\circ \xrightarrow{W_0} \circ\bullet$
$\uparrow\downarrow\downarrow \xrightarrow{W_0} \uparrow\uparrow\downarrow$	$\bullet\circ \xrightarrow{W_0} \circ\bullet$
$\downarrow\downarrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\uparrow$	$\circ\bullet \xrightarrow{W_0} \bullet\circ$
$\uparrow\uparrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\downarrow$	$\circ\bullet \xrightarrow{W_0} \bullet\circ$

Thus, after mapping there are only two types of transitions for inflow dynamics: $\bullet\bullet \to \circ\circ$ and $\circ\bullet \overset{W_0}{\leftrightarrow} \bullet\circ$. This mapping shows at once that for $W_0=0$ the dynamics is fully irreversible, in the sense that each spin flips at most once during the whole history of the sample.

It should be noticed that if we map the system under outflow dynamics onto the dimer model we have to take into account four particles, because changing the border spin influences the next particle. In this case four types of transitions are possible: $\circ\bullet\circ \to \circ\circ\bullet$, $\circ\bullet\bullet \to \circ\circ\circ$, $\bullet\circ\bullet \overset{W_0}{\leftrightarrow} \bullet\bullet\circ$, and $\bullet\bullet\bullet \overset{W_0}{\leftrightarrow} \bullet\circ\circ$ (to make it more clear the flipped spins are denoted by double arrows in the table below):

spins	particles
$\downarrow\downarrow\Uparrow\uparrow \to \downarrow\downarrow\Downarrow\uparrow$	$\circ\bullet\circ \to \circ\circ\bullet$
$\uparrow\uparrow\Downarrow\downarrow \to \uparrow\uparrow\Uparrow\downarrow$	$\circ\bullet\circ \to \circ\circ\bullet$
$\downarrow\downarrow\Uparrow\downarrow \to \downarrow\downarrow\Downarrow\downarrow$	$\circ\bullet\bullet \to \circ\circ\circ$
$\uparrow\uparrow\Downarrow\uparrow \to \uparrow\uparrow\Uparrow\uparrow$	$\circ\bullet\bullet \to \circ\circ\circ$
$\downarrow\uparrow\Uparrow\downarrow \xrightarrow{W_0} \downarrow\uparrow\Downarrow\downarrow$	$\bullet\circ\bullet \xrightarrow{W_0} \bullet\bullet\circ$
$\uparrow\downarrow\Downarrow\uparrow \xrightarrow{W_0} \uparrow\downarrow\Uparrow\uparrow$	$\bullet\circ\bullet \xrightarrow{W_0} \bullet\bullet\circ$
$\uparrow\downarrow\Uparrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\Downarrow\downarrow$	$\bullet\bullet\bullet \xrightarrow{W_0} \bullet\circ\circ$
$\downarrow\uparrow\Downarrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\Uparrow\uparrow$	$\bullet\bullet\bullet \xrightarrow{W_0} \bullet\circ\circ$
$\downarrow\uparrow\Downarrow\downarrow \xrightarrow{W_0} \downarrow\uparrow\Uparrow\downarrow$	$\bullet\bullet\circ \xrightarrow{W_0} \bullet\circ\bullet$
$\uparrow\downarrow\Uparrow\uparrow \xrightarrow{W_0} \uparrow\downarrow\Downarrow\uparrow$	$\bullet\bullet\circ \xrightarrow{W_0} \bullet\circ\bullet$
$\uparrow\downarrow\Downarrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\Uparrow\downarrow$	$\bullet\circ\circ \xrightarrow{W_0} \bullet\bullet\bullet$
$\downarrow\uparrow\Uparrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\Downarrow\uparrow$	$\bullet\circ\circ \xrightarrow{W_0} \bullet\bullet\bullet$

This mapping shows that for $W_0=0$ the outflow dynamics consists of two processes—diffusion of $\bullet$ particles in the sea of $\circ$ and annihilation of $\bullet\bullet$ pairs. Thus our model for W_0 with random sequential updating reduces to the analytically solvable reaction-diffusion system $A+A \to 0$ (denoting the empty place by $\circ$ and the A particle by $\bullet$).

For $W_0 \geq 0$ we can also easily use the mean-field approach (MFA). Mean-field results for the outflow dynamics without dimer mapping can be found in [23]. Within dimer mapping we take into account correlations between pairs of nearest neighbors. Thus if we apply the MFA to the mapped system we expect more correct results than are obtained within the MFA without mapping.

Let us denote the number of $\bullet$ particles by N_b and $\frac{N_b}{N}=b$.

In our case, in one time step τ, only two events are possible—the number of $\bullet$ particles decreases by $2/N$ with probability $\gamma(b)$ or remains constant.

For the inflow dynamics

$$\gamma^{in}(b) = b^2 \tag{7}$$

and for the outflow dynamics

$$\gamma^{out}(b) = (1-b)b^2 + W_0 b^3 = b^2[1 - b(1-W_0)]. \tag{8}$$

It is seen that the above results are not precise, since there is no dependence between $\gamma^{in}(b)$ and W_0 for the inflow dynamics and the only stable steady state in this case is $b=0$, i.e., the ferromagnetic state, which is true as long as $W_0>0$. However, as has been noticed this result is not correct for $W_0=0$. The same results can be obtained using the mean-field approach without mapping.

However, for the outflow dynamics the MFA with dimer mapping gives better results than the basic MFA presented in [23]. This is understandable, because in this case pairs of neighboring spins cause the changes in the system.

For $W_0=0$ there are two steady states, $b=0$, i.e., the ferromagnetic state, and $b=1$, i.e., the antiferromagnetic state. For $b\neq 0$ and $b\neq 1$ $\gamma^{out}(b)>0$ which implies that $b=0$ is an unstable steady state, while $b=1$ is a stable steady state. This result is in agreement with computer simulations [5]. For $W_0=1$ there is only one ferromagnetic steady state, which is also confirmed by the computer simulations [4].

As we see the differences between outflow and inflow dynamics are already seen if we apply the mean-field approach with mapping of the pairs of spins into single particles. In the next section we present simulation results which show more differences between these two dynamics.

V. SIMULATION RESULTS

The spin updating within both dynamics can be sequential or parallel. Within the parallel (or in other words synchronous) updating the system state at time step $t+1$ is given by its state at time step t. At every time step t we go systemically through the whole lattice and change spins according to the appropriate rule. In the random sequential (or in other words asynchronous) updating in each time step only one spin is selected at random and it adapts to its neighborhood. One Monte Carlo step (MCS) in this case consists of N time steps, while in the case of parallel updating one MCS is equivalent to a single time step.

In this paper we compare both dynamics for random sequential updating, parallel updating, and partially parallel updating. From now on we call the last case c-parallel updating. Within this updating the randomly chosen fraction c of spins is updated synchronously. Of course $c=1$ corresponds to parallel updating and $c=0$ to random sequential updating.

A. The number of persistent spins

One of the main quantities of interest in the nonequilibrium dynamics of spin systems at zero temperature is the fraction of spins $P(t)$ that persist in the same state up to some later time $t=N\tau$ (i.e., measured in Monte Carlo steps) [24,25]. In this paper we measure the fraction of persistent spins for both outflow and inflow dynamics with c-parallel updating for different values of c. The initial configuration consists of a randomly distributed fraction $p_+(0)$ of up spins. The number of persistent spins for the outflow dynamics with $W_0=0$ and random sequential updating has already been investigated by Stauffer and Oliveira [26] and found to agree with results for the inflow dynamics, i.e., decays with time t as $1/t^{-3/8}$. However, it was found that in higher dimensions the exponents for inflow and outflow dynamics are different [26]. Here we investigate the case of the Ising spin chain

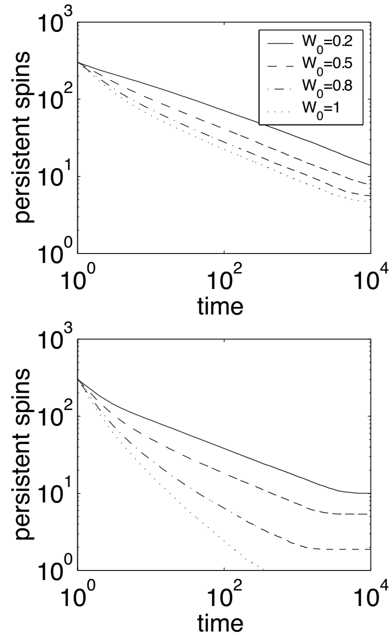

FIG. 1. The change in time of the number of persistent spins on the chain $N=300$ for random sequential updating (i.e., $c=0$) for the inflow dynamics (upper panel) and outflow dynamics (lower panel).

more carefully, i.e., for different values of W_0 and c.

The first difference between inflow and outflow dynamics is already seen for random sequential updating, i.e., $c=0$. For both dynamics the number of persistent spins decays initially as a power law $\sim t^{-\theta}$. However, for inflow dynamics the exponent is independent of W_0 until $W_0>0$, while for outflow dynamics the exponent is W_0 dependent, $\theta=\theta(c)$ (see Fig. 1). Moreover, for inflow dynamics the power law describes properly the decay of the number of persistent spins for the whole range of time, while within the outflow dynamics it is valid only for t smaller than a certain value of time $t^*(W_0)$ dependent on the flipping probability W_0. For $W_0\rightarrow 0$ we obtain $t^*(W_0)\rightarrow\infty$ and the evolution of the number of persistent spins is the same for outflow and inflow dynamics in agreement with the results obtained by Stauffer and Oliveira [26].

More differences are seen for partially synchronous updating with $c>0$. At each elementary time step τ the fraction c of spins is chosen randomly and the chosen group is changed synchronously. In such a case we have noticed that the number of persistent spins still decays as a power law for the inflow dynamics. However, for the outflow dynamics the power law is no longer valid. The number of persistent spins decays very fast in this case (see Fig. 2).

We may conclude this subsection in the following – the number of persistent spins is c sensible only for the outflow dynamics. For $W_0>0$ and any value of c the number of persistent spins in the inflow dynamics is described by the power law with nearly the same exponent.

B. The number of clusters

Probably the most natural way to investigate the relaxation process of the consensus dynamics is to look at the

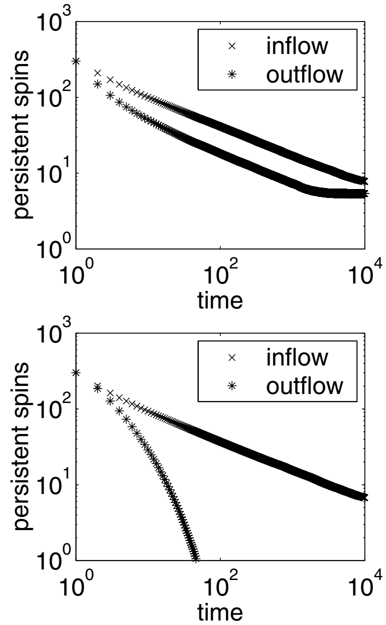

FIG. 2. The change in time of the number of persistent spins on the chain $N=300$ for partially synchronous updating for $W_0=1/2$. Upper panel presents results for $c=0$ and lower for $c=0.2$. It is seen that for $c>0$ (bottom case) the number of persistent spins decays very fast and cannot be described by a power law.

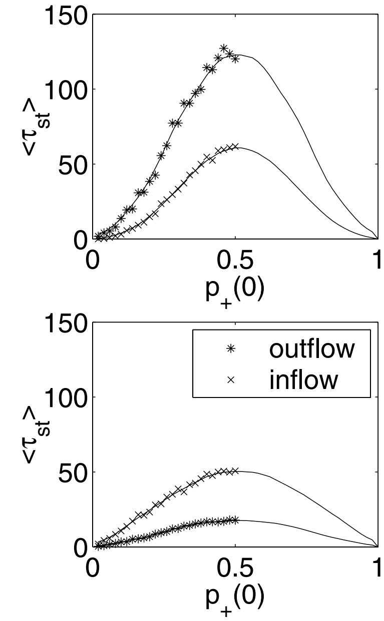

FIG. 3. The mean relaxation times from a random initial state consisting of $p_+(0)$ randomly distributed up spins for $W_0=0.2$. Upper panel corresponds to synchronous updating $c=1$, and bottom panel to $c=0.2$. It is seen that the relaxation under outflow dynamics is slower then under inflow for synchronous updating. On the contrary, the relaxation is much slower under the inflow dynamics than under the outflow dynamics for small c.

number of clusters (or number of domain walls between neighboring opposite spins) over time. A cluster consists of a group of spins, each of which is a nearest neighbor to at least one other spin in the cluster, with all spins having the same orientation. With such a definition consensus is reached when only one cluster is present in the system or when there is no domain wall. Because of the similarity of both dynamics (inflow and outflow) with random sequential updating to the VM, we expected, and have verified numerically, that the density of domain walls (as well as the number of clusters) decays as $t^{-1/2}$, analogously with the results for the MR model [8].

Moreover, for both inflow and outflow dynamics with c-parallel updating the number of clusters (and the number of domain walls) monotonically decays as $t^{-1/2}$ for any value of c. This result shows that the variation of the number of clusters in time, although is a very intuitive and natural measure of the relaxation, is not a good quantity for dynamics comparison.

C. The mean relaxation time

The differences between the dynamics can be observed clearly if we look at the mean relaxation time as a function of the initial fraction of randomly distributed up spins $p_+(0)$. Within 0-parallel updating (i.e., random sequential updating) the relaxation is much slower for the inflow dynamics than for the outflow dynamics. This is also true for the c-parallel updating with small c. On the contrary, within 1-parallel updating (i.e., synchronous updating) the relaxation under outflow dynamics is slower than under inflow (see Fig. 3).

In general, the relaxation time decays with W_0 growth, but the dependence between the mean relaxation time and W_0 is different for outflow and inflow dynamics. Two examples for $c=0.2$ and 0.5 for several values of W_0 are shown in Figs. 4 and 5, respectively. It can be noted that, for example, for $c=0.5$ and $W_0=0.8$ the dependence between the mean relaxation time and the initial concentration of up spins $p_+(0)$ is nearly the same.

In Fig. 6 we have presented the dependence between the mean relaxation times from a random initial state consisting of 50% randomly distributed up spins (maximal waiting time) and the flipping probability W_0 for the inflow and outflow dynamics. It is seen that the dependence on c is much stronger for the outflow dynamics. For the inflow dynamics the mean relaxation time is almost the same for all values of c. On the other hand for a given value of c the dependence between $\langle\tau\rangle$ and W_0 is stronger for the inflow dynamics.

D. The distribution of waiting times

In the paper [23] the mean-field approach for the outflow dynamics with $W_0=0$ was presented and the distribution of waiting times needed to reach the stationary state was found. Recall that for δ initial conditions the distribution of waiting times has an exponential tail [23]:

$$P_s t^{>}(\tau) \approx \frac{6}{4}(1-m_0^2)e^{-2\tau}, \quad \tau \to \infty . \tag{9}$$

Monte Carlo simulations confirm this prediction both on the complete graph and on the chain. In this paper we have

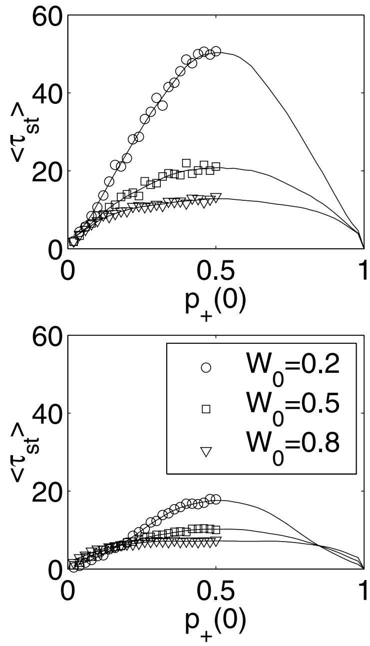

FIG. 4. The mean relaxation times from the random initial state consisting of $p_{+}(0)$ randomly distributed up spins for $c=0.2$ for the inflow (upper panel) and outflow (bottom panel) dynamics.

checked also the distribution of waiting times for different values of W_0 and c for both outflow and inflow dynamics. The distribution of waiting times has an exponential tail for any value of W_0 and c, although the exponent depends on these parameters. The example for $c=0$, showing comparison between inflow and outflow dynamics, is shown in Fig. 7.

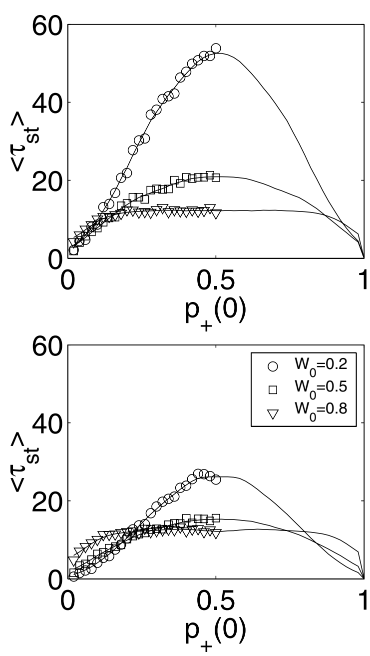

FIG. 5. The mean relaxation times from the random initial state consisting of $p_{+}(0)$ randomly distributed up spins for $c=0.5$ for the inflow (upper panel) and outflow (bottom panel) dynamics.

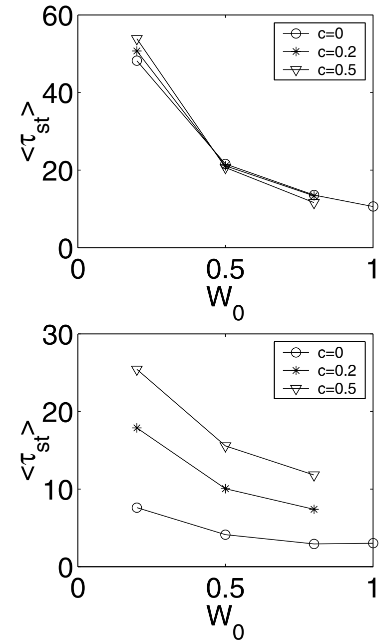

FIG. 6. The dependence between the mean relaxation times from the random initial state consisting of 50% randomly distributed up spins and the flipping probability W_0 for the inflow (upper panel) and outflow (bottom panel) dynamics for different values of c. It is seen that the dependence on c is much stronger for the outflow dynamics. On the other hand for a given value of c, the dependence between $\langle \tau \rangle$ and W_0 is stronger for the inflow dynamics.

VI. CONCLUSIONS

It has been suggested [6,7] that zero-temperature outflow and inflow dynamics for an Ising ferromagnet with nearest-

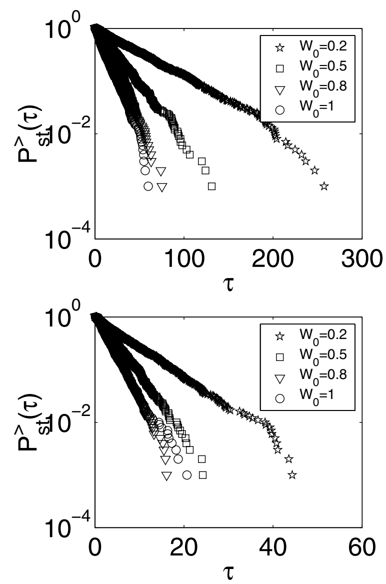

FIG. 7. Probabilities of reaching a steady state in time larger than τ on the chain $L=200$. The distribution of waiting times has an exponential tail for both dynamics—inflow (upper panel) and outflow (lower panel).

neighbor interactions are equivalent in one dimension. However, it is certainly not true for $W_0=0$. This particular value corresponds to the constrained zero-temperature Glauber dynamics where the only possible moves are flips of isolated spins and the system therefore eventually reaches a blocked configuration, where there is no isolated spin [2]. This can be also easily shown using the method of mapping the Ising spin system onto the RSA dimer model. On the other hand, the outflow dynamics leads to a ferromagnetic steady state for any value of W_0. This observation motivated us to compare both dynamics more carefully. We have made Monte Carlo simulations for both dynamics using random sequential updating, parallel updating, and c-parallel updating (a randomly chosen fraction c of spins is updated synchronously). We have measured, for different values of W_0 and c, the distribution of waiting times, the mean waiting time, the decay of the number of clusters, and the number of persistent spins in time.

A qualitative difference between inflow and outflow dynamics is not visible either in the number of clusters in time or in the distribution of waiting times. However, it should be noticed that the relaxation time is different for the two both dynamics. Nevertheless, for both dynamics the distribution of waiting times has an exponential tail and the number of clusters decays as $t^{-1/2}$ for any value of $W_0>0$ and c.

Differences between the dynamics appear if we look at the dependence between the mean relaxation time and the initial concentration of randomly distributed up spins for different values of W_0 and c. For $c=0$, which corresponds to random sequential updating, the mean relaxation time is shorter for the outflow dynamics (e.g., for $W_0=0.2$ and $p_0=0.5$ it is about ten times shorter) than for inflow. The mean relaxation time $\langle\tau\rangle$ decreases with W_0 growth for both dynamics, but the dependence between $\langle\tau\rangle$ and W_0 is different for outflow and inflow dynamics. Generally the mean relaxation time decays faster with growing W_0 for the inflow dynamics for any value of c. Moreover, with growing c the dependence between the mean relaxation time for the inflow dynamics and the outflow dynamics vanishes. As a result, for some values of c and W_0 (e.g., $c=0.5$ and $W_0=0.8$) the dependence between the mean relaxation times and the initial concentration of up spins is identical. Of course this suggests that for some values of parameters W_0 and c the relaxation under outflow dynamics is faster than under inflow dynamics. This is indeed true. In the case of $c=1$ (parallel updating), the relaxation is faster under the inflow dynamics for any value of W_0.

The second quantity that behaves differently for the two dynamics is the number of persistent spins in time. Mainly differences are seen for partially synchronous updating with $c>0$. In such a case we have noticed that the number of persistent spins decays as a power law for the inflow dynamics (as in the case of $c=0$). However, for the outflow dynamics the power law is no longer valid. The number of persistent spins decays very fast in this case.

Concluding, the inflow and outflow dynamics differ very clearly even in one dimension. There is an obvious, very strong difference for $W_0=0$, but also for $W_0>0$ the two dynamics are qualitatively different. In the case of random sequential updating the relaxation under outflow dynamics is much faster than under inflow dynamics. On the contrary in the case of parallel updating the outflow dynamics is much slower than the inflow.

In closing this paper we should mention that the outflow dynamics with $W_0=0$ with synchronous updating was investigated earlier and it was found that in such a case the possibility of reaching a consensus is reduced quite dramatically [27]. Also the number of persistent spins varies with c only for the outflow dynamics. For $W_0>0$ and any value of c the number of persistent spins in the inflow dynamics is described by a power law with nearly the same exponent.

Generally the outflow dynamics is much more influenced by the type of updating than the inflow dynamics. We believe that this result especially is very important in the various interdisciplinary applications of the zero-temperature single-spin-flip dynamics.

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Some new results on one-dimensional outflow dynamics

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Abstract – In this paper we introduce a modified version of the one-dimensional outflow dynamics in the spirit of the Sznajd model, which simplifies the analytical treatment. The equivalence between original and modified versions is demonstrated in simulations. Using the Kirkwood approximation, we obtain an analytical formula for the exit probability and we show that it agrees very well with computer simulations in the case of random initial conditions. Moreover, we compare our results with earlier analytical calculations obtained from the renormalization group method and from the general sequential probabilistic frame introduced by Galam and show that our result is superior to the others. Using computer simulations we investigate the time evolution of several correlation functions in order to check the validity of the Kirkwood approximation. Surprisingly, it turns out that the Kirkwood approximation gives correct results even for such initial conditions for which it cannot be easily justified.

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Introduction. – Opinion-dynamics models are among the most studied topics in the field of sociophysics [1]. The two-state, or "Ising-like" models have been used since the very beginning [2]. The interest in opinion dynamics was triggered by the works of Galam [3,4] and a large amount of works was produced, including the study of the voter [5,6], Sznajd [7] and majority rule [3,4,8] models. These models have two features in common. First, the complexity of real-world opinions is reduced to the minimum set of two options, $\sigma = +1$ or -1. Second, the individuals bearing these opinions are immobile; they are attached to the sites of a lattice, which may be linear chain, hypercubic lattice, random graph or any of other possibilities. The basic questions asked when studying these models are: what is the probability to reach consensus in opinions, say, all individuals having $\sigma = +1$? and what is the time necessary to reach such consensus?

More specifically, the Sznajd model can be characterised by the outflow dynamics. Contrary to the kinetic Ising model, the information does not spread from the neighbourhood of a chosen site towards that site but, conversely, from a cluster of sites to the neighbourhood of that cluster. In one dimension, the dynamics is defined as follows. If a pair of neighbours at sites x, $x+1$ agree in opinion, $\sigma(x) = \sigma(x+1)$, the two neighbours of the pair adopt the same opinion, *i.e.* $\sigma(x-1) \to \sigma(x)$ and $\sigma(x+2) \to \sigma(x)$. Otherwise the two neighbouring states are unchanged. In higher dimensions and on other lattices the rule is defined analogously.

By now, quite a few results have been accumulated (an interested reader may resort to reviews [1,9–12]). In this letter, we shall address the question: what is the probability P_+ that all of the individuals eventually reach consensus in state, say, +, provided that at the beginning the fraction of + opinions was p? This quantity is commonly called exit probability [13,14]. From the simulations [15], as well as from the exact solution on a complete graph [16] and a renormalisation-group calculation [17] it is known that it is a step function with discontinuity at $p = 0.5$, unless the lattice is a one-dimensional chain. In this case it is a continuous function [18]. Therefore, the one-dimensional case is singular and poses a problem of fundamental interest.

Several analytical approaches have been proposed. We have already mentioned the mean-field solution [16], which, however, is inapplicable in 1D. Later, Galam in [19] presented a general sequential probabilistic frame (GSPF), which extended a series of earlier opinion dynamics

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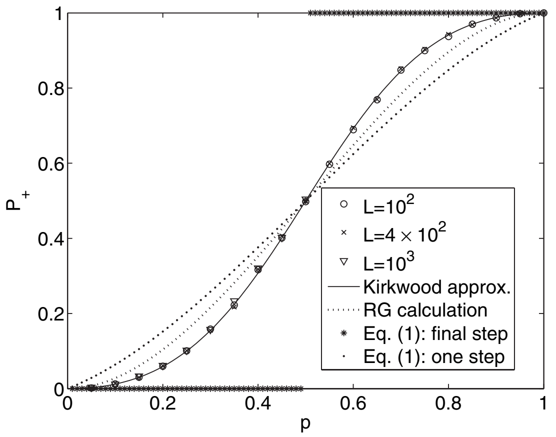

Fig. 1: Exit probability P_+ from the random initial state consisting of the fraction p of up-spins for the modified original outflow dynamics in one dimension for several lattice sizes L. It may be seen that results agree very well with the analytical formula given by eq. (22) obtained from the Kirkwood approximation (solid line). The renormalization group (RG) results obtained in [17] for growing networks and calculations made by Galam within his general sequential probabilistic frame (GSPF) given by eq. (1) agree with simulation results much worse. However, it should be noticed that a one-step update yields much more reasonable results than the final step function obtained by successive iterations of eq. (1). Results obtained for a modified version of the outflow dynamics in which only one neighbour of the central pair is changed are exactly the same. Results are averaged over 10^4 samples.

models. Within his frame, he was able to find analytic formulae for the probability $p(t+1)$ that a randomly chosen agent shares opinion + at time $t+1$ in terms of the same probability $p(t)$ at time t. Among several models, he considered also the same one-dimensional rule as we are about to study here and within the GSPF he found the following dynamical rule [19]:

$$p(t+1) = p(t)^4 + \frac{7}{2}p(t)^3[1-p(t)] + 3p(t)^2[1-p(t)]^2 + \frac{1}{2}p(t)[1-p(t)]^3. \quad (1)$$

Iterating this formula until the absorbing state is reached, one can find that the exit probability P_+ is a step function (see fig. 1).

In the paper [17] the real-space renormalization-group approach has been proposed to calculate the probability $P_+(p)$ for the outflow dynamics on a network. In the case of a growing network, either hierarchical or of Barabási-Albert type, the resulting formula was $P_+ = 3p^2 - 2p^3$, while in the case of a fixed network they have found that P_+ is a step function, just the same as for the complete graph [16].

In fig. 1 we can compare the exit probability obtained in our simulations of the 1D outflow dynamics with the results of Galam's GSPF and RG calculation of ref. [17] for growing networks. None of them are satisfactory. Note also that if we limited the process of Galam's GSPF to one iteration only, the agreement would be at least qualitatively correct.

So, we can see that currently there is no analytic argument which would satisfactorily explain the behaviour of the outflow dynamics in the Sznajd model in one dimension. Our intention is to fill this gap. In the rest of this paper we present analytical results obtained using the Kirkwood approximation following the method developed in [13] for the majority rule model. Anticipating the conclusions, we can see in fig. 1 that the agreement with simulations is very good.

Approximate solution in 1D. – We consider individuals having opinions represented as spins ± 1 occupying sites on a linear chain of length L. We use the following notation: $\sigma \in \{-1,+1\}^L$ denotes the state of the system and $\sigma(y)$ the state of the individual at site y if the system is in state σ. We also denote by σ^x the state which differs from σ by flipping the spin at site x. Therefore, $\sigma^x(y) = (1-2\delta_{xy})\sigma(y)$.

We introduce here a slight modification of the original outflow rule: we choose a pair of neighbours and if they both are in the same state, then we adjust one (instead of two) of its neighbours (chosen randomly with equal probability 1/2) to the common state. At most one spin is flipped in one step, while in the original formulation two can be flipped simultaneously. Therefore, the time must be rescaled by factor $\frac{1}{2}$. We measure the time so that the speed of all processes remains constant when $L \to \infty$, and thus normally one update takes time $\frac{1}{L}$. Here, instead, we consider also the factor $\frac{1}{2}$, so a single update takes time $\Delta t = \frac{1}{2L}$. Our modification eliminates some correlations due to simultaneous flip of opinions at distance 3. However, if we look at later stages of the evolution, where typically the domains are larger than 2, simultaneous flips occur very rarely. Therefore, we do not expect any substantial difference. Indeed, computer simulations confirm our expectations —only time has to be rescaled (see fig. 2).

On the other hand, the modification simplifies the analytical treatment. Indeed, the update rule can be equivalently formulated as follows: Choose randomly a spin x and side s ($s=1$ for right, $s=-1$ for left). The updated state is $\sigma(x;t+\Delta t) = \sigma(x+s;t)$ if $\sigma(x+s;t) = \sigma(x+2s;t)$, otherwise $\sigma(x;t+\Delta t) = \sigma(x;t)$.

Within such a formulation the probability of the flip $\sigma \to \sigma^x$ in one update is

$$W(\sigma \to \sigma^x) = \frac{1}{8L}[\sigma(x+2)\sigma(x+1) + \sigma(x-1)\sigma(x-2) - \sigma(x)(\sigma(x+2) + \sigma(x+1) + \sigma(x-1) + \sigma(x-2)) + 2]. \quad (2)$$

These flip probabilities are then inserted into the master equation:

$$P(\sigma; t+\Delta t) = \sum_{\sigma'} W(\sigma' \to \sigma) P(\sigma'; t), \quad (3)$$

fully describing the evolution of the system.

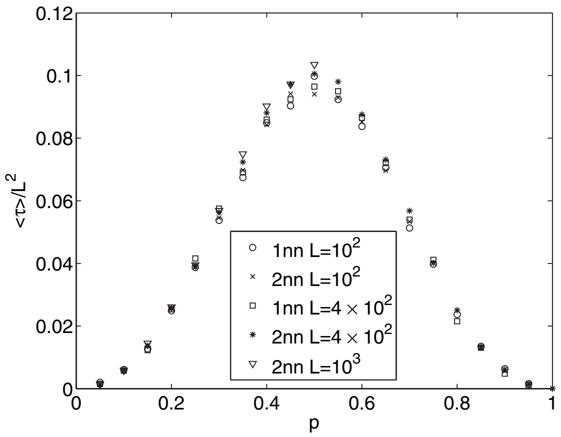

Fig. 2: The mean relaxation times from the random initial state consisting of p up-spins for the modified (1 nn) and original (2 nn) outflow dynamics in one dimension for several lattice sizes L. In the modified version at most one spin is flipped in one elementary step, while in original formulation two can be flipped simultaneously. Therefore, in the case of a modified version the time was rescaled by a factor $\frac{1}{2}$. It should be noticed that in computer simulations time is measured in Monte Carlo steps (MCS). As usual, one MCS consists of L elementary updating, *i.e.* in one MCS L times the pair of spins is randomly and independently selected with probability $1/L$, *i.e.* it may happen that one pair will be chosen several times in one MCS. Since we investigate the relaxation process we simulate the system as long as it reaches the final state with all spins up or down. The average number of MCSs needed to reach the final state depends on the initial concentration of up-spins and is proportional to L^2 analogously to the voter model [5,6,20]. The results presented here are averaged over 10^4 samples.

Now, we make the limit $L \to \infty$, which also implies the continuous time limit, as $\Delta t \to 0$. We also note that most of the transition probabilities $W(\sigma' \to \sigma)$ are zero, since only one spin flip is allowed in one step. Finally we end with

$$\frac{\mathrm{d}}{\mathrm{d}t} P(\sigma;t) = \sum_x \Big[w(\sigma^x \to \sigma) P(\sigma^x;t) - w(\sigma \to \sigma^x) P(\sigma;t) \Big], \tag{4}$$

where the transition rates are trivially related to transition probabilities (2) by a proportionality factor

$$w(\sigma^x \to \sigma) = 2NW(\sigma^x \to \sigma)\,. \tag{5}$$

(The sum is now over an infinite set of sites.)

It is hopeless to solve the master equation as it is. Instead, we write evolution equations for some correlation functions derived from it. We define:

$$\begin{aligned} C_0(t) &= \langle \sigma(y) \rangle \equiv \sum_\sigma \sigma(y) P(\sigma;t), \\ C_1(n;t) &= \langle \sigma(y)\sigma(y+n) \rangle, \\ C_2(n,m;t) &= \langle \sigma(y-n)\sigma(y)\sigma(y+m) \rangle, \\ C_3(n,m,l;t) &= \langle \sigma(y-n)\sigma(y)\sigma(y+m)\sigma(y+m+l) \rangle, \\ &\vdots \end{aligned} \tag{6}$$

Only two equations are relevant for us. The first is

$$\frac{\mathrm{d}}{\mathrm{d}t} C_0(t) = -C_2(1,1;t) + C_0(t) \tag{7}$$

and the second

$$\frac{\mathrm{d}}{\mathrm{d}t} C_1(1;t) = -C_3(1,1,1;t) - C_1(1;t) + C_1(3;t) + 1. \tag{8}$$

These two become a closed set of equations, if we apply the approximations described in the next section. Before going to it, it is perhaps instructive to show the intermediate results which lead to equations (7), (8), and analogically to others, for more complicated correlation functions.

Thus, for example, for the lowest correlation function —the average of one spin— we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \langle \sigma(y) \rangle = -2 \langle w(\sigma \to \sigma^y) \sigma(y) \rangle \tag{9}$$

and for the next one in the level of complexity

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \langle \sigma(y)\sigma(y+1) \rangle = &-2 \langle w(\sigma \to \sigma^y) \sigma(y)\sigma(y+1) \rangle \\ &-2 \langle w(\sigma \to \sigma^{y+1}) \sigma(y)\sigma(y+1) \rangle. \end{aligned} \tag{10}$$

The pattern is transparent. When computing the correlation function of spins at sites x_1, x_2, x_3, ..., on the RHS we have the sum of terms, in which we average the product of spins at sites x_1, x_2, x_3, ...with transition rate, which is derived from the spin configuration according to (2) and (5) for flip at positions x_1, x_2, x_3, ... As a formula, this sentence means

$$\frac{\mathrm{d}}{\mathrm{d}t} \left\langle \prod_i \sigma(x_i) \right\rangle = -2 \sum_j \left\langle w(\sigma \to \sigma^{x_j}) \prod_i \sigma(x_i) \right\rangle. \tag{11}$$

Kirkwood approximation. Now we shall discuss the approximations used for solving eqs. (7) and (8).

The first one is the usual Kirkwood approximation, or decoupling, which is used in various contexts and accordingly it assumes different names. For example in the classical quantum many-body theory of electrons and phonons in solids, it is nothing else than the Hartree-Fock approximation (but contrary to this theory, which may be improved systematically using diagrammatic techniques, here the systematic expansions are not developed). We use the name Kirkwood approximation, following the work [13].

In our case, the Kirkwood approximation amounts to

$$C_3(1,1,1;t) \simeq \big(C_1(1;t)\big)^2 \tag{12}$$

in eq. (8) and

$$C_2(1,1;t) \simeq C_1(1;t) C_0(t) \tag{13}$$

in eq. (7). While the latter assumption (13) enables us to relate equation (7) directly to (8) and therefore to

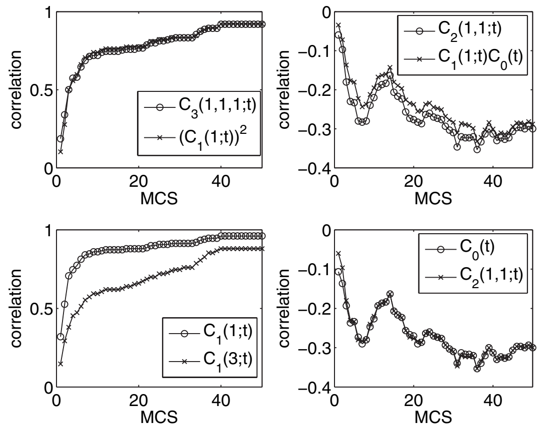

Fig. 3: Sample time evolution of several correlation functions given by eq. (6) for random initial conditions with fraction p of up-spins. The Kirkwood approximation given eqs. (12), (13) and assumption (14) are valid for later stages, although the assumption (12) agrees very well with simulation results from the very beginning (left upper panel). The data come from one single run (not averaged).

solve it as soon as we have the solution of (8), the approximation (12) does not make of (8) a closed equation yet. The point is that there is also the function $C_1(3;t)$ measuring the correlation at distance 3. So, we make also an additional approximation, which is also made in [13]. We suppose that $C_1(n;t)$ only weakly depends on the distance n, or else, that the decay of the correlations is relatively slow. If the spins are correlated to a certain extent on distance 1 (the neighbours), they are correlated to essentially the same extent also on distance 3 (next-next neighbours). This is also justified if the domains are large enough, *i.e.* at later stages of the evolution. So, we assume

$$C_1(3;t) \simeq C_1(1;t)\,. \tag{14}$$

In fig. 3 we present a sample (not averaged) time evolution of several correlation functions. The first assumption (12) agrees very well with simulation results from the very beginning and the second condition (13) agrees with simulations also quite well. On the other hand, the assumption (14) that the decay of the correlations is relatively slow is valid only at later stages of the evolution.

To sum it up, the approximations (12), (13), and (14) say that approximately

$$\begin{aligned} C_0(t) &\simeq \psi(t),\\ C_1(n;t) &\simeq \phi(t), \end{aligned} \tag{15}$$

where $\psi(t)$ and $\phi(t)$ satisfy the equations (the dot denotes the time derivative)

$$\begin{aligned} \dot{\psi} &= (1-\phi)\psi,\\ \dot{\phi} &= 1-\phi^2\,. \end{aligned} \tag{16}$$

The solution is straightforward. We assume initial conditions $\phi(0)=m_1$ and $\psi(0)=m_0$. First we solve the second equation from the set (16). This gives

$$\phi(t)=\frac{\sinh t + m_1\cosh t}{\cosh t + m_1 \sinh t} \tag{17}$$

and inserting that into the first of the set (16) we have

$$\psi(t)=\frac{2m_0}{1+m_1+(1-m_1)\,\mathrm{e}^{-2t}}\,. \tag{18}$$

The most important result is the asymptotics

$$\psi(\infty)=\frac{2m_0}{1+m_1}\,. \tag{19}$$

How to interpret this finding? The average $C_0(t)$ is the average magnetisation. In other terms, it determines the probability that a randomly chosen spin will have state $+1$ at time t. This probability is $p_+(t)=(C_0(t)+1)/2$. Therefore, $m_0=C_0(0)$ is the initial magnetisation. When we go to the limit $t\to\infty$, we know that ultimately the homogeneous state is reached. The asymptotic magnetisation $C_0(\infty)$ therefore says what the probability that the final state will have all spins $+1$ is. It is $(C_0(\infty)+1)/2$. So, (19) means that

$$C_0(\infty)\simeq\frac{2C_0(0)}{1+C_1(1;0)}\,. \tag{20}$$

If the initial state is completely uncorrelated, *i.e.* we set the spins at random, with the only condition that the average magnetisation is m_0, we have $C_1(1;0)=m_0^2$ and

$$C_0(\infty)\simeq\frac{2m_0}{1+m_0^2}\,. \tag{21}$$

Finally, we express this result in terms of the probability $p=(C_0(0)+1)/2$ to have a randomly chosen spin in state $+1$ at the beginning and the probability $P_+=(C_0(\infty)+1)/2$ that all spins are in state $+1$ at the end. We have

$$P_+\simeq\frac{p^2}{2p^2-2p+1}\,. \tag{22}$$

Computer simulations for random initial conditions, in which assumption $C_1(1;0)=m_0^2$ can be done, show very good agreement with analytical formula (22). In the next section we show how the results will change if we allow correlations in the initial conditions.

Correlated initial conditions. – Here we consider two examples of correlated initial conditions with the fraction p of up-spins:

1) Ordered initial state that consists of two clusters: pL-length of up-spins and $(1-p)L$-length of down-spins, for example in case of $L=10$:

$$\begin{aligned} p=0.5 &: \uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\\ p=0.4 &: \uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\\ p=0.3 &: \uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\\ &\dots\,. \end{aligned} \tag{23}$$

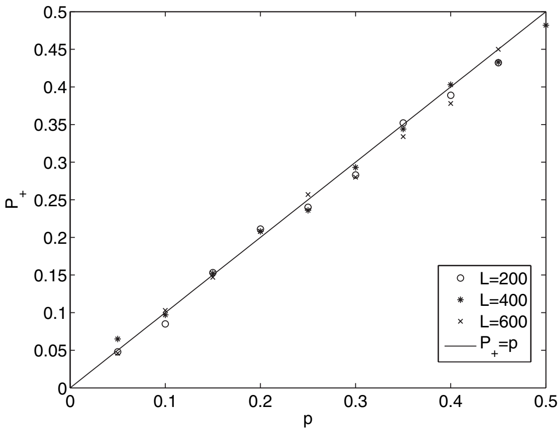

Fig. 4: Exit probability P_+ from the ordered initial state consisting of the fraction p of up-spins for the outflow dynamics in one dimension for several lattice sizes L. The initial state consists of two clusters: pL-length of up-spins and $(1-p)L$-length of down-spins. The results for original and modified rules are the same. The dependence between the initial ratio of up-spins p and the exit probability is given by the simplest linear function $P_+ = p$ as in the case of the voter model. An analytical result in this case can be obtained from eq. (26). Results are averaged over 10^3 samples.

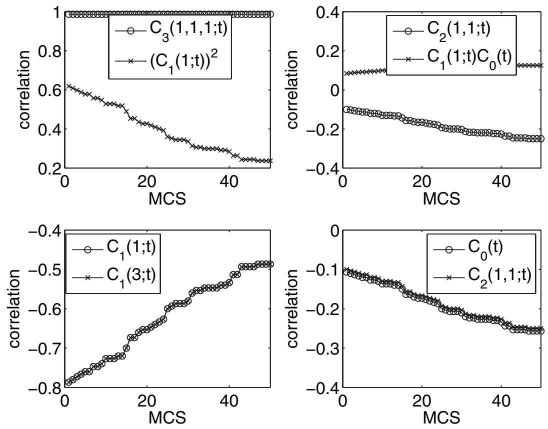

Fig. 5: Sample time evolution of several correlation functions given by eq. (6) for random ordered initial conditions with fraction p of up-spins. The initial state consists of two clusters: pL-length of up-spins and $(1-p)L$-length of down-spins. The Kirkwood approximation given eqs. (12) and (13) are not valid. The data come from one single run (not averaged).

2) Correlated, completely homogeneous, initial state. For such p that $1/p$ is an integer, we set $\sigma(n/p)=1$ for $n=0,1,2,3,\dots$ and $\sigma(x)=-1$ otherwise. For example, in the case of $L=8$:

$$\begin{aligned} p=0.5 &: \uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow \\ p=0.25 &: \downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow \\ &\dots . \end{aligned} \tag{24}$$

In both cases it is easy to calculate exactly the correlation function $C_1(1;0)$. In the first case of ordered initial conditions we obtain

$$C_1(1;0) = 1 - \frac{1}{L} \approx 1. \tag{25}$$

Thus, from eq. (20):

$$C_0(\infty) \simeq \frac{2C_0(0)}{1+C_1(1;0)} = \frac{2m_0}{1+1} = m_0 \Rightarrow P_+ = p. \tag{26}$$

Computer simulations show that indeed for such initial conditions $P_+ = p$ (see fig. 4).

As we can see, the Kirkwood approximation gives, surprisingly, correct results also in this case. At the same time, fig. 5 shows that eqs. (12) and (13) defining the Kirkwood approximation are not justified by computer simulations.

We have checked also the mean relaxation time in case of ordered initial conditions (fig. 6). It occurs that like for the random initial conditions the mean relaxation time scales with the system size as $\langle\tau\rangle \sim L^2$ (see figs. 2 and 6). The same scaling has been found in the voter model [5,6,20].

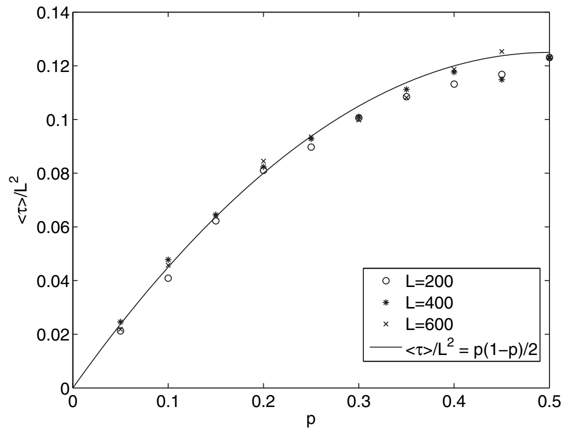

Fig. 6: The mean relaxation times for the outflow dynamics in one dimension for several lattice sizes L. The initial state consists of two clusters: pL-length of up-spins and $(1-p)L$-length of down-spins. The results for original and modified rules are the same. It is clearly visible that in the case of such an ordered initial state the dependence between the initial ratio of up-spins p and the mean relaxation time $\langle\tau\rangle$ is given by a simple parabola rather than by a bell-shaped curve. The data presented in the figure are averaged over 10^4 samples.

However, contrary to the random initial conditions for which a bell-shaped curve is observed, here the mean relaxation times is well described by simple parabola:

$$\frac{\langle\tau\rangle}{L^2} = \frac{1}{2}p(1-p). \tag{27}$$

It is quite easy to understand this result. In fact, in the initial condition there is only one domain wall, where $+1$ and -1 sites come into contact. During the evolution this domain wall performs a random walk and cannot disappear, unless it hits the left or right edge of the one-dimensional chain. The mean exit time for a random walker among two absorbing walls is well known and depends on the initial position of the walker, which is determined by the proportion p, exactly as indicated in formula (27). The same consideration of a random walker with two absorbing walls also explains the linear dependence of P_+ observed in fig. 4.

For the second case of correlated initial conditions, which are completely homogeneous, we observed in computer simulations that the exit probability is a step function with an unstable fixed point at $p = 0.5$, *i.e.*

$$\begin{aligned} P_+ &= 0, \quad \text{for } p < 0.5, \\ P_+ &= 1, \quad \text{for } p > 0.5, \\ \text{antiferromagnetic state}, &\quad \text{for } p = 0.5. \end{aligned} \tag{28}$$

In this case the two-spins correlation function can be also calculated easily. For $p = \frac{1}{n} < 0.5$, $n = 3, 4, \ldots, L$ we obtain

$$C_1(1;0) = p\left(1 \times \left(\frac{1}{p} - 2\right) + (-1) \times 2\right) = 1 - 4p. \tag{29}$$

Thus, from eq. (20)

$$C_0(\infty) \simeq \frac{2C_0(0)}{1 + C_1(1;0)} = \frac{4p - 2}{2 - 4p} = -1 \Rightarrow P_+ = 0, \tag{30}$$

which again agrees very well with computer simulations, although the Kirkwood approximation cannot be easily justified.

Conclusions. – We introduced a modified version of the one-dimensional outflow dynamics in which we choose a pair of neighbours and if they both are in the same state, then we adjust *one* (in the original version both) of its neighbours to the common state. We checked in computer simulations that in accord with our expectations the results in the case of a modified rule are the same as in the case of the original outflow dynamics, only the time must be rescaled by a factor $\frac{1}{2}$. In the modified version the analytical treatment is greatly simplified and allows to derive the master equation involving only single-flip events. Following the method proposed in [13] we wrote evolution equations for some correlation functions and used the Kirkwood approximation. This approach allowed us to derive the analytical formula for the final magnetisation and, equivalently, for the exit probability. In fact, just before finishing this paper, the same result was published by Lambiotte and Redner as a special case in the work [21] where a model interpolating the voter, the majority rule (or Sznajd) and the so-called vacillating voter dynamics was investigated, using also the Kirkwood approximation.

In the case of random initial conditions the Kirkwood approximation can be justified looking at the time evolution of simulated correlation functions. In this case our analytical results can be simplified to eq. (22) and agree very well with simulations, in contrast to earlier approaches [17,19]. We have checked also how the Kirkwood approximation works in the case of two types of correlated initial conditions. Although in both cases the Kirkwood approximation cannot be easily justified, surprisingly we obtained very good agreement of our formula (20) with computer simulations. The validity of the formula is much wider than the validity of the Kirkwood approximation used in its derivation.

* * *

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Phase transition in a one-dimensional Ising ferromagnet at zero temperature using Glauber dynamics with a synchronous updating mode

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In the past decade low-temperature Glauber dynamics for the one-dimensional Ising system has been several times observed experimentally and occurred to be one of the most important theoretical approaches in a field of molecular nanomagnets. On the other hand, it has been shown recently that Glauber dynamics with the Metropolis flipping probability for the zero-temperature Ising ferromagnet under synchronous updating can lead surprisingly to the antiferromagnetic steady state. In this paper the generalized class of Glauber dynamics at zero temperature will be considered and the relaxation into the ground state, after a quench from high temperature, will be investigated. Using Monte Carlo simulations and a mean field approach, discontinuous phase transition between ferromagnetic and antiferromagnetic phases for a one-dimensional ferromagnet will be shown.

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I. INTRODUCTION

Glauber dynamics for the Ising spin chain has been known for almost 50 years [1], but only recently it became a really hot topic, not only from a fundamental, but also an applicative point of view [2–8]. It is well known that a purely one-dimensional (1D) system exhibits long-range ordering only at zero temperature $T=0$ K. Nevertheless, in some situations long relaxation times for the magnetization reversal with decreasing temperature can be observed, and finally at significantly low temperatures, the material can behave as a magnet. The phenomenon of slow magnetic relaxation is considered as one of the most important achievements of molecular magnetism, opening exciting new perspectives including that of storing information [9,10]. Slow relaxation of the magnetization, predicted in the 1960s by Glauber in a chain of ferromagnetically coupled Ising spins [1], in materials composed of magnetically isolated chains was observed for the first time in 2001 [2]. In 2002, this new class of nanomagnets was named single-chain magnets (SCM) [3] (for a recent review see [8]) and the Glauber dynamics for the one-dimensional Ising spins system became one of the most important theoretical approaches for SCM.

Within the Glauber dynamics for Ising spins with a spin $s=1/2$, in a broad sense, each spin is flipped $S_i(t)\rightarrow -S_i(t+1)$ with a rate $W(\delta E)$ per unit time and this rate is assumed to depend only on the energy difference implied in the flip. In this paper we consider the generalize class of zero-temperature dynamics defined as

$$W(\delta E)=\begin{cases}1 & \text{if } \delta E<0,\\ W_0 & \text{if } \delta E=0,\\ 0 & \text{if } \delta E>0,\end{cases} \qquad (1)$$

which occurred to be very interesting not only from an applicative perspective, but also from a theoretical point of view as an example of nonequilibrium dynamical systems with many attractors [11]. The zero-temperature limits of the original Glauber dynamics [1] and Metropolis rates [12] (two the most popular choices) are respectively $W_0^G=1/2$ and $W_0^M=1$.

Glauber dynamics was originally introduced as a sequential updating (SU) process [1]. Also Monte Carlo method, used frequently for various models in statistical physics, as proposed originally by Metropolis *et al.* [12], is essentially SU process. Evolution under dynamics defined by Eq. (1) with random sequential updating is already well known in a case of one-dimensional system and can be derived analytically [11]. For any nonzero value of the rate W_0 ferromagnetic steady state is reached and the dynamics belongs to the universality class of the zero-temperature Glauber model [1]. The particular value $W_0=0$ corresponds to the constrained zero-temperature Glauber dynamics ([11] and references therein). In the constrained zero-temperature Glauber dynamics, the only possible moves are flips of isolated spins and therefore the system eventually reaches a blocked configuration, where there is no isolated spin [11], i.e., for $W_0=0$ the relaxation time to the ferromagnetic steady state is infinite.

The case of the synchronous updating, in which all units of the system are updated at the same time, is much more interesting. Moreover, clear evidence of a relaxation mechanism which involves the simultaneous reversal of spins has been shown experimentally for magnetic chains at low temperatures [15].

In [20] more general form of zero-temperature Glauber dynamics has been investigated than one defined by Eq. (1). They have studied a model with two parameters Γ and δ, which can be presented at $T=0$ analogously to Eq. (1) as

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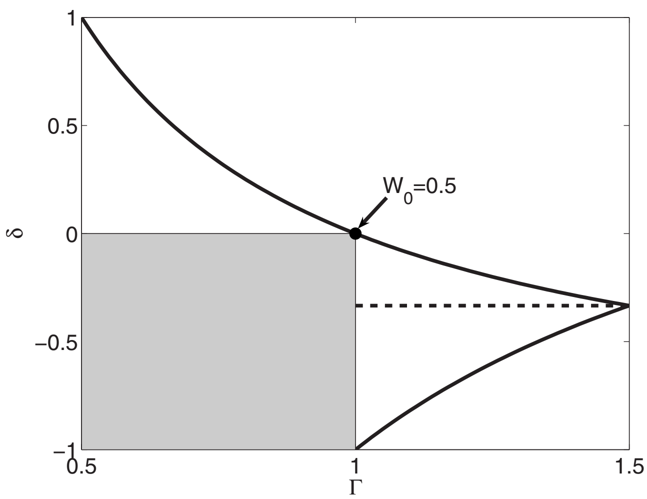

FIG. 1. Thick lines correspond to equations $\delta=(1-\Gamma)/\Gamma$ and $\delta=(\Gamma-2)/\Gamma$. The region between these two lines corresponds to the condition $W(\delta E)\in[0,1]$ (see Eq. (2)). In [20] the region denoted by the gray color has been investigated (i.e., $\delta<0$, $\Gamma\in(0,1)$). In this paper we consider one-parameter model defined by Eq. (1) and therefore we are able to investigate only upper bold line defined by Eq. (4).

$$W(\delta E)=\begin{cases}\Gamma(1+\delta) & \text{if } \delta E<0,\\ \frac{\Gamma}{2}(1-\delta) & \text{if } \delta E=0,\\ 0 & \text{if } \delta E>0,\end{cases} \tag{2}$$

where again $W(\delta E)$ denotes the flipping rate per unit time. To fulfill the condition $W(\delta E)\in[0,1]$, as seen from Eq. (2), the following relations have to be satisfied,

$$-1\le\delta\le\frac{1-\Gamma}{\Gamma},$$

$$\frac{\Gamma-2}{\Gamma}\le\delta\le 1. \tag{3}$$

Above relations correspond to the region between thick lines in Fig. 1. In [20] the region denoted by the gray color in Fig. 1 has been investigated [i.e., $\delta<0$, $\Gamma\in(0,1)$]. Comparing Eqs. (1) and (2) we can easily derive the following relations:

$$\Gamma=W_0+\frac{1}{2},$$

$$\delta=\frac{1/2-W_0}{1/2+W_0}, \tag{4}$$

which are parametric expression of the upper bold line of Fig. 1. In this paper we consider one-parameter model defined by Eq. (1) with $W_0\in[0,1]$ and therefore we are able to investigate only upper bold line of Fig. 1 defined by Eq. (4).

II. SIMULATION AND MEAN FIELD RESULTS

We consider the chain of L Ising spins $\sigma_i=\pm1$ $(i=1,2,\dots L)$ with the periodic boundary conditions. In the

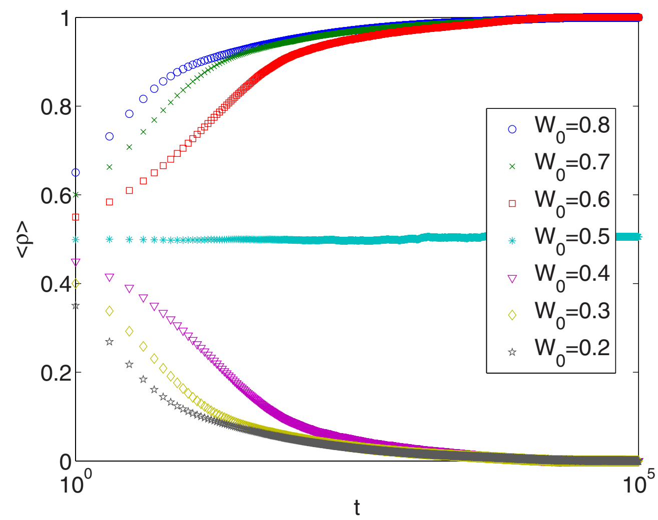

FIG. 2. (Color online) The time evolution of the mean value of the density of active bonds $\langle\rho\rangle$ measured in Monte Carlo steps for the lattice size $L=160$ is presented. Averaging was done over 10^4 samples. For $W_0<0.5$ the mean number of active bonds decreases in time to 0 (ferromagnetic steady state) and for $W_0>0.5$ increases to 1 (antiferromagnetic limit cycle).

initial state each lattice site is occupied independently by a randomly chosen value +1 or −1, both equally probable (high temperature situation). In every time step all spins are considered simultaneously, but each spin is flipped independently with probability $W(\delta E)$ defined by Eq. (1). It occurs that for all $W_0\in(0,1)$ system eventually reaches one of the two final states—ferromagnetic steady state or antiferromagnetic limit cycle. If we measure the density of active bonds (bond is active if connects two sites with opposite spins):

$$\rho=\frac{1}{2L}\sum_{i=1}^{L}(1-\sigma_i\sigma_{i+1}), \tag{5}$$

we obtain in the final state $\rho_{st}=1$ (antiferromagnetic state) or $\rho_{st}=0$ (ferromagnetic state).

The time evolution of the mean value (averaged over 10^4 samples) of the density of active bonds measured in Monte Carlo steps (MCS) is presented in Fig. 2. This is seen that for $W_0<0.5$ the average number of active bonds decreases in time and eventually the system reaches the ferromagnetic steady state $[\langle\rho(\infty)\rangle=\langle\rho_{st}\rangle=0]$, while for $W_0>0.5$ it increases and eventually antiferromagnetic limit cycle is reached $[\langle\rho(\infty)\rangle=\langle\rho_{st}\rangle=1]$. Results presented in Fig. 2 show that for $W_0=0.5$ there is a phase transition between ferromagnetic and antiferromagnetic phase.

This phase transition can be predicted using the mean field approximation (MFA) analogously as it was done in [20]. In [20] the mean field equations for the density of active bonds and magnetization have been derived,

$$\frac{d\rho}{dt}=2\delta\Gamma\rho(1-3\rho+2\rho^2),$$

$$\frac{dm}{dt} = -\delta\Gamma m(m^2-1). \tag{6}$$

Using relations (4) we can easily rewrite above equations in the case of our one-parameter model,

$$\frac{d\rho}{dt} = (1-2W_0)\rho(1-3\rho+2\rho^2),$$

$$\frac{dm}{dt} = \left(W_0 - \frac{1}{2}\right)m(m^2-1). \tag{7}$$

As we see there are three types of fixed points,

$$m_{st} = \pm 1 \quad \text{and} \quad \rho_{st} = 0,$$

$$m_{st} = \pm 0 \quad \text{and} \quad \rho_{st} = 1/2,$$

$$m_{st} = \pm 0 \quad \text{and} \quad \rho_{st} = 1.$$

In [20] only two first types have been considered:

(i) $\rho_{st}=0$ (ferromagnetic state with $m_{st}=-1,1$)

(ii) $\rho_{st}=1/2$ (so called active phase).

However, there is a third fixed point $\rho_{st}=1$, $m_{st}=0$, which corresponds to antiferromagnetic steady state found in our computer simulations.

Let us first consider stability of the magnetization fixed points. For $W_0<0.5$ we can easily check that $|m_{st}|=1$ (ferromagnetic order) is an absorbing state, since from Eq. (7),

$$\frac{dm}{dt} < 0 \quad \text{for} \quad m \in (-1,0) \to m_{st} = -1,$$

$$\frac{dm}{dt} > 0 \quad \text{for} \quad m \in (0,1) \to m_{st} = 1. \tag{8}$$

Analogously, for $W_0>0.5$ we obtain from Eq. (7) that $m_{st}=0$,

$$\frac{dm}{dt} > 0 \quad \text{for} \quad m \in (-1,0) \to m_{st} = 0,$$

$$\frac{dm}{dt} < 0 \quad \text{for} \quad m \in (0,1) \to m_{st} = 0. \tag{9}$$

Therefore, MFA equation for magnetization predicts the phase transition for $W_0=0.5$ between ferromagnetic phase $|m_{st}|=1$ and phase with magnetization equal zero.

Now we can check stability of the MFA equation for active bonds. For $W_0<0.5$ we obtain from Eq. (7) that $\rho_{st}=1/2$ is the stable point (active phase [20]):

$$\frac{d\rho}{dt} > 0 \quad \text{for} \quad \rho \in (0,1/2) \to \rho_{st} = 1/2,$$

$$\frac{d\rho}{dt} < 0 \quad \text{for} \quad \rho \in (1/2,1) \to \rho_{st} = 1/2. \tag{10}$$

Analogously, for $W_0>0.5$ we can easily check that

$$\frac{d\rho}{dt} < 0 \quad \text{for} \quad \rho \in (0,1/2) \to \rho_{st} = 0,$$

$$\frac{d\rho}{dt} > 0 \quad \text{for} \quad \rho \in (1/2,1) \to \rho_{st} = 1. \tag{11}$$

As we see there is a contradiction in a simple MFA equations. Considering only equation for m one can easily check that for $W_0<0.5$ there is a ferromagnetic absorbing state $|m_{st}|=1$, while for $W_0>0.5$ we obtain $m_{st}=0$, which might be associated with antiferromagnetic phase (if simultaneously $\rho_{st}=1$) or active phase (if simultaneously $\rho_{st}=0$). However, if one considers the MFA equation for ρ it occurs that for $W_0<0.5$ $\rho_{st}=0.5$ (active phase), while for $W_0>0.5$ $\rho_{st}=0$ in a case of ρ between 0 and 0.5 (ferromagnetic phase) or $\rho_{st}=1$ in a case of ρ between 0.5 and 1 (antiferromagnetic phase).

Inconsistency in equations is clearly visible for $W_0<0.5$, in which $|m_{st}|=1$ and simultaneously $\rho_{st}=0.5$ (instead of $\rho_{st}=0$, which is valid for ferromagnetic phase). For $W_0>0.5$ MFA results are more reasonable, since $m_{st}=0$ and $\rho_{st}=0$ or 1. Of course only the second possibility is consistent and corresponds to antiferromagnetic phase. Contradiction which is present in MFA equations follows from MFA equation for the density of active bonds. This is understandable since, due to the Eq. (5), correlations between neighboring sites (which are not considered in a simple MFA) are essential for ρ.

Nevertheless, summing up above considerations, MFA equations suggest discontinuous phase transition for $W_0=0.5$ between ferromagnetic and antiferromagnetic phase. This should be noticed that the transition value $W_0=0.5$ corresponds to the original Glauber dynamics [1].

In the case of discontinuous phase transition one would expect the phase coexistence. We have provided computer simulations to confirm this mean field result and indeed coexistence of ferro- and antiferromagnetic phases can be observed near the transition point $W_0=0.5$ (see Fig. 3). For $W_0=0.5$ both types of clusters (ferro- and antiferromagnetic) are nearly the same size and after a long-time competition between them eventually one of two possible steady states is reached. Because for $W_0=0.5$ both of them are equally probable we see the constant value of the average density of active bonds in Fig. 2. Let us now investigate the phase transition more quantitatively using Monte Carlo simulations.

Following [14,20], we use as an order parameter the mean value of the density of active bonds. We provide Monte Carlo simulations and wait until the system reaches the final stationary state. Dependence between order parameter in the stationary state $\langle\rho_{st}\rangle$ and the flipping probability W_0 is presented in Fig. 4, showing again clearly discontinuous phase transition for $W_0=0.5$ in agreement with the mean field result. In the case of $W_0<0.5$ the ferromagnetic steady state is obtained with probability 1 (for the infinite system $L=\infty$). For $W_0>0.5$ the antiferromagnetic state is always reached, i.e., the stationary states losses any remnants of the ferromagnetic Ising interactions.

One of the most important issues connected with the coarsening is the relaxation time τ, i.e., time needed to reach the ground state. In this paper we measure the relaxation time starting from the random initial conditions and counting how many Monte Carlo steps is needed to reach the steady state ($\rho=1$ or $\rho=0$). We average over $N=10^4$ samples and calculate the mean relaxation time,

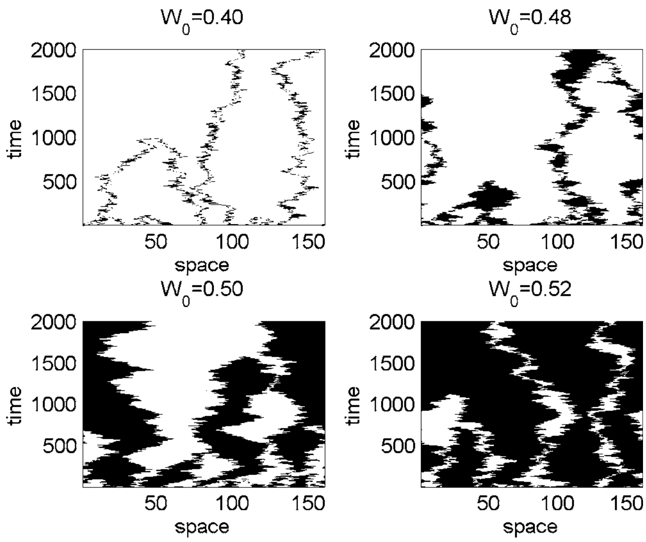

FIG. 3. The time evolution of the Ising spins chain of the length $L=160$ is presented. Black points represent active bonds and thus black regions correspond to antiferromagnetic and white to ferromagnetic clusters. Coexistence of both types of clusters is visible for $W_0 \approx 0.5$. For $W_0=0.5$ both types of clusters are nearly the same size and there is a long-time competition between them leading eventually to one of two possible steady states (ferromagnetic or antiferromagnetic)

$$\langle \tau \rangle = \frac{1}{N}\sum_{i=1}^{N} \tau_i, \tag{12}$$

where τ_i is the relaxation time of ith sample. In Fig. 5 $\langle \tau \rangle$ divided by the square of the lattice size L as a function of the flipping probability W_0 is shown. This is seen that for W_0 =0.5 the mean relaxation time scales as $\langle \tau \rangle \sim L^2$, which is well known result in a case of sequential updating [16,17]. The dependence between the mean relaxation time $\langle \tau \rangle$ and the flipping probability W_0 is nonmonotonical. For $W_0 \to 0$ the relaxation time grows rapidly [18,19], which can be understood recalling that $\langle \tau \rangle$ if infinite for $W_0=0$ [11]. For increasing W_0 the mean relaxation time decreases up to a certain point $W_0^{\min}(L)$. However, due to the phase transition in $W_0=0.5$, for $W_0 \in [W_0^{\min}(L), 0.5]$ it grows again, resulting nonmonotonic behavior shown in Fig. 5. The maximum peak is narrower with the growing lattice size, which is expected behavior for the phase transition. The minimal value $W_0^{\min}(L)$ depends on the system size L as $W_0^{\min}(L) = -2.5/L + 0.5$ and therefore $\lim_{L\to\infty} W_0^{\min}(L) \to 0.5$. The mean relaxation time for this minimal value scales with the system size as $\langle \tau(W_0^{\min}) \rangle \sim L^2$, i.e., with the same exponent as for the transition point $W_0=0.5$.

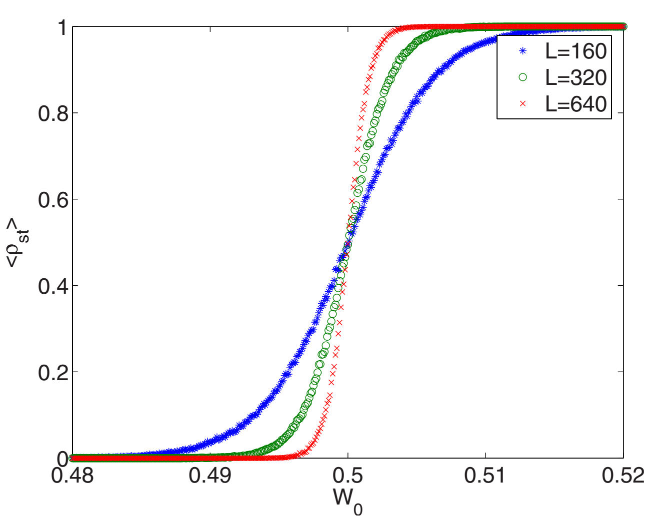

FIG. 4. (Color online) Density of active bonds ρ_{st} in stationary state as a function of flipping probability W_0 (so called exit probability) averaged over 10^4 samples. In the thermodynamical limit $L \to \infty$ for $W_0 < 0.5$ ferromagnetic steady state is reached with probability one ($\rho_{st}=0$) and for $W_0 > 0.5$ antiferromagnetic steady state is reached with probability one ($\rho_{st}=1$). Note that, the transition value $W_0=0.5$ corresponds to the original Glauber dynamics.

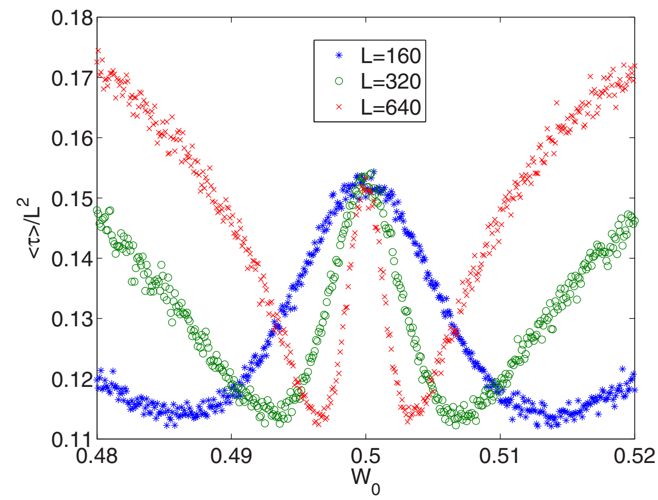

FIG. 5. (Color online) The mean relaxation times $\langle \tau \rangle$ divided by the square of lattice size L as a function of flipping probability $W_0 \in [0.48, 0.52]$. Averaging was done over 10^4 samples. Note that for $W_0=0.5$ relaxation time scales with the system size as $\langle \tau \rangle \sim L^2$. However, for $W_0 \neq 0.5$ scaling exponent differs from known value $\alpha=2$ (see Fig. 6).

The most important question here is the one concerning the origin of the phase transition. As it was mentioned above, in the case of Metropolis flipping rate ($W_0=1$) the system reaches antiferromagnetic limit cycle, instead for the ferromagnetic steady state [13,14]. It can be easily understood, because for the flipping probability $W_0=1$, the case of synchronous updating is fully deterministic (see an example below):

$$\begin{aligned}
&\cdots\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\cdots,\\
&\cdots\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\cdots,\\
&\cdots\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\cdots,\\
&\cdots\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\cdots,\\
&\cdots\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\cdots.
\end{aligned} \tag{13}$$

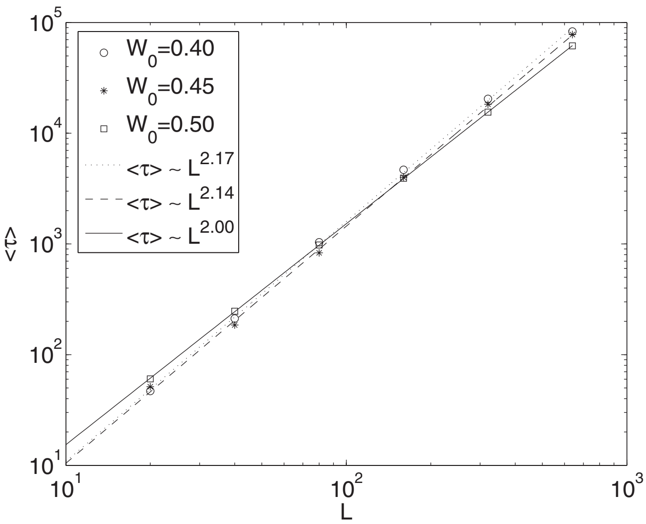

FIG. 6. The mean relaxation time $\langle\tau\rangle$ over the system size L for several values of W_0. For all values of W_0 the mean relaxation time scales with the system size nearly as $\langle\tau\rangle \sim L^{\alpha}$ with $\alpha \approx 2$. However, for different values of W_0 the scaling exponent α slightly varies. In the Fig. 7 the dependence between the scaling exponent α and parameter W_0 is presented.

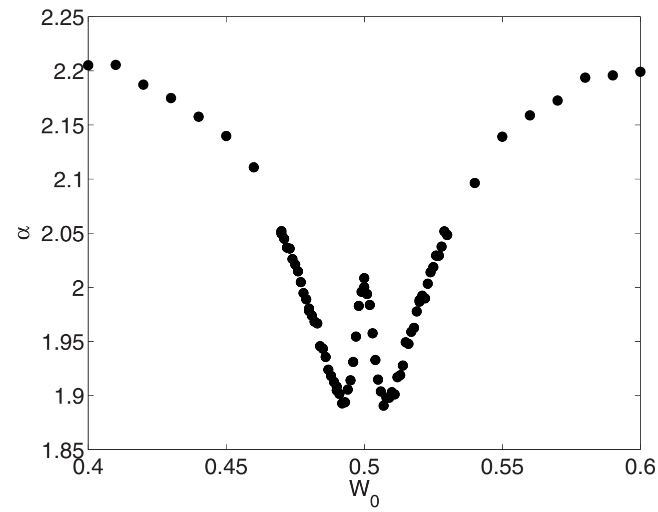

FIG. 7. The mean relaxation time $\langle\tau\rangle$ scales with the system size as $\langle\tau\rangle \sim L^{\alpha}$. For $W_0=0.5$ the scaling exponent $\alpha=2$, which is well known result in the case of sequential updating. However, in general scaling exponent depends on the flipping probability W_0, i.e., $\alpha=\alpha(W_0)$. Dependence between scaling exponent α and the flipping probability W_0 is shown. Simulations were done for the system size $L \in [20,1280]$ and averaged over 10^4 samples.

On the other hand, only for $W_0=1$ updating is really synchronous. For decreasing W_0 only isolated spins are concerned really synchronously, since in the case of isolated spins $\delta E<0$ [see Eq. (1)] the flip is provided with the probability 1. Flipping of isolated spins leads clearly to growth of ferromagnetic domains. Let us introduce for a while a notation $L_{\delta E=0}$ for the number of spins that flipping would not change the energy and $L_{\delta E<0}$ for the number of spins that flipping would decrease the energy. The flip for $\delta E=0$ is realized with the probability W_0 and for $\delta E<0$ with the probability 1, which means that on average $L_{\delta E<0}+W_0 L_{\delta E=0}$ is flipped in a single time step. In the case of $W_0=1$, as mentioned above, the antiferromagnetic order is reached. On the other hand, for $W_0=1/L_{\delta E=0}$ on average only one not isolated spin (i.e., with $\delta E=0$) is flipped in a single time step, similarly to the case of the sequential updating for the system without isolated spins. Thus, because in the case of sequential updating ferromagnetic steady state is reached, one can expect also ferromagnetic order in the case of synchronous updating for small values of W_0. Clearly the phase transition must occur somewhere between the antiferromagnetic order, preferred by a fully synchronous updating ($W_0=1$), and the ferromagnetic steady state, preferred by sequential updating ($W_0=1/L_{\delta E=0}$).

As mentioned above, for $W_0=0.5$ and $W_0=W_0^{\min}(L)$ the mean relaxation time scales with a system size as $\sim L^2$. We have checked also the scaling for other values of W_0. For all values of W_0 the mean relaxation time scales with the system size nearly as $\langle\tau\rangle \sim L^{\alpha}$ with $\alpha \approx 2$ (see Fig. 6). However, for different values of W_0 the scaling exponent α slightly varies. The dependence between scaling exponent and the flipping probability is presented in Fig. 7. The shape of the curve $\alpha(W_0)$ mimic the shape of $\langle\tau(W_0)\rangle$.

III. SUMMARY

In this paper we have been investigating the relaxation of the Ising spins chain under the generalized class of Glauber dynamics at zero-temperature. Within such a dynamics, the flipping probability in a case of conserved energy is given by arbitrary value of $W_0 \in [0,1]$ (review in a case of sequential updating can be find in [11]). We have proposed to use synchronous updating for such a generalized class of zero-temperature dynamics. Our motivation for this work came from recent experiments showing slow relaxation in magnetic chains at low temperatures [2–8,15]. We have shown by Monte Carlo simulations that there is a phase transition for $W_0=0.5$, which correspond to the value originally proposed by Glauber [1]:

(i) for $W_0<0.5$ ferromagnetic fixed point ($m_{st}=\pm 1$, $\rho_{st}=0$) is stable

(ii) for $W_0>0.5$ antiferromagnetic fixed point ($m_{st}=0$, $\rho_{st}=1$) is the stable one.

Following [20] we were able to obtain the mean field result which also suggests phase transition between ferro- and antiferromagnetic phases for $W_0=0.5$.

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Phase transition in the Sznajd model with independence

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Abstract – We propose a model of opinion dynamics which describes two major types of social influence —conformity and independence. Conformity in our model is described by the so-called outflow dynamics (known as Sznajd model). According to sociologists' suggestions, we introduce also a second type of social influence, known in social psychology as independence. Various social experiments have shown that the level of conformity depends on the society. We introduce this level as a parameter of the model and show that there is a continuous phase transition between conformity and independence.

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Introduction. – Opinion dynamics is one of the most studied subjects in the field of sociophysics. Among a number of models that have been proposed (for a recent review see [1]), simple models based on Ising spin variables are particularly interesting. In all these models, from the voter model [2], majority rule [3,4] to the Sznajd model [5], opinions are described by discrete variables $S = \pm 1$. The ferromagnetic state is an attractor for all three models in one and two dimensions, as well as in the case of complete graphs [1]. Obviously, in real social systems complete unanimity is never reached. Moreover, in real systems the public opinion does not reach any fixed point and permanently changes. To make models of opinion dynamics more realistic, Galam has proposed two modifications:

– Contrarian behavior [6] —with a certain probability an agent adopts the choice opposite to the prevailing choice of the others, whatever this choice is.

– Inflexibles [7] —inflexible agents keep their opinion always unchanged.

As shown by Galam, for a low concentration of contrarians a new mixed phase is stabilized, with a coexistence of both opinions, *i.e.* minority persists. Moreover, there is a phase transition into a new disordered phase with no dominating opinion. It has been shown that introduction of contrarians make the tendency towards extremism of the original model weaker also in the case of the CODA model [8], in which agents have internal continuous opinions, but exchange information only about a binary choice. In the case of the Sznajd model, contrarian behavior has been studied for the first time by de la Lama *et al.*, [9] and the same results have been obtained. As suggested by Galam [6], these results may be put in parallel with "hung elections" in America (2000) and Germany (2002).

In the field of social psychology, contrarian behavior, introduced by Galam in [6], is nothing more than anticonformity —a particular type of non-conformity [10]. There are two widely recognized types of non-conformity: anticonformity and independence. From a social point of view, it is very important to distinguish between independence and anticonformity [11]. The term "independence" implying the failure of attempted group influence. Independent individuals evaluate situations independently of the group norm. On the contrary, anticonformists are similar to conformers in the sense that both take cognizance of the group norm —conformers agree with the norm, anticonformers disagree. As noticed in [11]: *This behaviour is a bit of a paradox, because in order to be vigilant about not doing what is expected, one must always be aware of what is expected. In contrast, truly independent people are oblivious to what is expected.* Numerous studies have shown that the level of conformity/anticonformity depends on the society (culture, age, etc.) [11,12]. Moreover, the degree of individualism represents one of the 5 dimensions of cultures, *i.e.* aspect of a culture that

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can be measured relative to other cultures [13]. It has been measured that among 50 countries surveyed, United States has the highest Individualism Index Value (IDV = 91), whereas Guetemala the lowest (IDV = 6). High IDV, which varies between 0 and 100, indicates a loose connection with people. This means that in spite of individual differences, global thinking of relationship between the individual and the group is dramatically different in each country [13]. Independence is not identical with IDV, since it is defined on the individual level. However, one could expect that independent behavior is more common in the society with high IDV. For example, the level of independent behavior is much lower in Japan than in America [11] and simultaneously IDV = 46 in Japan, which is over two times less than in US [13].

In this paper we examine the influence of independence on the Sznajd model in one and two dimensions, as well as on a complete graph. We show that contrarians are not needed in order to obtain phase transitions as in [6,9] and similar results can be observed in the presence of independent behavior. Below a certain critical independence value p_c, the minority opinion coexists with the majority and above p_c there is no majority in the society, the system is in the so-called stalemate state. This is not surprising, since both contrarian and independent behavior plays the role of noise and one of the main effects of noise is to induce an order-disorder transition. The same result has been obtained recently for the Deffuant *et al.* model for continuous opinion dynamics [14].

To generalize our model and make it more realistic from a social point of view, we introduce also a flexibility factor f, which describes what is the probability of an opinion change in the case of independent behavior. We show, both analytically (for complete graphs) and using Monte Carlo simulations, that the critical threshold of independence p_c decays with the flexibility factor f. This means that in an inflexible (conservative) society the critical independence factor is high. This result implicates that in conservative societies, even if the level of independence is high, there is always a majority in the system —in the case of democratic voting, one of the two options wins.

Model. – We consider a set of N individuals, which are described by the binary variables: $S = 1$ (↑) or $S = -1$ (↓). At each elementary time step, a group of people is chosen randomly and it influences its surrounding individuals. In the original model [5] only one type of social influence (conformity) was considered:

1) On a complete graph, two individuals are chosen at random and they influence a third randomly chosen individual [15].

2) In one dimension (1D), a pair of neighboring individuals S_iS_{i+1} is chosen and it influences two neighboring sites S_{i-1}, S_{i+2}. In this paper, to be consistent with the case of a complete graph, we will use the modified version in which only one of the two (left S_{i-1} or right S_{i+2}), chosen randomly, will be changed. This kind of modification has been introduced for the first time by Slanina [16].

3) Several possibilities of generalization to the square lattice were proposed by Stauffer *et al.* [17]. Here we use probably the most popular rule —a 2×2 panel of four neighbors is chosen randomly and influences its surroundings. In this paper we use a modified version to be consistent with the rules above —only one out of the 8 neighbors of the panel is randomly chosen to be changed.

As mentioned earlier, there are many factors that affect the likelihood of conformity, among them culture is one of the most important. Therefore, in this paper we introduce a second type of social response, known as independence. With probability p, an individual S_k chosen to be changed will not follow the group, but act independently; with probability f it will flip, *i.e.* $S_k \to -S_k$ and with probability $1-f$ stay unchanged, *i.e.* $S_k \to S_k$. The parameter f is called flexibility, since it describes how often an individual will change its opinion in case of independent behavior. With probability $1-p$ the individual will follow the usual Sznajd conformity rules, described above. It should be repeated here that although independence is an individual psychological trait, it is connected with cultural dimension called individualism. Therefore, the value of parameter p can be understood as a mean value of independence within a particular society.

Model on a complete graph. – We consider a set of N Ising spins $S_i = \pm 1$, $i = 1, \ldots, N$ on a complete graph. In each elementary time step t two spins S_i and S_j are chosen randomly. They will influence a third randomly chosen spin S_k in the following way:

– conformity (original Sznajd rule), with probability $1-p$: $S_k(t+\mathrm{d}t) = S_i(t)$ if $S_i(t) = S_j(t)$, otherwise $S_k(t+\mathrm{d}t) = S_k(t)$,

– independence, with probability p: $S_k(t+\mathrm{d}t) = -S_k(t)$, with probability f or $S_k(t+\mathrm{d}t) = S_k(t)$, with probability $1-f$.

Time $t \to t+1$ after N elementary time steps, *i.e.* $\mathrm{d}t = 1/N$.

In the case of a complete graph, the state of the system is completely described by the magnetization (or public opinion from a social point of view) defined as

$$m(t) = \frac{1}{N}\sum_{i=1}^{N} S_i(t). \tag{1}$$

Let us denote by $N_\uparrow$ the number of spins "up" and by $N_\downarrow$ the number of spins "down". We can easily derive the formula for the probability of choosing randomly a spin $S = +1$:

$$P_+(t) = \frac{N_\uparrow}{N} = \frac{1+m(t)}{2}. \tag{2}$$

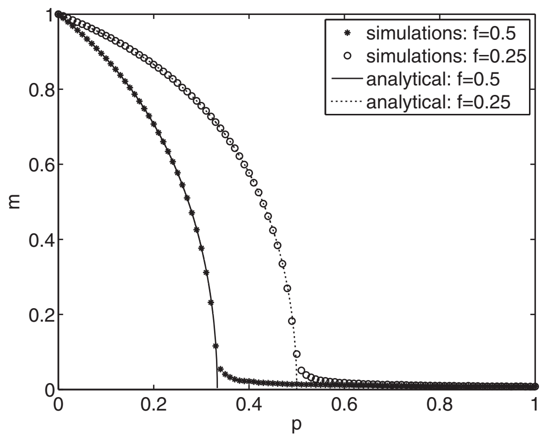

Fig. 1: Stationary value of magnetization for two values of the flexibility factor, $f = 0.5$ (solid line) and $f = 0.25$ (dotted line), from analytical calculations and from Monte Carlo simulations for the system size $N = 10^3$. Averaging was done over 10^3 samples.

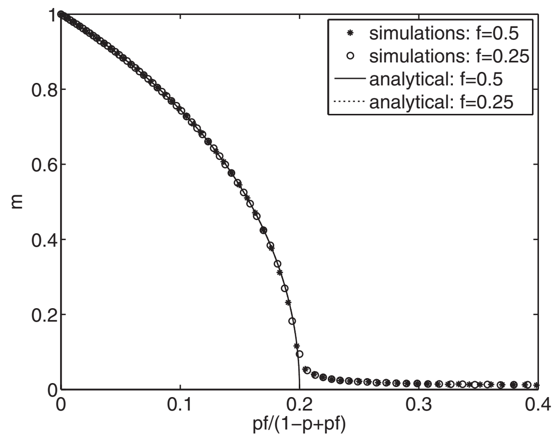

Fig. 2: Rescaled results from fig. 1.

The evolution of the system is described by the following equation:

$$P'_+ - P_+ = pfP_- - pfP_+ + (1-p)P_+^2 P_- - (1-p)P_-^2 P_+, \tag{3}$$

where we use the notation $P'_+ \equiv P_+(t + \mathrm{d}t)$ and $P_+ \equiv P_+(t)$ and the probability of choosing a "down" spin is $P_- = 1 - P_+$. We look for fixed points of the above transformation:

$$P'_+ - P_+ = 0. \tag{4}$$

The equation above has the following solutions:

$$\begin{aligned} P_+^0 &= 1/2 && \text{for} \quad p \in [0,1] \\ P_+^{1,2} &= \frac{1 - p \pm \sqrt{(\Delta)}}{2(1-p)} && \text{for} \quad p < \frac{1}{1+4f}, \end{aligned} \tag{5}$$

where $\Delta = (1-p)(1-p-4pf)$. Fixed points for the magnetization can be easily calculated from relation (2) and are presented in fig. 1. There is a continuous phase transition at $p_c = 1/(1+4f)$ – for $p < p_c$ minority coexists with majority and for $p > p_c$ there is a stalemate (status quo) situation. To confirm our analytical results, we have provided Monte Carlo simulations on a complete graph, for several lattice sizes. Initially the system has been ferromagnetically ordered ($m(0) = 1$) and then evolved according to the algorithm described in this section. These results are presented in fig. 1 and agree with the analytical prediction.

Scaling. – Although the model considered in this paper has 2 parameters, independence p and flexibility f, it can be shown that in fact results depend on the ratio $pf/(1-p+pf)$ (see fig. 2). To show this scaling let us first sum up the rules of the model as:

– *Rule I*: the usual Sznajd rule,

– *Rule II*: a site is flipped ($S_i \to -S_i$),

– *Rule III*: nothing happens.

At each iteration, we choose a group of sites that will attempt to convince one of their neighbours, chosen at random, then one of the 3 rules above is chosen at random to be followed by this chosen neighbour, with the rules I, II and III being followed with probability $1-p$, pf and $p(1-f)$, respectively. Firstly, we note that rule III does not change the state of the system (the opinions of each of the sites) and that the parameters p and f are not important in a given iteration, once the rule to be followed is chosen. Suppose now that we keep records of the state of the system after the iterations where either rule I or rule II was followed. These records would form a sequence of states, whose statistical properties depend only on the initial conditions and on the ratio between the probabilities of following rule I and II:

$$r = \frac{pf}{1-p}. \tag{6}$$

So if two models have parameters (p, f) and (p', f') such that $r = r'$, then they would both generate statistically similar sequences, given the same initial conditions. In order to compare both models, we need to make simulations with lengths t and t' such that the mean number of records is the same. As we follow either rule I or rule II with probability $1 - p + pf$, this means that $t(1-p+pf) = t'(1-p'+p'f')$.

We are interested in the behavior of the magnetization $m(t,p,f)$, so if we define $M(t,p) = m(t,p,1)$, we can peek $f' = 1$ and solve for p' and t':

$$\begin{cases} t' = t(1-p+pf), \\ p' = {}^{pf}/_{(1-p+pf)}. \end{cases} \tag{7}$$

It follows that plotting $m(t/(1-p+pf), p, f)$ against $pf/(1-p+pf)$ should collapse the curves, as

$$m\left(\frac{t}{1-p+pf}, p, f\right) = M\left(t, \frac{pf}{1-p+pf}\right). \tag{8}$$

We have used the scaling given by eq. (7) and indeed the data collapses in the case of a complete graph (see fig. 2). In the next sections we will see that the scaling derived here is valid also in the case of one- and two-dimensional lattices.

In fig. 2 we can see a phase transition between the situation where a majority exists and the stalemate situation. This transition happens for a value of independence p_c, that depends on the flexibility value f. We can apply this scaling to find that this dependence must be of the form

$$p_c = \frac{1}{1+\alpha f}, \quad \text{where} \quad \alpha = \frac{1}{P_c} - 1 \tag{9}$$

and P_c is the critical independence for $f = 1$. This means that if there is a phase transition, then the critical value of independence decreases with increasing flexibility. We will give some interpretations for this further down, when the model in a two-dimensional lattice is analyzed.

Model on a one-dimensional lattice. – We consider a chain of length N with periodic boundary conditions. Each site $i = 1, \ldots, N$ of the chain is occupied by an Ising spin $S_i = \pm 1$. At each time step we choose randomly a spin S_i and side s ($s = 1$ for right, $s = -1$ for left). The updated state is

- conformity (original Sznajd rule), with probability $1-p$: $S_i(t+\mathrm{d}t) = S_{i+s}(t)$ if $S_{i+s}(t) = S_{i+2s}(t)$, otherwise $S_i(t+\mathrm{d}t) = S_i(t)$,
- independence, with probability p: $S_i(t+\mathrm{d}t) = -S_i(t)$, with probability f or $S_i(t+\mathrm{d}t) = S_i(t)$, with probability $1-f$.

Time $t \to t+1$ after N elementary time steps, *i.e.* $\mathrm{d}t = 1/N$. We have chosen as an initial condition, ferromagnetic order ($m(0) = 1$). We made Monte Carlo simulations for several lattice sizes $N = 4\times 10^2$, 9×10^2, 64×10^2, 10^4, 4×10^4, but here we present results for only one selected lattice size, $N = 10^4$. We have measured the magnetization of the system after various "termalization" times $\tau \in [10, 10^4]$. The averaging has been done over 10^3 samples. First of all, we notice that the scaling found in the previous section and given by eq. (7) is still valid for one-dimensional systems —see fig. 3.

The results presented in fig. 3 suggest the existence of a phase transition, which is unexpected for one-dimensional systems with short-range interactions. However, one should remember that the initial state in our simulations was ordered ($m(0) = 1$). Starting from this ordered state, the system evolves toward a stationary state that depends on the ratio $pf/(1-p+pf)$. After a short "termalization" time, like in fig. 3, the system might be still ordered but finally it will reach its real steady state, which is expected to be disordered. To check the validity of our expectations let us now present the dependence between the magnetization m and the independence factor p, for several "termalization" times τ and a given value of flexibility $f = 1/2$ (fig. 4). It has been seen that with an increasing τ the threshold value p^*, below which the system is ordered, decreases, suggesting the lack of the phase transition in one dimension. For the infinite system, $N \to \infty$ and $\tau \to \infty$, order is present in the system only for $pf/(1-p+pf) = 0$. This is an expected result, because only short-range interactions are present in the model and the independence $pf/(1-p+pf)$ plays the role of a temperature.

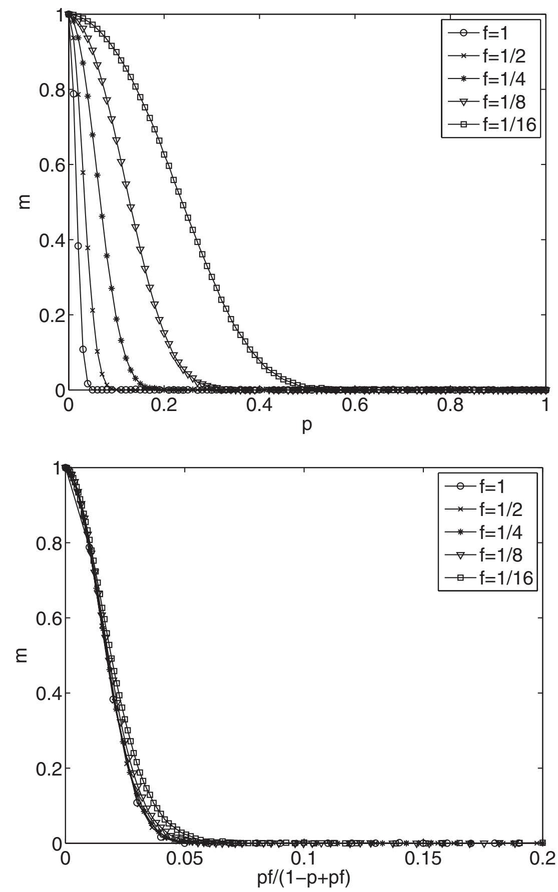

Fig. 3: Relationship between the magnetization m and the independence factor p for several values of the flexibility factor f, for a one-dimensional system of size $N = 10^4$ (upper panel). The system evolved ("termalized") from an initially ordered state, $m(0) = 1$. "Termalization" time was $\tau = 500$ MCS and averaging was done over 10^3 samples. The scaling found in the case of a complete graph still works (bottom panel).

Model on a square lattice. – In this section we consider a square lattice $L \times L$ with periodic boundary conditions. Each lattice site is occupied by an individual, characterized by a binary opinion $S_i = +1$ (in favor) or $S_i = -1$ (against). In each elementary time step t, a 2×2 box of four neighboring spins is chosen randomly and influences one of the 8 neighboring sites of the box, denoted as S_i. In this paper, we use a modified version of

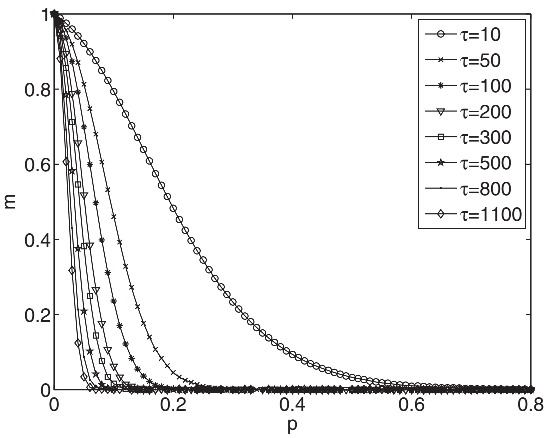

Fig. 4: Dependence between the magnetization m and the independence factor p, for a flexibility factor $f = 0.5$, several "termalization" times, τ, and a one-dimensional system of size $N = 10^4$. The system evolved ("termalized") from an initially ordered state, $m = 1$. Averaging was done over 10^3 samples. With an increasing τ, the critical value p_c is decreasing, suggesting the lack of the phase transition in one dimension.

the two-dimensional model introduced to study duopoly markets in [18], therefore we update the state in the following way:

- Conformity, with probability $1 - p$: If all four spins in the box have the same value, they will convince one of the eight nearest neighbors S_i, changing its orientation in the direction of the spins in the box. If one of the spins in the box has the opposite orientation to the other three spins, then the neighbor changes its orientation to the orientation of the majority, with probability 3/4. In the case when there is no majority, *i.e.* two spins in the panel are up and two are down, nothing changes, *i.e.* $S_i(t + \mathrm{d}t) = S_i(t)$.
- Independence, with probability p: $S_i(t + \mathrm{d}t) = -S_i(t)$, with probability f or $S_i(t + \mathrm{d}t) = S_i(t)$, with probability $1 - f$.

We measure again the magnetization as a function of the independence factor p for several values of the flexibility factor f (see fig. 5). In the case of the square lattice there is no doubt that there is a well defined continuous phase transition (see fig. 6) at $p = p_c$. As in the other cases, scaling (given by eq. (7)) is valid and the critical value p_c of independence increases with decreasing flexibility f.

This means that in the case of high f (non-conservative societies) consensus is possible only in the case of low independence (high conformity). For example (see fig. 5), for a relatively conservative society with $f = 1/16$ the critical independence is $p_c \approx 0.8$. This means that for a level of independence up to 4/5 there is a majority in the society. If the society is less "conservative", *e.g.*, $f = 1/4$ then the critical value of independence is $p_c \approx 0.5$. This means that if the level of independence were, for example, $p = 0.7$ there would be no majority in the system

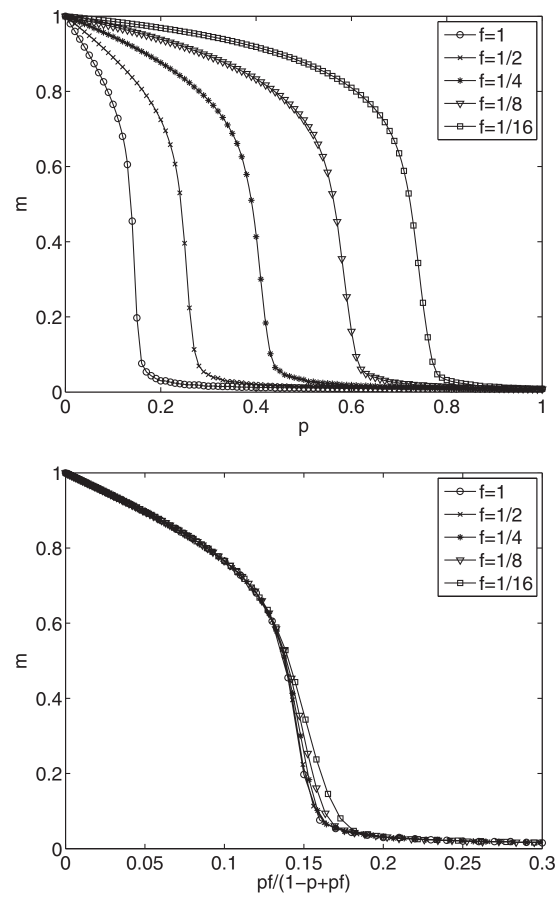

Fig. 5: Dependence between the magnetization m and the independence factor p for several values of the flexibility factor f, in the case of a square lattice, 101×101. The system evolved ("termalized") from an initially ordered state, $m(0) = 1$. "Termalization" time was $\tau = 500\,\mathrm{MCS}$ and averaging was done over 10^3 samples. The critical value of independence p_c decays with flexibility f (upper panel). The scaling found in the case of a complete graph still works (bottom panel).

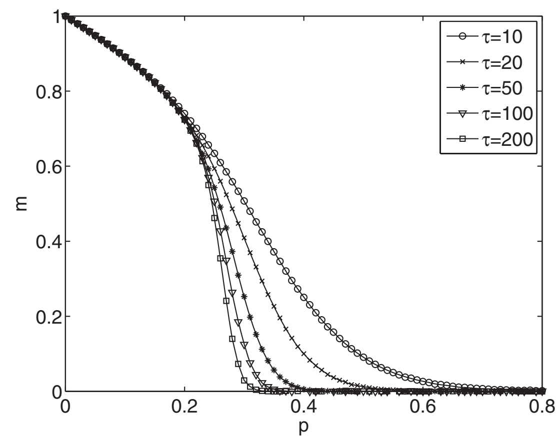

Fig. 6: Dependence between the magnetization m and the independence factor p, for a flexibility factor $f = 0.5$, several "termalization" times, τ, and a square lattice, 101×101. The system evolved ("termalized") from an initially ordered state, $m(0) = 1$. Averaging was done over 10^3 samples.

(status quo), while for the same value of p in the case of $f = 1/16$ consensus would be possible. This result could be one of the possible explanations for why the status-quo situation is more and more common in modern societies. Another explanation, connected with the public debate, has been proposed several years ago by Galam [6].

Summary. – In this paper we have introduced a modified version of the original Sznajd model, in which two types of social influence were considered —independence (with probability p) and conformity (with probability $1-p$). Conformity in our model has been modeled analogously to the Sznajd model, *i.e.* in the case of conformal behavior, individuals have followed the group norm. Of course, this type of social influence could be modeled also by the voter or majority models. In the case of independent behavior individuals take actions (change opinion) independently of the group norm. Of course, even in the case of independent behavior an individual can change opinion, but it does not depend on the social norm. Therefore, we have introduced a flexibility factor f, which denotes the probability of opinion changes in the case of independent behavior —varying f we can model the level of conservatism in the society.

We have studied the model in three cases: complete graphs, one-dimensional systems and two-dimensional square lattices. We found that, in the case of the complete graph and of the two-dimensional system, there is phase transition for a critical value of independence $p_c = \frac{1}{1+\alpha f}$, where α is a constant that depends on the lattice. In the case of a complete graph it can be calculated analytically as $\alpha = 4$. Below the critical value of independence, $p < p_c$, the majority coexists with the minority, *i.e.* the public opinion is $m \neq 0$. Therefore, in the case of a democratic voting, one of the two options wins. For high independence, $p > p_c$, there is a stalemate situation in the society, *i.e.* $m = 0$.

This is particularly interesting result from the social point of view (both marketing and cultural studies). Various empirical studies correlates two cultural dimensions —Individualism (IDV), that has been already described in the introduction, and Power/Distance (PD) indexes [13]. PD measures the extent to which the less powerful individuals accept that power is distributed unequally in a society. For example, Germany has PD = 35 on the cultural scale of Hofstede's analysis, whereas in Arab countries PD = 80 and Austria PD = 11. It has been shown in empirical studies that societies with low IDV accepts an unequal distribution of power (high PD) [13]. Empirical studies show also the influence of the cultural dimensions on the market shares. For example, it occurred recently that the distribution of major smart phone applications differ across the 10 countries studied (Korea, US., India, Indonesia, England, Canada, Japan, France, China, and Mexico) accordingly to cultural dimensions. Again, segregation is lower in Asian countries that showed high preferences for education-related applications [19]. All these results suggest that the level of individualism is anticorrelated with homogeneity of the society —with the increasing level of individualism the heterogeneity of society (in various aspects) also increases. In the case of our model for a completely homogenous society the public opinion $|m| = 1$, while for the most heterogenous one $m = 0$. With increasing level of individualism p the public opinion m decreases, which agrees with empirical observations [13,19].

In our opinion another interesting result is the relationship between the critical value of independence and flexibility, $p_c = \frac{1}{1+\alpha f}$ —the critical value of independence decreasing with increasing flexibility f. This means that in the case of high f (non-conservative societies) consensus is possible only in the case of low independence (high conformity). On the other hand, in conservative societies, even in the case of high independence, consensus is possible. In modern societies the value of tradition and hence the level of conservatism seems to be decreasing. This, according to our model, could be one of the possible explanations for why the status-quo situation observed by Galam [6] is more and more encountered nowadays.

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Exit probability in a one-dimensional nonlinear q-voter model

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We formulate and investigate the nonlinear q-voter model (which as a special case includes the linear voter and the Sznajd model) on a one-dimensional lattice. We derive an analytical formula for the exit probability and show that it agrees perfectly with Monte Carlo simulations. The puzzle that we deal with here may be summarized by a simple question: Why does the mean-field approach give the exact formula for the exit probability in the one-dimensional nonlinear q-voter model? To answer this question, we test several hypotheses proposed recently for the Sznajd model, including the finite size effects, the influence of the range of interactions, and the importance of the initial step of the evolution. On the one hand, our work is part of a trend of the current debate on the form of the exit probability in the one-dimensional Sznajd model, but on the other hand, it concerns the much broader problem of the nonlinear q-voter model.

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I. INTRODUCTION

The linear voter model [1], one of the most recognized in a field of nonequilibrium phase transitions, is not only a toy model of an Ising spin system but also a caricature of opinion dynamics. One of the main reasons for its importance is the fact that the linear voter model (VM) is solvable in an arbitrary spatial dimension. However, from a social point of view it is definitely too simplified and therefore several other models of opinion dynamics, based on Ising spins, have been introduced (for an excellent recent review, see [2]), e.g., the Sznajd model [3] or the majority model [4,5]. Of course, it happens that seemingly different models give the same results or even can be formulated in such a way that they appear to be identical. For example, it has been shown that the original one-dimensional Sznajd model [3] can be rewritten as a classical voter model [6]. However, the most commonly used version, in which only the unanimous pair changes the state of the system, differs significantly from the VM. Nevertheless, it has been suggested that this case could be described by a broader class of nonlinear voter models [7,8].

Recently, a particularly interesting nonlinear variant of the voter model, the q-voter model, has been introduced [8]. In the proposed model q randomly picked (with possible repetitions) neighbors influence a voter to change its opinion. If all q neighbors agree, the voter takes their opinion; if they do not have a unanimous opinion, a voter can still flip with probability ϵ. For $q = 2$ and $\epsilon = 0$ the model is almost identical to the Sznajd model on a complete graph [9]. The only difference is that in the q-voter model, repetition in choosing neighbors is possible. However, for $q = 2$ and reasonably large lattice size, this difference is negligible. In this paper we formulate and investigate the q-voter model on a one-dimensional lattice for $\epsilon = 0$. We show that an analytical formula for the exit probability can be derived and that several approaches (among them the simple mean-field approach) appear to lead to the same result. Moreover, the received analytical formula agrees perfectly with Monte Carlo simulations. On the one hand, our work is part of a trend of the current debate on the form of the exit probability in the one-dimensional Sznajd model [10–13]. On the other hand, it concerns the much broader problem of the nonlinear q-voter model. The puzzle that we deal with here may be summarized by a simple question: Why does the mean-field approach give the exact formula for the exit probability in the one-dimensional nonlinear q-voter model?

We have to admit here that our question has been strongly inspired by the initial twofold question of Galam and Martins: Why does the mean-field approach give the exact formula for the exit probability in the one-dimensional modified Sznajd model, or why do the Monte Carlo simulations incorrectly give a mean-field result? Their recent paper [13] concludes with, Therefore the question is open for future work to settle: either there is an explanation of why the system studied here exhibits mean-field behavior or why different simulations of the same system for different sizes all produced, incorrectly, a similar mean-field result. We extend their question to the generalized q-voter model, and show by computer simulation which explanation of the puzzle given in [13] is more probable.

II. NONLINEAR Q-VOTER MODEL IN ONE DIMENSION

We consider a system of L spins $S_i = \pm 1$ located on a one-dimensional ring. At each elementary time step t a panel of q neighboring spins $S_i, S_{i+1}, \ldots, S_{i+q-1}$ is picked at random. If all q neighbors are in the same state, they influence surrounding spins; if all spins in the q-panel are not equal then nothing changes. Two versions of the model are considered:

(1) *Both sides.* The q panel influences R neighbors on the left side and the right side of the panel simultaneously—all spins $S_{i-R}(t + \Delta t), S_{i-R+1}(t + \Delta t), \ldots S_{i-1}(t + \Delta t)$ and $S_{i+q+1}(t + \Delta t), S_{i+q+2}(t + \Delta t), \ldots S_{i+q+R-1}(t + \Delta t)$ take the state of the panel, i.e., $\rightarrow S_i(t)$. It is easy to notice that for $R = 1$ and $q = 2$ we deal with the original Sznajd model.

(2) *Random.* The q panel influences R neighbors only on the one randomly chosen side (left or right)—with probability $1/2$ spins $S_{i-R}(t + \Delta t), S_{i-R+1}(t + \Delta t), \ldots S_{i-1}(t + \Delta t) \rightarrow S_i(t)$ or, with the same probability, $S_{i+q+1}(t + \Delta t), S_{i+q+2}(t + \Delta t), \ldots S_{i+q+R-1}(t + \Delta t) \rightarrow S_i(t)$. In this case for $R = 1$ and $q = 2$ we deal with the modified version of the Sznajd model, introduced by Slanina [11]. For $R = 1$ and $q = 1$ we obtain the original linear voter model [14].

After one elementary step time increases by $\Delta t = 1/L$. Therefore the time unit corresponds to one Monte Carlo step (MCS).

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We have introduced both versions to be consistent with several other papers that deal with the problem of exit probability in the Sznajd model. In some of them the "both sides" version is used [12], whereas others deal with the "random" version. However, as we will show, there is no difference between the exit probability for both versions; therefore, either of them can be used.

III. EXIT PROBABILITY

Let us consider a finite system with an initial fraction $\rho(0)$ of randomly distributed spins in the $+1$ state. Two scenarios for the q-voter model on a one-dimensional ring are possible:

(i) If there is no cluster of size $\geqslant q$, the system is deadlocked and no evolution is possible, since only a unanimous panel of size q is able to change the state of the system. Obviously the number of deadlocks grows with q. For $q = 1$ no configuration is deadlocked and for $q = 2$ the only deadlocked configuration is the antiferromagnetic state.

(ii) If there is at least one cluster of size $\geqslant q$, then the system will evolve and eventually reach the ferromagnetic state—with probability $E(\rho)$ the "all spins $+1$" state is obtained, whereas with probability $1 - E(\rho)$ the "all spins -1" state is reached. $E(\rho)$ is called the exit probability, and it is one of the most important first-passage properties [14].

The exit probability for the one-dimensional Sznajd model (which corresponds to the nonlinear voter model with $q = 2$) has been calculated analytically [10,11]:

$$E(\rho) = \frac{\rho^2}{\rho^2 + (1-\rho)^2}. \tag{1}$$

This result agrees perfectly with Monte Carlo simulations, which is quite puzzling since calculations were not exact but based on the Kirkwood approximation decoupling scheme [10,11]. Recently it has been shown that even a much less sophisticated method, the simple mean-field approach, leads to the same result [13]. A question arises: Why do the approximate methods give the exact result in this case? Some suggestions related to the importance of choosing the first pair have appeared very recently [12,13].

In [12] the Sznajd model of range R [SM(R)] has been studied. As usual, at each time step, a pair of nearest neighbors S_i, S_{i+1} is chosen at random. If $S_i = S_{i+1}$ then R neighbors to the left and R neighbors to the right change value to S_i. This corresponds to the "both sides" version of the nonlinear q-voter model (with $q = 2$) introduced in the preceding section. Remarkably, in the case studied the exit probability $E(\rho)$ turned out to be completely independent of the range of the interaction R. Based on this result $E(\rho)$ was derived in the following way: the exit probability is given by the probability that a pair of sites in the $+1$ state is chosen before any pair of sites in the -1 state [12]:

$$E(\rho) = \rho^2 \sum_{n=0}^{\infty} [1 - \rho^2 - (1-\rho)^2]^n \tag{2}$$

$$= \frac{\rho^2}{\rho^2 + (1-\rho)^2}. \tag{3}$$

Another idea was presented in [13]: the quasideterministic procedure in which the only random step is the selection of the initial pair, and then the process is deterministic. These types of considerations have led again to the same formula for the exit probability (1).

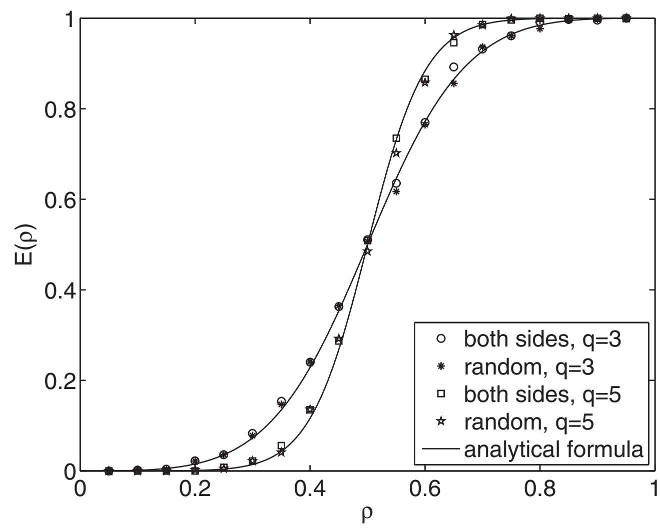

FIG. 1. Exit probability in the case of the "random" and "both sides" nonlinear q-voter model on a one-dimensional ring of length $L = 100$. Averaging was done over 10^4 samples. Results are identical for both versions of the model.

In this paper we show that the mean-field approach gives the formula that agrees perfectly with Monte Carlo simulations for an arbitrary value of q, and we also examine possible explanations as to why approximate methods give the exact result for a one-dimensional nonlinear q-voter model.

We start with showing by computer simulation that the exit probabilities for both versions of the model ("random" and "both sides") are identical. In Fig. 1 we present results for lattice size $L = 100$ and two values of q. Analogous results were obtained earlier for the Sznajd model (i.e., $q = 2$) [11]. Given that both versions of the model give identical results, we will concentrate in a later work on the "random" version. However, we have checked all results presented in this paper for both cases.

Now we are ready to derive an analytical formula for the q-voter model. Let us denote by $\langle N_q^+(t)\rangle$ the average value of the number of q panels with all spins $+1$, and by $\langle N_q^-(t)\rangle$ the average value of the number of q panels with all spins $+1$. Obviously,

$$\langle N_q^+(t)\rangle = \rho(t)^q L, \quad \langle N_q^-(t)\rangle = [1-\rho(t)]^q L, \tag{4}$$

where $\rho(t)$ is the fraction of spins $+1$ at time t. Let us now introduce the quantity

$$m_q(t) = \frac{\langle N_q^+(t)\rangle - \langle N_q^-(t)\rangle}{\langle N_q^+(t)\rangle + \langle N_q^-(t)\rangle} = 2\frac{\rho^q(t)}{\rho^q(t) + (1-\rho)^q(t)} - 1, \tag{5}$$

which for $q = 1$ (linear voter model) is simply average magnetization:

$$m_1(t) = \frac{\langle N_1^+(t)\rangle - \langle N_1^-(t)\rangle}{L} = 2\rho(t) - 1. \tag{6}$$

It is known that for the linear voter model average magnetization is constant, i.e., $m_1(t) = m_1(0)$ [14]. Knowing this, it is very easy to derive the exit probability,

$$m_1(\infty) = E(\rho) - [1 - E(\rho)] = 2E(\rho) - 1, \tag{7}$$

where we use the notation $\rho(0) \equiv \rho$. Because

$$m_1(\infty) = m_1(0) = 2\rho - 1, \tag{8}$$

we obtain

$$E(\rho) = \rho. \tag{9}$$

Now we can generalize this reasoning assuming that

$$m_q(\infty) = m_q(0). \tag{10}$$

With such an assumption we obtain the result for an arbitrary value of q:

$$E(\rho) = \frac{\rho^q}{\rho^q + (1-\rho)^q}. \tag{11}$$

Indeed substituting into (11) the value of $q = 1$ we obtain the known result $E(\rho) = \rho$, and for $q = 2$ we obtain formula (1) derived independently in four papers [10–13]. Of course, since we did not show any proof that our assumption (10) is valid, formula (11) can be treated as a guess. Let us start with checking validity of formula (11) by performing Monte Carlo simulations. In Fig. 2 we present both analytical and Monte Carlo results for several values of q. As seen there is perfect agreement, though the number of averaging is not very large (10^4 samples).

Now we will follow the reasoning presented in [12]. We check how the exit probability depends on the range of interactions R. Up until now we focused only on $R = 1$, but we have also provided simulations for $R = 2,3,4,5$, and 10. Results for $R = 1$ and $R = 5$ are presented in Fig. 3. Analogous results were also obtained for other values of R. It occurred that the exit probability for the q-voter model

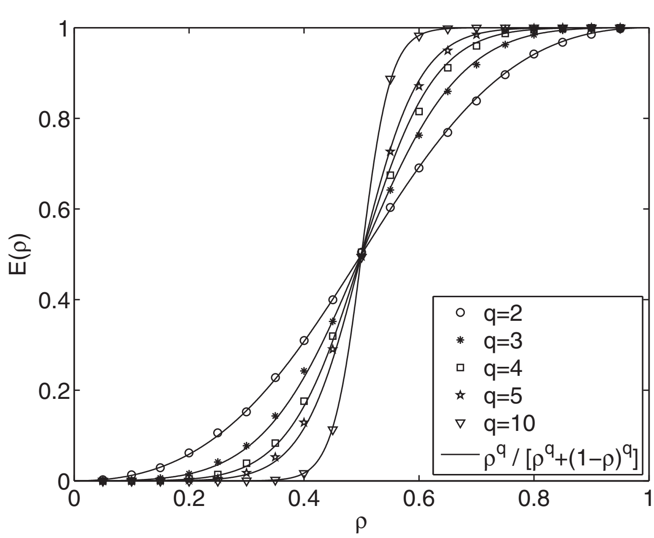

FIG. 2. Exit probability for the nonlinear q-voter model on a one-dimensional ring of length $L = 100$. Analytical formula agrees with the Monte Carlo results for any value of q. Averaging was done over 10^4 samples.

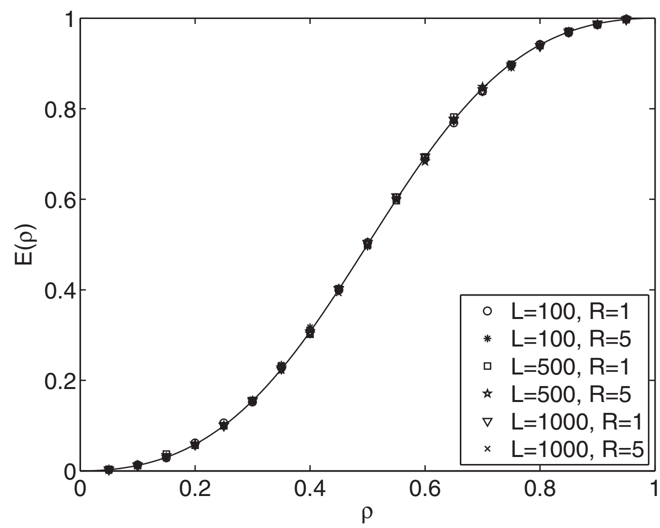

FIG. 3. Exit probability for nonlinear ($q = 2$)-voter model on a one-dimensional ring of length L. Exit probability changes neither with the lattice size L nor with the range of interactions. Analogous results have been obtained in [10–12]. Analogous results are valid for any value of q. Averaging was done over 10^4 samples.

is identical for any value of R. The same results have been obtained for the Sznajd model (i.e., $q = 2$) in [12]. Remarkably, the size of the system also does not influence the exit probability (see Fig. 3). The same results have been obtained earlier for the Sznajd model [11,12]. This is quite intriguing, since in the case of a q-voter model on a complete graph the size of the system influences the exit probability $E(\rho)$ [8]. Using the fact that the range of interactions does not change the exit probability, we can derive an analytical formula for $E(\rho)$ in the same way as in [12]. For $R \geqslant (L - q)/2$ (in the case of "both sides") or $R \geqslant L - q$ (for "random"), the system is fully ordered after the first unanimous q panel is chosen. Therefore the exit probability is equal to the probability of choosing a q panel of "up" spins:

$$E(\rho) = \frac{\rho^q}{\rho^q + (1-\rho)^q}. \tag{12}$$

This type of reasoning coincides, in a sense, with the idea presented in the work of Galam and Martins [13]. They have proposed simple quasideterministic procedure that drives the system to the absorbing ferromagnetic state with the same exit probability as in the Sznajd model. It their procedure the only probabilistic step is the choice of the first pair and further evolution is entirely deterministic. To see whether the first choice actually affects the probability of the final state, we performed two types of simulations. In the first step we picked at random a q-panel of "up" spins and then the system was evolving under the standard procedure, or we picked at random a q-panel of "down" spins. We have denoted the exit probability in the first case by $E^+(\rho)$, and in the second case by $E^-(\rho)$. If the first choice does not influence the exit probability we should obtain $E^+(\rho) = E^-(\rho)$. We have plotted in Fig. 4 the difference $E^+(\rho) - E^-(\rho)$ for several values of q. As seen, the difference grows with q, which is quite understandable. For large values of q it is difficult to find

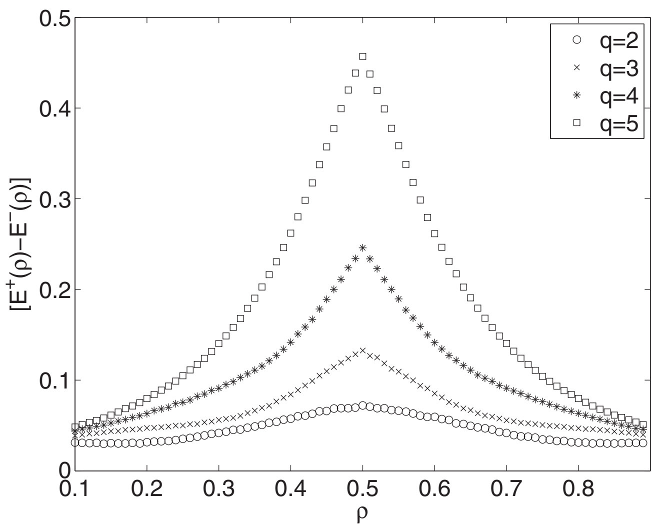

FIG. 4. Difference between two exit probabilities $E^+(\rho)$ and $E^-(\rho)$ for the lattice size $L = 100$ and the range of interaction $R = 1$. In the case of $E^+(\rho)$ a cluster of q "up" spins has been chosen in the first step of simulation, whereas in the case of $E^-(\rho)$ a cluster of q "down" spins has been chosen in the first step. It is seen that the first choice influences the exit probability. Averaging was done over 10^6 samples.

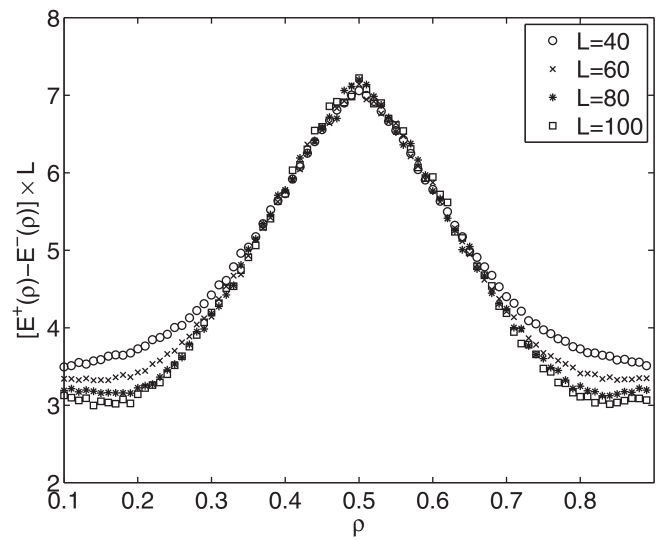

FIG. 5. Difference between two exit probabilities $E^+(\rho)$ and $E^-(\rho)$ for several lattice sizes L and the range of interaction $R = 1$. In the case of $E^+(\rho)$ a cluster of $q = 2$ "up" spins has been chosen in the first step of the simulation, whereas in the case of $E^-(\rho)$ a cluster of $q = 2$ "down" spins has been chosen in the first step. Results scale trivially with the system size L. Averaging was done over 10^6 samples.

a unanimous q panel, especially for $\rho \to 0.5$ (where only small clusters are present in the system). The probability of finding a q panel of "up" spins is equal to ρ^q, whereas the probability of a q panel of "down" spins is equal to $(1-\rho)^q$. This explains the shapes of the curves in Fig. 4. However, for $q = 2$ the importance of the first choice is not too high, and therefore the quasideterministic procedure proposed by Galam and Martins [13] cannot explain the analytic formula for $E(\rho)$. In addition, the results concerning the importance of the interaction's range R should be treated rather statistically. The fact that R does not influence the exit probability does not mean that it is not important at all. In the other case the first choice would determine the final state of the system, which is obviously not true (see Figs. 4 and 5).

Although the choice of the first pair does not determine the result in 100% of cases, the first steps of the simulation are actually the most important. To see this, let us present the probability $\rho(t)$ of choosing a q panel of "up" spins as a function of initial probability ρ for several values of t (see Fig. 6). As we can already see, after several Monte Carlo steps $\rho(t)$ coincides with $E(\rho)$. Why do the latter steps not change the form of $E(\rho)$?

To understand this, we should mention here that formula (11) is valid only for random initial conditions. If we start from two clusters—the first cluster of spins $+1$ and length ρL and the second one of spins -1 and length $(1-\rho)L$—we obtain the final state of all spins $+1$ with probability ρ. This is easy to understand because the probability in such a case of choosing a q-panel of "up" spins is equal to ρ (at least for the infinite system size L). The same result has been obtained for the Sznajd model in a more sophisticated way using the Kirkwood approach [11]. Therefore, as soon as the system orders to several domains, $E(\rho)$ will no longer continue to change.

IV. SUMMARY

A recent work [13] pointed to the puzzle emphasizing the mean-field character of the formula for the exit probability in the Sznajd model obtained in [10,11]. It has been shown that the continuous shape of the exit probability is a direct outcome of a mean-field treatment [13]. Two possible explanations have been given: most likely the finite size effects in the simulations, or as a alternative, the irrelevance of the fluctuations in the system. In this paper we have extended the puzzle to the nonlinear q-voter model on a one-dimensional lattice. We have proposed an analytical formula for the exit probability and checked its validity by Monte Carlo simulations for several

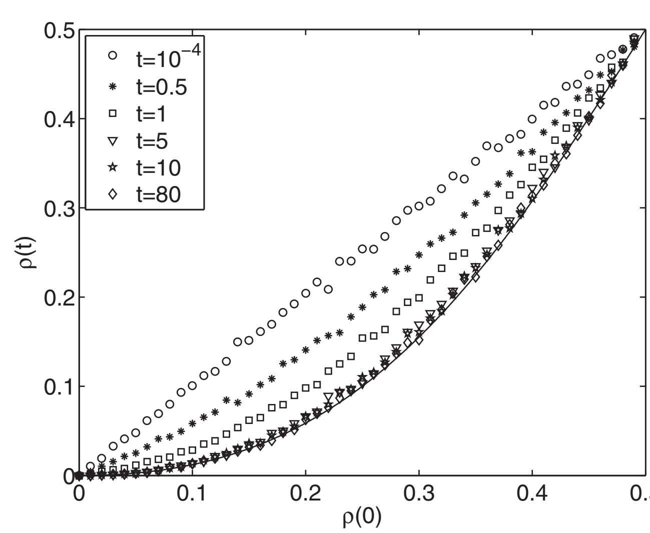

FIG. 6. Fraction of "up" spins after time t measured in the Monte Carlo Steps.

values of q. In particular, we have shown that neither the range of interactions nor the size of the system influence the exit probability. Moreover, we have checked the importance of the initial evolution of the system, as suggested for the Sznajd model in [12,13], to understand the proposed analytical formula. It should be stressed that the results presented here contain the well-known cases of the linear voter and Sznajd models.

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Phase diagram for a zero-temperature Glauber dynamics under partially synchronous updates

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We consider generalized zero-temperature Glauber dynamics under a partially synchronous updating mode for a one-dimensional system. Using Monte Carlo simulations, we calculate the phase diagram and show that the system exhibits phase transition between the ferromagnetic and active antiferromagnetic phases. Moreover, we provide analytical calculations that allow us to understand the origin of the phase transition and confirm simulation results obtained earlier for synchronous updates.

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I. INTRODUCTION

In the last decade renewed interest in Glauber dynamics [1] has been observed, especially at zero temperature [2–14]. This is partially caused by recent experiments with so-called single-chain magnets (for a recent review, see [15]) but is also due to the development of the nonequilibrium statistical physics. From this point of view one-dimensional systems at zero temperature are especially interesting [9].

The dynamical rules of stochastic models, such as Glauber dynamics, can be defined in terms of various update schemes, the most important ones being parallel (synchronous) and random-sequential (asynchronous) updates [16]. Although Glauber dynamics was originally introduced as a sequential updating process, interesting theoretical results can be obtained also using a synchronous updating mode [4,8,10,12, 13,17]. Moreover, clear evidence of a relaxation mechanism which involves the simultaneous reversal of spins has been shown experimentally for magnetic chains at low temperatures [18]. In computer simulations under the synchronous updating mode all units of the system are updated at the same time. However, in real systems one can expect that simultaneous reversal of spins concerns only a part of the system. From this point of view partially synchronous updates are the most realistic.

We introduced such a partially synchronous updating scheme in 2006 [12] to investigate the differences between Glauber and Sznajd dynamics for a chain of L Ising spins. Within such an update in each elementary time step we visit all sites and select each of them with probability c as a candidate to get flipped, i.e., on average, cL randomly chosen spins are considered in a single time step [12]. Of course $c = 1$ corresponds to the synchronous updating scheme and $c = 1/L$ corresponds to random sequential updates. Partially synchronous updates were also used in 2007 by Radicchi *et al.* [13] to investigate the Ising spin chain at zero temperature for the Metropolis algorithm [19]. They observed, as a function of c, a critical phase transition between two phases: a ferromagnetic phase and the so-called active phase. A similar phase transition had already been observed earlier for the generalized zero-temperature Glauber dynamics by Menyhard and Odor in the case of a synchronous updating scheme [4].

It should be noticed that the Metropolis algorithm at zero temperature is a special case of a broader class of zero-temperature Glauber dynamics. Within the Glauber dynamics for Ising spins with a spin $s = 1/2$, in a broad sense, each spin is flipped $S_i(t) \to -S_i(t+1)$ with a rate $W(\delta E)$ per unit time, and this rate is assumed to depend only on the energy difference implied in the flip. At zero temperature it can be defined as [9]

$$W(\delta E) = \begin{cases} 1 & \text{if} \quad \delta E < 0, \\ W_0 & \text{if} \quad \delta E = 0, \\ 0 & \text{if} \quad \delta E > 0. \end{cases} \tag{1}$$

The zero-temperature limits of the original Glauber dynamics [1] and Metropolis rates [19] (two of the most popular choices) are respectively $W_0^G = 1/2$ and $W_0^M = 1$.

Very recently, generalized Glauber dynamics defined by (1) under a synchronous updating mode have been studied [17]. It has been shown that the system exhibits a phase transition for $W_0 = W_c = 1/2$ between ferromagnetic and antiferromagnetic phases. As an order parameter, the density ρ of active bonds has been used:

$$\rho = \frac{1}{2L} \sum_{i=1}^{L} (1 - \sigma_i \sigma_{i+1}), \tag{2}$$

where L is the number of spins and $\sigma_i = \pm 1$ is the Ising spin variable at the ith site on the one-dimensional chain with the periodic boundary condition. Starting from a randomly disordered initial state (high-temperature situation), the system eventually approaches one of two steady states: fully ferromagnetic, $\rho_{st} = 0$, or fully antiferromagnetic, $\rho_{st} = 1$. In a previous paper [17] it has been suggested that for $W_c = 1/2$, in the case of synchronous updating, the system undergoes a discontinuous phase transition between two types of order. However, very recently, it has been claimed that the observed phase transition is rather continuous [20]. It has been shown that the dependence between the mean value of ρ_{st} and the control parameter W_0 scales with the system size L with scaling exponents $\beta = 0$ and $\nu = 1$ [20]. Moreover, the mean exit time needed to reach the stationary state also scales with the system size with the dynamical scaling exponent $z = 2$. According to [20], both scaling laws indicate continuous phase transition, contrary to the suggestion made in [17]. However, it should be noticed that trivial scaling exponents $\beta = 0$ and $\nu = 1/d$ (where d denotes spatial dimension, i.e., $d = 1$ in our case) are typical for the first order phase transitions, as shown both analytically by Fisher and Berker [21] and using Monte Carlo simulations by Binder and Landau [22]. We will come back to this problem in Sec. III.

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In this paper we consider zero-temperature Glauber dynamics defined by (1) under a partially synchronous updating mode. We show that both parameters W_0 and c are responsible for the phase transition between ferromagnetic and antiferromagnetic phases. We construct the phase diagram in (c,W_0) space based on the Monte Carlo simulations. Moreover, we provide exact analytical calculations for a simple case with only three active bonds. Such a simple approach allows us to understand the origin of the phase transition and shows that, indeed, for $c = 1$ the critical value $W_0 = 1/2$, which confirms the results obtained in [17,20].

II. THE MODEL

As mentioned above, we consider a one-dimensional chain of L Ising spins $\sigma = \pm 1$ with the periodic boundary condition, described by the Hamiltonian

$$H = -J \sum_{i=0}^{L} \sigma_i \sigma_{i+1}, \quad (3)$$

where $J > 0$, which means that we are dealing with a ferromagnetic system. We consider the system at temperature $T = 0$, and therefore we use the generalized Glauber dynamics defined by (1).

In our computer simulations we use partially synchronous updates, parametrized by $c \in [1/L,1]$, which allows us to tune the algorithm from a sequential ($c = 1/L$) to synchronous ($c = 1$) updating scheme. At time t we visit all sites of the chain and select each of them with probability c as a candidate to get flipped. Each of the selected sites is then updated according to the zero-temperature Glauber dynamics defined by (1). After one step of the algorithm, the time increases as $t \to t + c$. As usual, one Monte Carlo step (MCS) passes when the average number of update events equals the total number of sites L. We investigate quench from $T = \infty$ to $T = 0$, i.e., an initial state is disordered: at each site i there is a randomly chosen value of spin $\sigma_i = \pm 1$, and both values $\sigma_i = +1$ and $\sigma_i = -1$ are equally probable.

III. MONTE CARLO RESULTS

In the Monte Carlo simulations, relaxation processes in magnetic or reaction-diffusion systems are usually investigated by measuring the time evolution of so-called active bonds (domain walls) [16,23–25]. As already mentioned, under a synchronous updating scheme ($c = 1$), the system described by dynamical rule (1) eventually approaches one of two steady states: fully ferromagnetic, $\rho_{st} = 0$, or fully antiferromagnetic, $\rho_{st} = 1$. We start by clarifying the problem of the type of the phase transition between ferromagnetic and antiferromagnetic orders that occurs at $W_0 = 1/2$. As written above, very recently, it has been claimed that the observed phase transition is continuous [20], contrary to what has been suggested in [17]. It is true that discontinuous phase transitions are rare in one-dimensional, even nonequilibrium systems, but there are several lattice models that exhibit discontinuous absorbing phase transition in one dimension [16]. There are several phenomena attributed to discontinuous phase transitions, such as phase coexistence, hysteresis cycles, and trivial critical exponents, in particular, $\beta = 0$, which indicates a jump of an order parameter. As has been shown for $c = 1$, there is a phase coexistence at $W_0 = 1/2$ [17]. Moreover, it has been shown that critical exponents $\beta = 0$ and $\nu = 1$ [20], which is typical for the first order phase transitions [21,22].

To distinguish ultimately between continuous and discontinuous phase transitions, the hysteresis loop should be observed. To measure the hysteresis we have decided to start with two types of initial conditions: (1) for a disordered ferromagnet, we disturb the ferromagnetic order by flipping one spin: $\cdots \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \cdots$. (2) For a disordered antiferromagnet, we disturb the antiferromagnetic order by flipping one spin: $\cdots \uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow \cdots$.

In Fig. 1 the dependence between $\langle\rho_{st}\rangle$ and control parameter W_0 is shown for two values of c. It is clear that the phase transition for $c = 1$ is qualitatively different from the phase transition in the case of $c < 1$. In the latter case the initial condition does not influence significantly the asymptotic state, while in the case of synchronous updating a hysteresis loop can be observed.

Now we are ready to discuss results for $c < 1$. As we have already seen in Fig. 1, there is no hysteresis loop for $c < 1$. This result suggests that in this case the continuous phase transition probably occurs. In such a case we should observe the continuous change of order parameter $\langle\rho_{st}\rangle$, and therefore the antiferromagnetic state should not be an absorbing state for $c < 1$. To check these predictions let us start by presenting the time evolution of the average density of active bonds $\langle\rho(t)\rangle$. From Fig. 2 we see that starting from disordered initial conditions ($\langle\rho(0)\rangle = 0.5$), the number of active bonds rapidly decreases to zero below some threshold value W_c, and for $W_0 > W_c$ it increases to a certain stationary value $\langle\rho_{st}\rangle$ that depends on both c and W_0. This means that for $c < 1$ there is a phase transition between the ferromagnetic order and the so-called active phase [16]. To check if the phase transition is indeed continuous even for $c \lesssim 1$ we have conducted detailed simulations for $c = 0.9, 0.99, 0.999, 1$.

In Figs. 3 and 4 results for $c = 0.9$ and $c = 0.99$ are presented. The phase transition between ferromagnetically

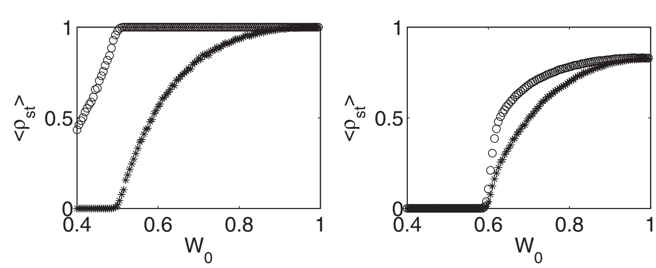

FIG. 1. The dependence between $\langle\rho_{st}\rangle$ and $W_0 \in [0,1]$ in the case of synchronous updating (left) $c = 1$ and (right) $c = 0.95$ for the lattice size $L = 100$ from two different initial states: open circles denote the antiferromagnetic initial state disturbed by flipping one spin, and stars denote the ferromagnetic initial state disturbed by flipping one spin. It is seen that for synchronous updating (left) there is a hysteresis loop: different steady states are reached for different initial conditions. For $c < 1$ (right) there is no hysteresis loop: for $W_0 < 0.6$ the ferromagnetic steady state is reached independently of the initial state, and for $W_0 > 0.6$ there is an active steady state with $\rho_{st} > 0$.

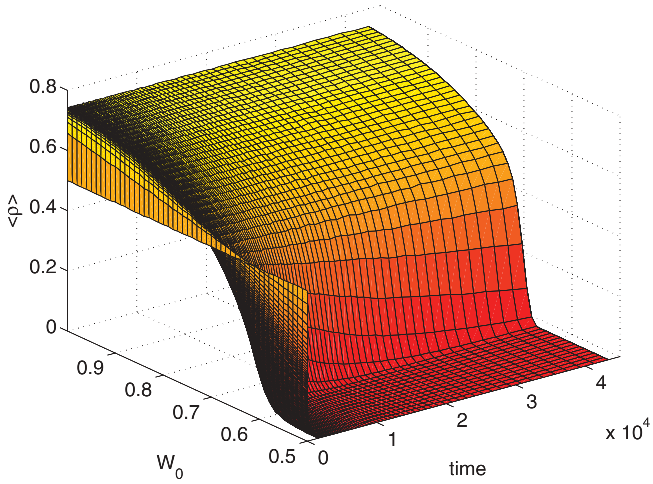

FIG. 2. (Color online) The average density of active bonds $\langle\rho\rangle$ as a function of time and W_0 for $c = 0.9$. It is seen that starting from disordered initial conditions ($\langle\rho(0)\rangle = 0.5$), the number of active bonds rapidly decreases to zero below some threshold value of W_0, and above this threshold value it increases to a certain stationary value $\langle\rho_{st}\rangle$ that depends both on c and W_0.

ordered and active phases is clearly visible, and the critical value of $W_0 = W_0(c)$, as well as the scaling exponents, can be estimated from the finite size scaling (see Table I). For all values of c the critical exponent $\nu = 1$, whereas $\beta = \beta(c)$ and for $c \to 1$ decreases with increasing c.

Finding precise values of critical exponents for all values of $c \in [0,1]$ is tedious but could be done. Here we were more interested in answering whether $c = 1$ is the only point at which the transition is discontinuous, and therefore we investigated $c \to 1$. According to our results, indeed, the discontinuous phase transition is observed only for $c = 1$, where generated clusters become compact (see Fig. 8). For $c < 1$ the transition is continuous, and β increases with the distance from the upper terminal point $c = 1$.

The exceptional behavior at the terminal point is due to the symmetry between ferromagnetic and antiferromagnetic states. Similar behavior is observed also in the Domany-Kinzel (DK) model and is usually referred to as compact directed percolation, which may be, in fact, misleading because the dynamics at this special point is the same as in the

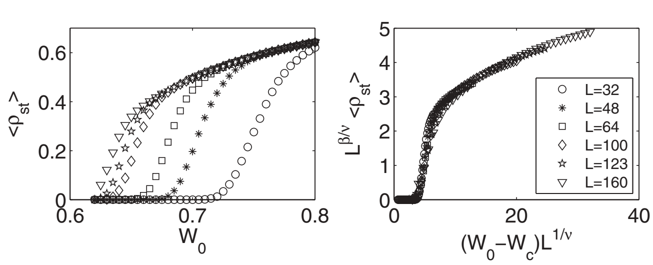

FIG. 3. (left) The average density of active bonds in the stationary state $\langle\rho_{st}\rangle$ as a function of W_0 for $c = 0.9$ and several lattice sizes L. The phase transition is clearly seen, and the critical value of W_0 can be found from the finite size scaling $W_c \approx 0.6$. (right) Results from the left panel are rescaled, showing clearly critical behavior with critical exponents $\nu = 1$ and $\beta \approx 0.4$.

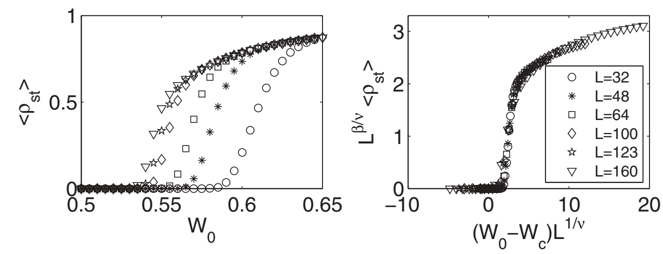

FIG. 4. (left) The average density of active bonds in the stationary state $\langle\rho_{st}\rangle$ as a function of W_0 for $c = 0.99$ and several lattice sizes L. The phase transition is clearly seen, and the critical value of W_0 can be found from the finite size scaling $W_c \approx 0.53$. (right) Results from the left panel are rescaled, showing clearly critical behavior with critical exponents $\nu = 1$ and $\beta \approx 0.25$.

$(1 + 1)$-dimensional Glauber-Ising model at zero temperature, or, equivalently, the voter model [16]. It should be recalled here that a DK model is a stochastic cellular automaton, and therefore it evolves by parallel updates, which for our model corresponds to $c = 1$, whereas the Glauber-Ising and voter model evolves by random sequential updating ($c = 1/L$). Therefore it is much easier to find direct correspondence between DK and our model with $c = 1$ than between our model and, e.g., the voter model. The DK model is characterized by two parameters, p_1 and p_2; p_1 is the probability that a site is activated if only one of two neighboring sites is active, and p_2 is the probability that the site is activated if both neighboring sites are active. In our model $p_2 = 1$ and p_1 corresponds to W_0. In the DK model for $p_2 = 1$ there is a discontinuous phase transition at $p_1 = 1/2$, which agrees exactly with the results obtained for our model with $c = 1$.

As we have written, the average density of active bonds in the stationary state $\langle\rho_{st}\rangle$ depends both on c and W_0. Up to now we have presented only the dependence between $\langle\rho_{st}\rangle$ and W_0 for several values of $c \to 1$. The average density of active bonds in the stationary state $\langle\rho_{st}\rangle$ as a function of W_0 and c is presented in Fig. 5. The transition line between ferromagnetically ordered ($\langle\rho_{st}\rangle = 0$) and active phases ($\langle\rho_{st}\rangle > 0$) is clearly visible. We have presented here results for a relatively small lattice size $L = 64$, although simulations were conducted also for larger systems, as presented in Figs. 3 and 4. Simulating smaller lattices allows us to measure the first passage time to one of the fully ordered states, i.e., with $\rho = 0$ or $\rho = 1$. As indicated, these two states are absorbing only for $c = 1$, and for $c < 1$ only $\rho = 0$ is an absorbing steady state. However, the small system still has nonzero probability to enter the antiferromagnetic state, although after it escapes from this state. Therefore we have decided to measure the mean time to enter one of the fully ordered states for the first

TABLE I. Approximate values of critical flipping probabilities and critical exponent β for several values of c.

c	W_c	β	ν
0.9	0.6	0.4	1.0
0.99	0.53	0.25	1.0
0.999	0.51	0.1	1.0
1	0.5	0	1.0

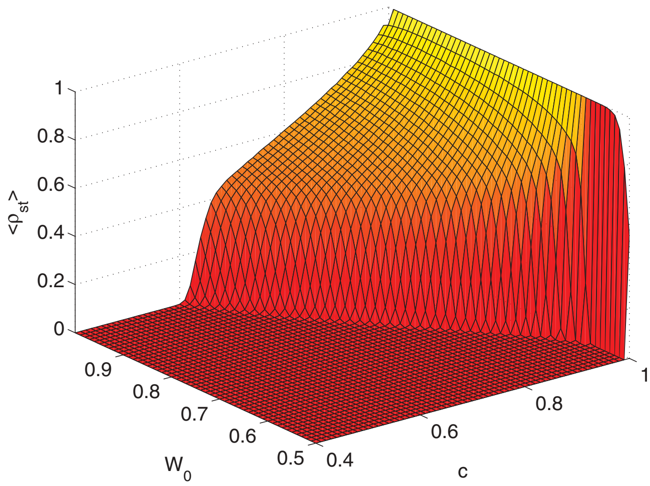

FIG. 5. (Color online) The average density of active bonds in the stationary state $\langle \rho_{\mathrm{st}} \rangle$ as a function of W_0 and c for lattice size $L = 64$. Simulations were conducted for 5×10^5 MCS, and averaging was done over 5×10^3 samples.

time $\langle \tau \rangle$ and see if any interesting behavior related to $\langle \tau \rangle$ will be seen along the transition line.

The mean first passage time $\langle \tau \rangle$ to reach one of the two types of fully ordered states (the so-called exit time [26]), ferromagnetic ($\rho = 0$) or antiferromagnetic ($\rho = 1$), as a function of W_0 and c for the lattice size $L = 64$ is presented in Fig. 6. It is seen that $\langle \tau \rangle$ dramatically increases approaching the transition line, which is an expected behavior. However,

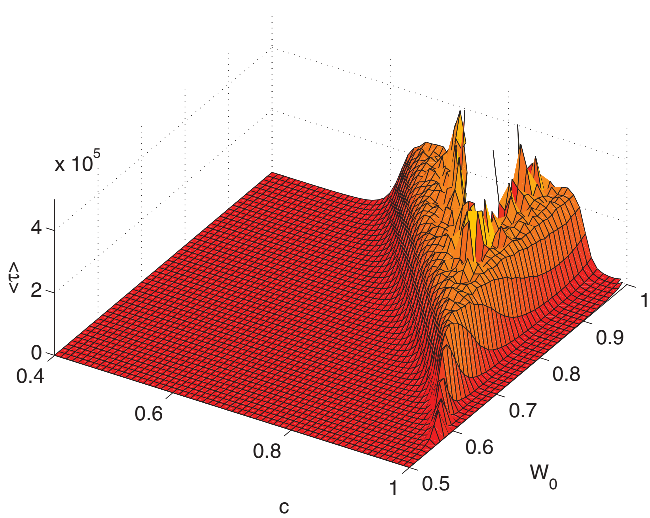

FIG. 6. (Color online) The mean exit time $\langle \tau \rangle$ to reach one of the two types of fully ordered states, ferromagnetic ($\rho = 0$) or antiferromagnetic ($\rho = 1$), as a function of W_0 and c for lattice size $L = 64$. Simulations were conducted for 5×10^5 MCS, and averaging was done over 5×10^3 samples. It is seen that below the transition line the system reaches the ordered ferromagnetic state quickly. Similarly, significantly above the transition line (large values of c) the system quickly reaches the ordered antiferromagnetic state, although for $c < 1$ this is not an absorbing state. The shape of a triangle, in which $\langle \tau \rangle$ dramatically increases, is seen. The hole inside the triangle indicates that none of the ordered states have been reached in 10^6 Monte Carlo steps.

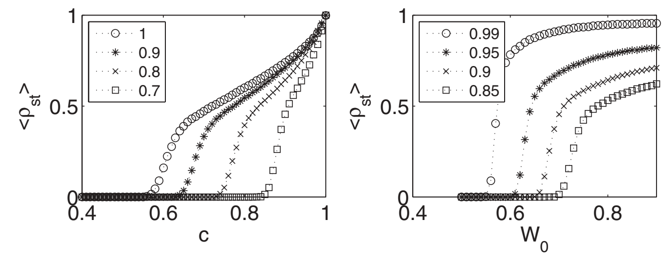

FIG. 7. The average density of active bonds in the stationary state $\langle \rho_{\mathrm{st}} \rangle$ (left) as a function of c for several values of W_0 and (right) as a function of W_0 for several values of c. The difference between the transitions with respect to c and W_0 is visible.

what is even more interesting is that it increases also along the transition line. For $c \to 1$ the mean first passage time $\langle \tau \rangle$ is relatively short, and it increases with the distance from the upper terminal point $c = 1$. Let us recall here that the same behavior is related to a critical exponent β. Colloquially speaking, the exit time increases with an increase in the continuity (β) of the transition. Therefore, although the antiferromagnetic state is not absorbing any longer for $c < 1$, the mean exit time $\langle \tau \rangle$ is a useful characteristic of an observed phase transition.

The last interesting feature connected to the phase transition seen in Fig. 5 is a difference between the transition along axis W_0 and c. The differences between the transitions with respect to c and W_0 are visible also in Fig. 7. For $c = 1$ the average density of active bonds in the stationary state $\langle \rho_{\mathrm{st}} \rangle$ is 1 for any $W_0 > 0.5$, whereas for $W_0 = 1$ the average density of active bonds in the stationary state $\langle \rho_{\mathrm{st}} \rangle$ depends on c. The transition with respect to c is much more gentle than the transition with respect to W_0.

The difference between phase transitions with respect to c and W_0 can also be seen from the time evolution of active bonds presented in Fig. 8. The phase transition for the Metropolis algorithm, i.e., $W_0 = 1$, which is induced by changing c, reminds us of typical annihilation: a branching process (right panel in Fig. 8). On the other hand, in the case of synchronous updating the growth of the antiferromagnetic domain from a single active bond can be observed (left panel in Fig. 8). In this case a kind of phase coexistence can be observed: ferromagnetic and antiferromagnetic clusters are present in the system.

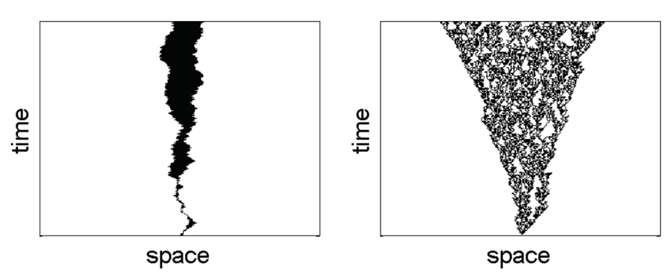

FIG. 8. Time evolution of active bonds near the phase transition (left) in the case of synchronous updating $c = 1$ induced by W_0 and (right) in the case of the Metropolis algorithm $W_0 = 1$ induced by c. In the initial state only one bond was active.

IV. THE ORIGIN OF THE PHASE TRANSITION

We have shown in the previous section that a one-dimensional system of Ising spins with generalized Glauber dynamics under partially synchronous updates exhibits well-defined phase transition between a stable ferromagnetic phase and an active phase. Moreover, it has been shown that for the synchronous updating scheme ($c = 1$) the system exhibits phase transition for $W_0 = 1/2$ between two absorbing stable states: ferromagnetically and antiferromagnetically ordered [17,20]. Although some mean field calculation has been provided [17], the origin of the investigated phase transition has not yet been.

The model considered here belongs to a broad class of so-called branching-annihilating random walks [16]; that is, three processes are possible: diffusion, branching, and annihilation of active bonds. It should also be mentioned that these processes conserve the number of active bonds modulo 2, which is usually called *parity conserving*. However, it has been shown that the conservation of the parity is not very relevant [16,27].

Let us first consider the simplest case, a chain of length L with a single active bond, i.e., $\cdots$ ↑↑↓↓ $\cdots$, at time t. Since we deal with a zero-temperature situation, changes are possible only on the domain wall (active bond). Therefore at time $t + c$ the single-bond system can evolve to

$\cdots$ ↑⇓⇑↓ $\cdots$ with probability $c^2 W_0^2 P_{L-2}$,
$\cdots$ ↑↑⇑↓ $\cdots$ with probability $[c(1-c) + c^2(1-W_0)]P_{L-2}$,
$\cdots$ ↑⇓↓↓ $\cdots$ with probability $[c(1-c) + c^2(1-W_0)]P_{L-2}$,
$\cdots$ ↑↑↓↓ $\cdots$ with probability $[(1-c)^2 + c^2(1-W_0)^2]P_{L-2}$,

where ⇑ and ⇓ denote spins that were flipped and

$$P_{L-2} = \sum_{k=0}^{L-2} \frac{(L-2)!}{k!(L-2-k)!} c^k (1-c)^{L-2-k} \tag{4}$$

denotes the sum of probabilities of all possible choices of remaining $L - 2$ spins.

Clearly, only the first process, which occurs with the probability $P_b = c^2 W_0^2 P_{L-2}$, leads to the growth of antiferromagnetic domains. The remaining three situations do not change the number of active bonds since the annihilation of a single active bond is impossible. Therefore the single-bond system can either remain unchanged or evolve to the system that consists of three neighboring active bonds, $\cdots$ ↑↑↓↑↓↓ $\cdots$. Analyzing all possible transitions in such a system (see Table II), we can calculate the probability of annihilation (P_a), branching (P_b), and diffusion (P_d):

$$\begin{aligned} P_a &= c^4 W_0^2 - c^2(1+2W_0) + 2c, \quad P_b = c^4 W_0^2, \\ P_d &= -c^4 W_0(2+W_0) - 2c^3\left(W_0^2 - 2W_0 - 1\right) \\ &\quad + c^2\left(W_0^2 - 4\right) + 2c. \end{aligned} \tag{5}$$

Of course there is also the possibility of no change in the system:

$$P_{no} = 1 - (P_a + P_b + P_d). \tag{6}$$

TABLE II. All possible outcome configurations from initial state ↑↑↓↑↓↓. Here ⇑ and ⇓ denote spins that were flipped. The constant factor P_{L-4}, which multiplies the right sides of the Eqs. (5), has been omitted to simplify notation. The initial state has three bonds.

After flip	Bonds	Probability
↑↑⇑↑↓↓	1	$c(1-c)^3 + c^3(1-W_0)^2(1-c)$ $+2c^2(1-c)^2(1-W_0)$
↑↑↓⇓↓↓	1	$c(1-c)^3 + c^3(1-W_0)^2(1-c)$ $+2c^2(1-c)^2(1-W_0)$
↑⇓↓↑↓↓	3	$cW_0(1-c)^3 + c^2W_0(1-c)^2(1-W_0)$
↑↑↓↑⇑↓	3	$cW_0(1-c)^3 + c^2W_0(1-c)^2(1-W_0)$
↑⇓⇑↑↓↓	3	$c^2W_0(1-c)^2 + c^3W_0(1-c)(1-W_0)$
↑↑↓⇓⇑↓	3	$c^2W_0(1-c)^2 + c^3W_0(1-c)(1-W_0)$
↑⇓↓⇓↓↓	1	$c^2W_0(1-c)^2 + c^3W_0(1-c)(1-W_0)$
↑↑⇑↑⇑↓	1	$c^2W_0(1-c)^2 + c^3W_0(1-c)(1-W_0)$
↑↑⇑⇓↓↓	1	$c^2(1-c)^2 + c^4(1-W_0)^2$
↑⇓↓↑⇑↓	3	$c^2W_0^2(1-c)^2$
↑⇓⇑⇓↓↓	3	$c^3W_0(1-c) + c^4W_0(1-W_0)$
↑↑⇑⇓⇑↓	3	$c^3W_0(1-c) + c^4W_0(1-W_0)$
↑⇓↓⇓⇑↓	3	$c^3W_0^2(1-c)$
↑⇓⇑↑⇑↓	3	$c^3W_0^2(1-c)$
↑⇓⇑⇓⇑↓	5	$c^4W_0^2$

The constant factor

$$P_{L-4} = \sum_{k=0}^{L-4} \frac{(L-4)!}{k!(L-4-k)!} c^k (1-c)^{L-4-k}, \tag{7}$$

which multiplies the right sides of the above equations, has been omitted to simplify the notation.

Now we can ask what the dependence is between parameters c and W_0 for which annihilation and branching are equally probable:

$$P_a = P_b \to -c^2(1+2W_0) + 2c = 0. \tag{8}$$

This means that annihilation and branching are equally probable for $c = 0$ or

$$c = \frac{2}{1+2W_0}. \tag{9}$$

From Eq. (9) we find that for synchronous updating, i.e., $c = 1$, the critical value of $W_0 = 1/2$, which confirms results obtained recently in [17,20]. Moreover, for $W_0 = 1$ we obtain the critical value of $c = 2/3$, which is also very close to the value obtained from Monte Carlo simulations (see Fig. 5). Therefore it seems that the phase transition between the ferromagnetic phase and antiferromagnetic active phase appears when annihilation and branching are equally probable.

Let us now present the dependence between probabilities (5) and parameter c for a given value of W_0. We focus on the Metropolis algorithm, i.e., $W_0 = 1$ (the case considered also in [13]). Results are presented in Fig. 9. Several interesting features of our system are visible.

(1) The value of c for which annihilation and branching are equally probable, i.e., $P_a = P_b$, agrees quite well with the critical value of c obtained from Monte Carlo simulations.

(2) The probability of diffusion has a maximum for the same value of c, for which $P_a = P_b$.

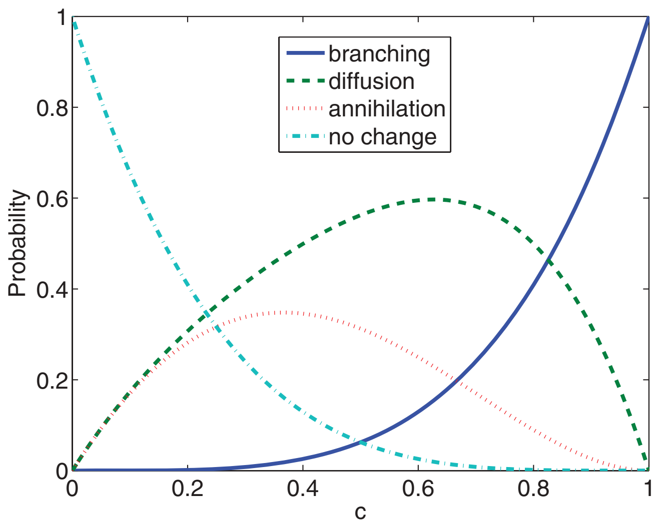

FIG. 9. (Color online) Probabilities of annihilation (P_a), branching (P_b), diffusion (P_d), and no change (P_{no}) as a function of updating scheme c for $W_0 = 1$.

(3) The probability of annihilation grows with c to a certain value, $c \sim 0.4$, and for $c > 0.4$ it decays. Simultaneously with decreasing P_a, the probability of branching grows, although it is still smaller then annihilation for $c < 2/3$. Therefore, one expects that eventually the system will still reach a ferromagnetic state, although branching of active bonds should be visible during time evolution. It should be mentioned here that in [13] the case of $W_0 = 1$ has also been studied, and the authors claimed that a system with partially synchronous updates exhibits phase transition for $c \sim 0.4$. However, our results (both types of Monte Carlo simulations and the simple analytical approach) suggest that the value of $c \sim 0.4$ corresponds merely to the situation in which the probability of annihilation starts to decay and branching appears.

V. SUMMARY

In this paper we have investigated one-dimensional systems of Ising spins driven by the generalized zero-temperature Glauber dynamics with a partially synchronous updating mode (tuned from sequential to synchronous by parameter c). It has been shown that for the synchronous updating mode, which corresponds to $c = 1$, there is a discontinuous phase transition between two ordered phases (ferromagnetic and antiferromagnetic). Three signatures of a discontinuous phase transition have been found in this case: (1) jump of an order parameter ($\beta = 0$), (2) phase coexistence, and (3) hysteresis cycles. Similar behavior has been observed in a one-dimensional Glauber-Ising model at zero temperature in a magnetic field, which is also known as compact directed percolation [16]. On the other hand, finding the precise values of critical exponents for $c < 1$ turned out to not be so easy a task. Nevertheless, the results obtained in this paper suggest that for any value of $c < 1$ there is a continuous order-disorder transition (between the ferromagnetic and so-called active phases). Using the finite scaling technique, we have shown that the critical exponent β has no single value along the transition line, i.e., $\beta = \beta(c)$, and it increases with the distance from the upper terminal point $c = 1$, at least for $c \to 1$. Finding the dependence between critical exponent β and c along the whole line, i.e., for $c \in [0,1]$, is quite tedious. Moreover, we were more interested in answering the question of whether $c = 1$ is the only point at which the transition is discontinuous and what the type of transition is for $c < 1$. Therefore we investigated $c \to 1$. The numerical findings of critical exponents are often difficult, and one should be careful when drawing conclusions only from simulations. However, it seems that the discontinuous phase transition for $c = 1$, similar to the Domany-Kinzel model [16,28], is exceptional due to an additional symmetry between active and inactive bonds.

Another interesting problem that could be investigated, but was not the subject of this paper, is the phase transition with respect to c for a given W_0. We have presented the general dependence between an order parameter $\langle \rho_{st} \rangle$ and parameters W_0 and c. We have also discussed briefly the differences between transitions with respect to W_0 and c. However, a detailed analysis has not been provided. The only results connected to this issue were obtained for $W_0 = 1$ in [13]. In this paper it has been shown that the phase transition can also be observed for any other value of $W_0 > 0.5$. It would be interesting to investigate this problem more precisely in the future.

To understand the origin of the phase transition we have provided a simple analytical approach and showed that transition occurs when branching and annihilation are equally probable, which is fulfilled for $W_0 = (2 - c)/2c$. Again, this confirms results from [17,20] since for $c = 1, W_0 = 1/2$, which was obtained earlier by Monte Carlo simulations and a simple mean field approach.

To conclude this work we would like to highlight one important issue that justifies the subject of the paper. As mentioned in the Introduction, clear evidence of a relaxation mechanism which involves the simultaneous reversal of spins has been shown experimentally for magnetic chains at low temperatures [18]. However, in [18] it has been suggested that the probability of simultaneous reversal of L spins scales as q^L (with certain parameter $q < 1$), which is not the same kind of macroscopic reversal which is assumed in this paper. Moreover, in [18] the simultaneous reversal of spins in a single segment has been considered, which is also very different from our approach. To be honest, we were not able to find any other example of a physical experiment that shows an evidence of simultaneous changes. One should also remember that Glauber dynamics, which has been introduced as a sequential updating process, satisfies the detailed balance condition and therefore ensures the existence of an equilibrium. There is thus a natural question of whether the model with partially simultaneous updating is merely another mathematical toy. Let us stress here that we strongly believe in toy models. They help to explore new regions and develop new fields even without meeting any reality. On the other hand, we understand the skepticism of people who would like to have even the smallest hope that the model would turn into something useful. We are not sure if partially synchronous or fully synchronous updating can describe a real physical experiment. On the other hand, the problem of updating methods is widely discussed in a recent work on cellular automata, Boolean networks, neural networks, and the so-called agent-based modeling in ecology and sociology [29–32]. It has been shown that the updating scheme can have an enormous influence on the model output [30]. It is also suggested that "the updating effects will be particularly marked in models with increasing interaction complexity such as models of interaction between many trophic levels." In this paper we show that the effect of the type of updating is clearly visible even within extremely simple model, which might be instructive, taking into account that many models of opinion dynamics are inspired by the Ising model [33]. In a world of agent-based modeling, asynchronous and synchronous updating are treated as two contrasting methods [33], and we see no reason why either of these two would be better than partially synchronous updating. As stated in [13], "Probably neither a completely synchronous nor a random asynchronous update is realistic for natural systems."

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Phase transitions in the q-voter model with two types of stochastic driving

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We study a nonlinear q-voter model with stochastic driving on a complete graph. We investigate two types of stochasticity that, using the language of social sciences, can be interpreted as different kinds of nonconformity. From a social point of view, it is very important to distinguish between two types nonconformity, so-called anticonformity and independence. A majority of work has suggested that these social differences may be completely irrelevant in terms of microscopic modeling that uses tools of statistical physics and that both types of nonconformity play the role of so-called social temperature. In this paper we clarify the concept of social temperature and show that different types of noise may lead to qualitatively different emergent properties. In particular, we show that in the model with anticonformity the critical value of noise increases with parameter q, whereas in the model with independence the critical value of noise decreases with q. Moreover, in the model with anticonformity the phase transition is continuous for any value of q, whereas in the model with independence the transition is continuous for $q \leqslant 5$ and discontinuous for $q > 5$.

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I. INTRODUCTION

Recently, various microscopic models of opinion dynamics have been proposed and widely studied by physicists and social scientists (for reviews see [1–4]). In the world of social studies this kind of modeling is known as agent-based modeling (ABM). It has been noted recently that despite the power of ABM in modeling complex social phenomena, widespread acceptance in the highest-level economic and social journals has been slow due to the lack of commonly accepted standards of how to use ABM rigorously [2,5]. As has been pointed out by Macy and Willer [3], one of the main problems in the field of social simulations is that there has been "little effort to provide analysis of how results differ depending on the model designs."

A similar problem is visible in a field of sociophysics. For example, to study opinion dynamics under conformity (one of the major paradigms of social response), a whole large class of models based on binary opinions $S = \pm 1$ has been proposed, among them the voter model [6,7], majority rule [8,9], the Sznajd model [10], and nonlinear voter models [11,12]. For all these models the ferromagnetic state is an attractor [1]. On one hand, this is expected since the conformity is the only factor influencing opinion dynamics in these models. On the other hand, this is obviously not realistic for real social systems. To make models of opinion dynamics more realistic several modifications has been proposed, among them the introduction of contrarians [13,14], inflexibles [15], and zealots [16]. From the social point of view all these modifications describe another major paradigm of social response—so-called nonconformity [17]. There are two widely recognized types of nonconformity: anticonformity and independence. From a social point of view, it is very important to distinguish between these two types of nonconformity [17,18]. The term "independence" implies the failure of attempted group influence. Independent individuals evaluate situations independently of the group norm. From this point of view both zealots, introduced by Mobilia [16], as well as inflexibles, introduced by Galam [15], describe a particular type of independent behavior. In contrast, anticonformists are similar to conformers in the sense that both take cognizance of the group norm—conformers agree with the norm, while anticonformers disagree. Therefore, the contrarians introduced by Galam in [13] or the stochastic driving proposed by de la Lama *et al.* [14] describe anticonformity.

Although differences between two types of nonconformity are very important for social scientists, the results obtained so far indicate that differences may be irrelevant from the physical point of view. Both contrarian and independent behaviors play the role of social temperature, which induces an order-disorder transition [13,14,19,20]. However, addressing the problem posed by Macy and Willer [3] we would like to check rigorously the differences between these two types of nonconformity under the framework of a possibly general model of opinion dynamics. In a class of models with binary opinions such a general model has been recently introduced in Ref. [12] under the name of the "q-voter model." As special cases this model consists of both the linear voter model as well as the Sznajd model. In this paper we investigate this model in the presence of different types of nonconformity and check whether results for anticonformity and independence are qualitatively the same, according to our first expectation. It should be mentioned that another general class of opinion dynamics, known as majority rule [8,9], would also be a good candidate to test the differences between these two types of nonconformity. However, introducing a general type of independence is not so straightforward in this case.

The paper is organized as follows. In the next section we introduce the generalized model with two types of nonconformity on a complete graph (topology, which is particularly convenient for analytical calculations). In Sec. III we analyze the time evolution of the system described by the master equation. In this section we will already see differences between models with anticonformity and independence, in contrast to our first prediction. In Sec. IV we calculate analytically the stationary values of public opinion in a case of an infinite system. The results presented in Secs. III and IV indicate clearly a phase transition between phases with

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and without majority. Therefore in Sec. V we find the phase diagram and calculate the transition point as a function of the model's parameters. The results presented in this section show clearly important qualitative differences between the two types of nonconformity. In Sec. VI we apply the approach that has been used to study nonequilibrium systems with two (Z_2) symmetric absorbing states in Refs. [21,22] to understand more deeply these differences and in particularly the origin of the discontinuous phase transition in the case of nonconformity. We conclude the paper in the last section.

II. MODEL

We consider a set of N individuals on a complete graph, which are described by the binary variables $S = \pm 1$. At each elementary time step q individuals $S_1, \dots, S_q$ (denoted by $\uparrow$ for $S_i = 1$ or $\downarrow$ for $S_i = -1$, where $i = 1, \dots, q$) are picked at random and form a group of influence, called a q lobby. Then the next individual ($\Uparrow$ or $\Downarrow$) that the group can influence, called the voter, is randomly chosen.

The part of a model described above is a special case of the nonlinear q-voter model introduced in Ref. [12]. In the original q-voter model, if all q individuals are in the same state, the voter takes their opinion; if they do not have a unanimous opinion, still a voter can flip with probability ϵ. For $q = 2$ and $\epsilon = 0$ the model is almost identical with Sznajd's model on a complete graph [23]. The only difference is that in the q-voter model repetitions in choosing neighbors are possible. In Ref. [24] the q-voter model with $\epsilon = 0$ and without repetition has been considered on a one-dimensional lattice. In this paper we also deal with a q-voter model with $\epsilon = 0$ and without repetition, but additionally we introduce a certain type of noise to the model. The original voter model describes only conformity, whereas noise is introduced to describe nonconformity.

In our model conformity and anticonformity take place only if the q lobby is homogeneous i.e., all q individuals are in the same state. In the case of conformity the voter takes the same decision as the q lobby (like in the original q-voter model), whereas in a case of anticonformity the voter takes the opposite opinion to that of the group. In the case of independent behavior, the voter does not follow the group but acts independently—with probability 1/2 it flips to the opposite direction, i.e., $S_{q+1} \to -S_{q+1}$.

To check the differences in results that are caused by different types of nonconformity we consider three versions of the model:

Anticonformity I: With probability p_1 the voter behaves like a conformist and with p_2 like an anticonformist. This type of anticonformity has been investigated in a case of the Sznajd model on a complete graph in Ref. [20]. Because results depend only on the ratio $p = p_1/p_2$, in this paper we consider $p_1 = 1$ and $p_2 = p$. In this version of the model the following changes are possible:

$$\underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Downarrow \xrightarrow{p_1=1} \underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Uparrow,$$
$$\underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Uparrow \xrightarrow{p_1=1} \underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Downarrow,$$
$$\underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Uparrow \xrightarrow{p_2=p} \underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Downarrow,$$
$$\underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Downarrow \xrightarrow{p_2=p} \underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Uparrow. \quad (1)$$

In other cases nothing changes.

Anticonformity II: With probability p the voter behaves like an anticonformist and with $1 - p$ like a conformist. This type of anticonformity has been investigated in a case of the Sznajd model on several networks in Ref. [14] and results were qualitatively the same as in Ref. [20]. Indeed it is quite easy to notice that anticonformity II is a special case of anticonformity I. However, for the record we consider here both cases and show that indeed differences are only quantitative. In this case the following changes are possible:

$$\underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Downarrow \xrightarrow{1-p} \underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Uparrow,$$
$$\underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Uparrow \xrightarrow{1-p} \underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Downarrow,$$
$$\underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Uparrow \xrightarrow{p} \underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Downarrow,$$
$$\underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Downarrow \xrightarrow{p} \underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Uparrow. \quad (2)$$

In other cases nothing changes.

Independence: With probability p the voter behaves independently and with $1 - p$ like a conformist. In the case of independent behavior an individual changes to the opposite state with probability 1/2. The following changes are possible:

$$\underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Downarrow \xrightarrow{1-p} \underbrace{\uparrow\uparrow\dots\uparrow}_{q}\Uparrow,$$
$$\underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Uparrow \xrightarrow{1-p} \underbrace{\downarrow\downarrow\dots\downarrow}_{q}\Downarrow,$$
$$\underbrace{\dots}_{q}\Downarrow \xrightarrow{p/2} \underbrace{\dots}_{q}\Uparrow,$$
$$\underbrace{\dots}_{q}\Uparrow \xrightarrow{p/2} \underbrace{\dots}_{q}\Downarrow. \quad (3)$$

In other cases nothing changes.

III. TIME EVOLUTION

In a single time step Δ_t, three events are possible: the number of "up" spins, $N_\uparrow$, increases, decreases by 1, or remains constant. Of course, all three events can be rewritten for the number of "down" spins $N_\downarrow$ as $N_\uparrow + N_\downarrow = N$. Also the concentration $c = N_\uparrow/N$ of spins up increases or decreases by $\Delta_N = 1/N$ or remains constant:

$$\begin{aligned}\gamma^+(c) &= \text{Prob}\{c \to c + \Delta_N\},\\ \gamma^-(c) &= \text{Prob}\{c \to c - \Delta_N\},\\ \gamma^0(c) &= \text{Prob}\{c \to c\} = 1 - \gamma^+(c) - \gamma^-(c).\end{aligned} \quad (4)$$

The time evolution of the probability density function of c is given by the master equation [7]

$$\begin{aligned}\rho(c,t+\Delta_t) &= \gamma^+(c-\Delta_N)\rho(c-\Delta_N,t)\\ &\quad + \gamma^-(c+\Delta_N)\rho(c+\Delta_N,t)\\ &\quad + [1-\gamma^+(c)-\gamma^-(c)]\rho(c,t). \end{aligned} \tag{5}$$

Of course, an analogous formula can be written for $N_\uparrow$. The exact forms of the probabilities $\gamma^+(c)=\gamma^+(N_\uparrow)=\gamma^+$ and $\gamma^-(c)=\gamma^-(N_\uparrow)=\gamma^-$ depend on the version of the model; for a finite system they are the following: For anticonformity I,

$$\begin{aligned}\gamma^+ &= \frac{N_\downarrow\prod_{i=1}^{q}(N_\uparrow-i+1)+p\prod_{i=1}^{q+1}(N_\downarrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)},\\ \gamma^- &= \frac{N_\uparrow\prod_{i=1}^{q}(N_\downarrow-i+1)+p\prod_{i=1}^{q+1}(N_\uparrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)};\end{aligned} \tag{6}$$

for anticonformity II,

$$\begin{aligned}\gamma^+ &= \frac{(1-p)N_\downarrow\prod_{i=1}^{q}(N_\uparrow-i+1)+p\prod_{i=1}^{q+1}(N_\downarrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)},\\ \gamma^- &= \frac{(1-p)N_\uparrow\prod_{i=1}^{q}(N_\downarrow-i+1)+p\prod_{i=1}^{q+1}(N_\uparrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)};\end{aligned} \tag{7}$$

and for independence,

$$\begin{aligned}\gamma^+ &= \frac{(1-p)N_\downarrow\prod_{i=1}^{q}(N_\uparrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)}+\frac{pN_\downarrow}{2N},\\ \gamma^- &= \frac{(1-p)N_\uparrow\prod_{i=1}^{q}(N_\downarrow-i+1)}{\prod_{i=1}^{q+1}(N-i+1)}+\frac{pN_\uparrow}{2N}.\end{aligned} \tag{8}$$

For an infinite system the above formulas take much simpler forms: For anticonformity I,

$$\begin{aligned}\gamma^+ &= (1-c)c^q + p(1-c)^{q+1},\\ \gamma^- &= c(1-c)^q + pc^{q+1};\end{aligned} \tag{9}$$

for anticonformity II,

$$\begin{aligned}\gamma^+ &= (1-p)(1-c)c^q + p(1-c)^{q+1},\\ \gamma^- &= (1-p)c(1-c)^q + pc^{q+1};\end{aligned} \tag{10}$$

and for independence,

$$\begin{aligned}\gamma^+ &= (1-p)(1-c)c^q + p(1-c)/2,\\ \gamma^- &= (1-p)c(1-c)^q + pc/2.\end{aligned} \tag{11}$$

Solving analytically master equation (5) is not an easy task, but exact formulas for γ^+ and γ^- allow for a numerical solution of the equation. For an arbitrary initial state the system reaches the same steady state. In the case of an infinite system the probability density function is a sum of delta functions, $\rho_{st}(c)=\delta(c-c_1)+\delta(c-c_2)+\cdots+\delta(c-c_k)$, whereas for a finite system $\rho_{st}(c)$ has maxima (peaks) for the $c=c_j$ $j=1,\ldots,k$, which are getting higher and more narrow with the system size, approaching the deltas for the infinite system. The number of peaks, k, and values $c_1,\ldots,c_k$ depend on the version of the model, as well as on the model's parameters p and q.

Examples of the stationary probability density functions for the q lobby of size $q=7$ and a system size of $N=200$ are presented in Figs. 1 (anticonformity I) and 2 (nonconformity). As seen from Figs. 1 and 2, for small values of noise p (whether the noise is introduced as independence or anticonformity) the system is polarized, whereas for large values of p there is no majority in the system. However, the transition from the phase with majority to the phase without majority is very different for each type of noise. In the case with anticonformity we observe a continuous phase transition for arbitrary values of q, whereas in the case with nonconformity there is a continuous phase transition for $q \leqslant 5$ and a discontinuous phase transition for $q > 5$.

In the case with anticonformity two states with majority, represented by two equally high peaks, are stable below the critical value of p^*. As $p < p^*$ increases the two peaks approach each other and eventually for $p = p^*$ they form a single peak, which is a typical picture for a continuous phase transition (see Fig. 1) [25,26]. In the case with independence this picture is valid only for the lobby $q \leqslant 5$. For $q > 5$

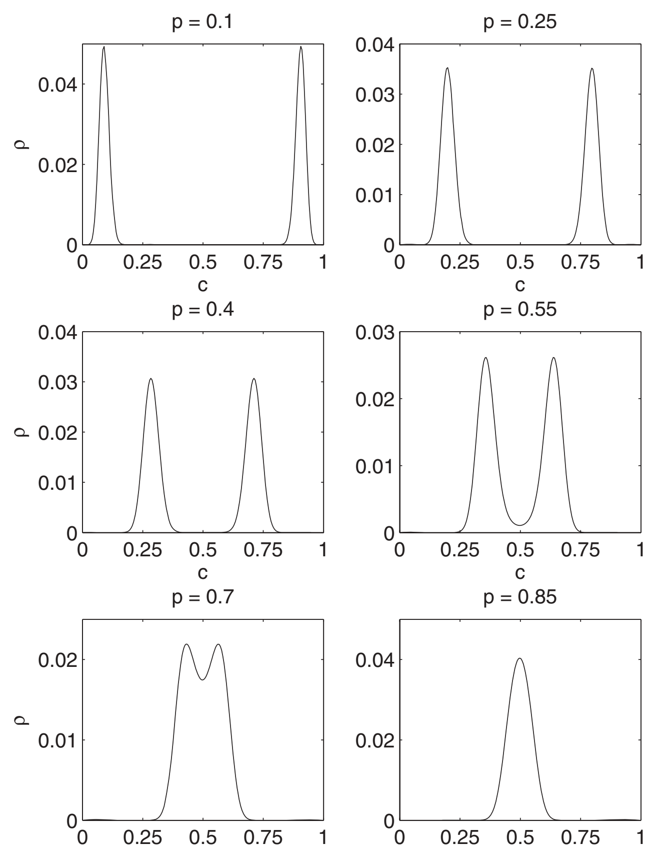

FIG. 1. Stationary probability density function of the concentration of up spins for the q-voter model with anticonformity I for a system of $N=200$ individuals and a lobby size of $q=7$. As seen for small values of anticonformity p the system is polarized, but for large values of p there is no majority in the system. For $p=0$ (the case without anticonformity) the system consists of all spins up or all spins down. With increasing p maxima are getting lower and approaching each other. Eventually they form a single maximum. This is typical behavior for a continuous phase transition. The critical value of p can be found analytically (see Sec. V) and depends on q.

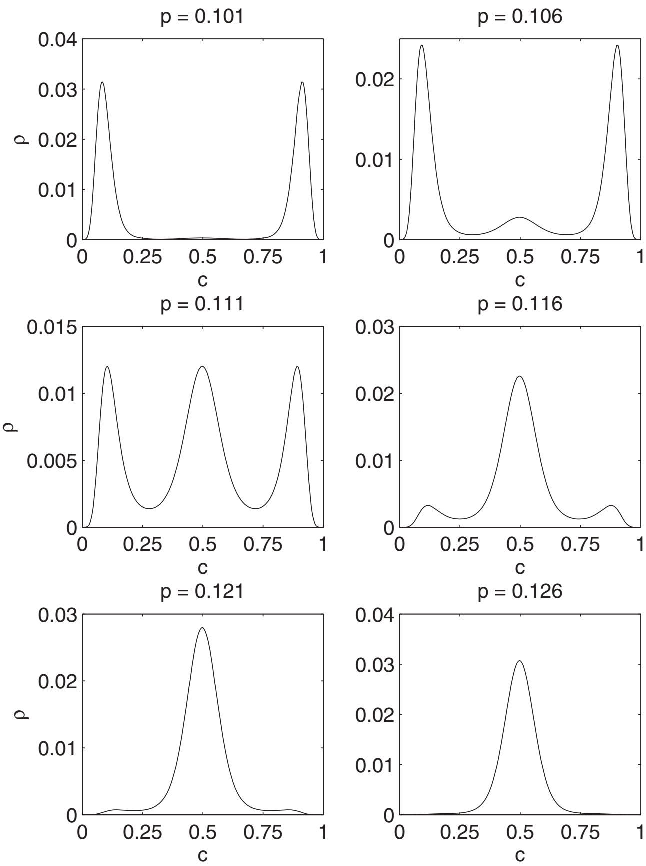

FIG. 2. Stationary probability density function of the concentration of up spins for the q-voter model with independence for a system of $N = 200$ individuals and a lobby size of $q = 7$. As seen for small values of independence p the system is polarized, but for large values of p there is no majority in the system. For $p = 0$ (the case without independence) the system consists of all spins up or all spins down. For larger values of p the third maximum appears at $c = 1/2$ (no majority). This maximum increases with p while the remaining two maxima are decreasing. Above a certain value of p there is only one maximum for $c = 1/2$. This is typical behavior for a discontinuous phase transition for which we can observe the phase coexistence.

the transition is very different. Again for small values of p there are two peaks but with increasing p they are not approaching each other. Instead, for $p = p_1^*$ the third peak appears at $c = 1/2$ (see Fig. 2). The third peak is initially lower than the remaining two peaks, which means that it represents a metastable state. As $p > p_1^*$ increases the third peak grows and for $p = p_2^*$ all three peaks have the same height. For $p > p_2^*$ the central peak dominates over the other two, which means that the state $c = 1/2$ is stable and the remaining two are metastable. Finally, for $p = p_3^*$ the side peaks disappear and only the center peak remains. This is a typical picture for a discontinuous phase transition, which takes place at $p = p_2^*$ [25,26]. Two values of the independence parameter, $p = p_1^*$ and $p = p_3^*$, demarcate the existence of metastability (spinodal lines) [27,28]. Values p_1^*, p_2^*, and p_3^* depend on the size of the lobby q, which will be shown exactly in Sec. V.

Before moving on to the analytical results for the infinite system and determining the points of phase transitions, let us present the time evolution. We stop for a moment to focus on the case of a finite system. Having exact formulas for transition probabilities γ^+ and γ^- we are able not only to calculate numerically the stationary density function $\rho_{st}(c)$ but also to generate sample trajectories of concentration (Figs. 3–6). In the case of a finite system spontaneous transitions between states are possible. In the case with anticonformity transitions between two states, which correspond to peaks in the probability density function $\rho_{st}(c)$, are possible below a critical value of p. Because both peaks are equally high the system spends the same amount of time on average in each state.

This is also true in the case with independence and $q \leqslant 5$ (see Fig. 4). However, as we have already written for $q > 5$ there is a discontinuous phase transition between states with and without majority and for $p \in (p_1^*, p_3^*)$ there are three possible states. Therefore for $q > 5$ we expect spontaneous transitions among three states.

Such transitions are indeed observed. For $p \in (p_1^*, p_2^*)$ the state with majority is stable and the state without majority is metastable. Therefore, the system spends more time in states with majority. For $p \in (p_2^*, p_3^*)$ the situation is exactly the opposite—the state without majority is stable. At a transition point $p = p_2^*$ all three states are stable and the system spends the same time on average in each of three states (see Figs. 5 and 6).

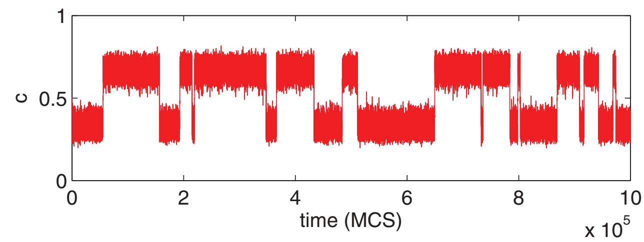

FIG. 3. (Color online) Time evolution of the concentration of up spins for the model with anticonformity with lobby $q = 7$ and level of anticonformity $p = 0.5$. The system size is $N = 200$. Spontaneous transitions between two stationary states are visible.

IV. STATIONARY CONCENTRATION

In the stationary state we expect that the probability of growth, γ^+, should be equal to the probability of loss, γ^-, and

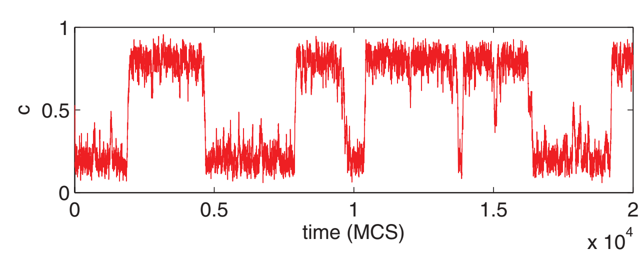

FIG. 4. (Color online) Time evolution of the concentration of up spins for the model with independence with lobby $q = 5$ and level of anticonformity $p = 0.175$. The system size is $N = 200$. Spontaneous transitions between two stationary states are visible.

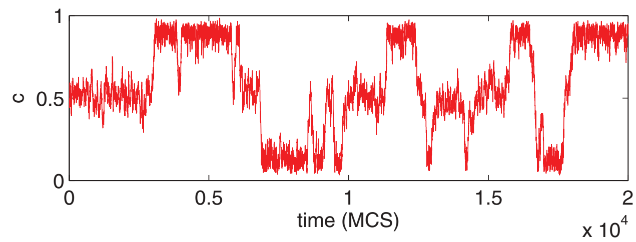

FIG. 5. (Color online) Time evolution of the concentration of up spins for the model with independence with lobby $q = 7$ and level of anticonformity $p = 0.111$ (where for this value all three states are stable). The system size is $N = 200$. Spontaneous transitions among three stationary states are visible.

therefore

$$F(c,q,p) = \gamma^{+}(c,q,p) - \gamma^{-}(c,q,p) = 0, \qquad (12)$$

where $F(c,q,p)$ can be treated as an effective force, γ^{+} drives the system to the state spins up, and γ^{-} drives them to spins down. Therefore we can easily calculate also an effective potential:

$$V(c,q,p) = -\int F(c,q,p)\,dc. \qquad (13)$$

To calculate stationary values of concentration we simply solve the equation

$$F(c,q,p) = 0, \qquad (14)$$

or, alternatively, find the minima of the potential V. Although the first possibility is more straightforward, we will see in the next section that knowing the form of the potential will help us to calculate the transition points.

The exact forms of the force F and the potential V for an infinite system are as follows: For anticonformity I,

$$\begin{aligned} F &= (1-c)c^{q} + p(1-c)^{q+1} - c(1-c)^{q} - pc^{q+1},\\ V &= -\frac{1}{q+1}(c^{q+1} + (1-c)^{q+1})\\ &\quad + \frac{p+1}{q+2}(c^{q+2} + (1-c)^{q+2}); \end{aligned} \qquad (15)$$

for anticonformity II,

$$\begin{aligned} F &= (1-p)(1-c)c^{q} + p(1-c)^{q+1}\\ &\quad - (1-p)c(1-c)^{q} - pc^{q+1},\\ V &= -\frac{1-p}{q+1}(c^{q+1} + (1-c)^{q+1})\\ &\quad + \frac{1}{q+2}(c^{q+2} + (1-c)^{q+2}); \end{aligned} \qquad (16)$$

and for independence,

$$\begin{aligned} F &= (1-p)(1-c)c^{q} + \frac{p(1-c)}{2}\\ &\quad - (1-p)c(1-c)^{q} - \frac{pc}{2},\\ V &= -\frac{1-p}{q+1}(c^{q+1} + (1-c)^{q+1})\\ &\quad + \frac{1-p}{q+2}(c^{q+2} + (1-c)^{q+2}) - \frac{p}{2}c(1-c). \end{aligned} \qquad (17)$$

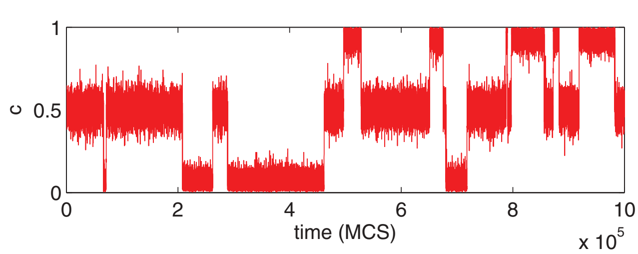

FIG. 6. (Color online) Time evolution of the concentration of up spins for the model with independence with lobby $q = 9$ and level of anticonformity $p = 0.0685$. The system size is $N = 200$. Spontaneous transitions among three stationary states are visible.

Solving analytically Eq. (14), i.e., finding c_{st} as a function of p for an arbitrary value of q, is impossible, but we can easily derive the opposite relations satisfying Eq. (14): For anticonformity I,

$$p = \frac{c_{st}(1-c_{st})^{q} - (1-c_{st})c_{st}^{q}}{(1-c_{st})^{q+1} - c_{st}^{q+1}}, \qquad (18)$$

for anticonformity II,

$$p = \frac{c_{st}(1-c_{st})^{q} - (1-c_{st})c_{st}^{q}}{(1-c_{st})^{q+1} + c_{st}(1-c_{st})^{q} - (1-c_{st})c_{st}^{q} - c_{st}^{q+1}}, \qquad (19)$$

and for independence,

$$p = \frac{c_{st}(1-c_{st})^{q} - (1-c_{st})c_{st}^{q}}{(1-c_{st})/2 + c_{st}(1-c_{st})^{q} - (1-c_{st})c_{st}^{q} - c_{st}/2}. \qquad (20)$$

We have used the above formulas to plot the dependence between steady value of concentration c_{st} and the level of noise p for several values of q (see Fig. 7). Although only the relation $p(c_{st})$ is calculated analytically and the opposite relation is unknown, we plot $c_{st}(p)$ by simply rotating the figure with the relation $p(c_{st})$. Clear differences between the two types of noise are visible—in a case with anticonformity the transition value of p increases with q and in a case with independence it decreases with p. Moreover, the type of transition is the same for arbitrary values of q in the case with anticonformity, whereas in the case with independence the transition between phases with and without majority changes its character for $q > 5$.

It should be also noticed that formulas (18)–(20) have been obtained from condition (14); i.e., they correspond to extreme values of potentials (15)–(17). However, only the minima of the potential correspond to the stable value of concentration. Therefore, in Figs. 7 and 8 we have denoted unstable values that correspond to the maxima of potentials by dotted lines. Moreover, we have presented the flow diagram for chosen values of q to show precisely which state is reached from given initial conditions. Particularly interesting behavior is related with independence (bottom panel in Fig. 8). Starting from two different initial concentrations disorder or order can be reached as a steady state (hysteresis).

In the next section we derive analytically transition points using knowledge of the effective potentials V in Eqs. (15)–(17).

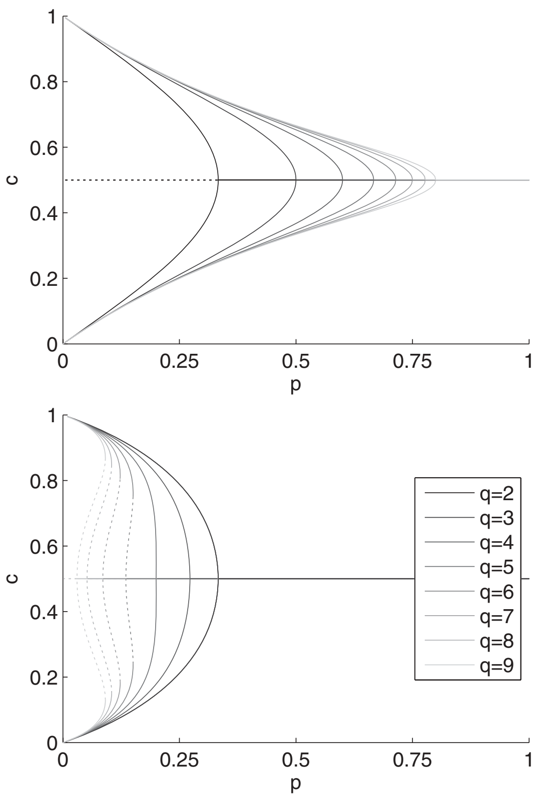

FIG. 7. Phase diagram for the models with anticonformity (top panel) and independence (bottom panel). Dependencies between steady values of concentration c_{st} and the level of noise p for several values of q are plotted using formulas (18)–(20). Although only relation $p(c_{st})$ is calculated analytically and the opposite relation is unknown, we plot $c_{st}(p)$ simply by rotating the figure. Dotted lines have been used to mark instability. Although both types of line (solid and dotted) are obtained from Eq. (14), i.e., correspond to extreme values of potentials (15) and (17), only solid lines denotes stable values, i.e., correspond to the minima of potentials (see also Fig. 8). A clear difference between two types of noise is visible—in a case with anticonformity the transition value of p increases with q and in a case with independence it decreases with p. Moreover, the type of transition is the same for arbitrary values of q in a case with anticonformity, whereas in a case with independence the transition between phases with and without majority changes its character for $q > 5$.

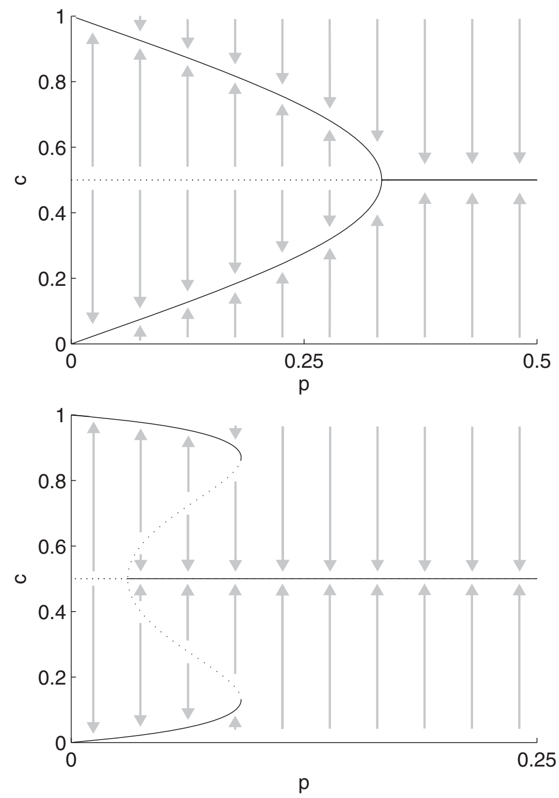

FIG. 8. Flow diagrams for the models with anticonformity for $q = 2$ (top panel) and independence for $q = 9$ (bottom panel). Particular values of q have been chosen just as examples and the dependencies between stationary values of c and parameter p for other values of q are seen in Fig. 7. Here solid lines denote stable (attracting) steady values of concentration that correspond to the minima of potentials (15)–(17), whereas dotted lines denote unstable values of c that correspond to maxima of potentials. Arrows denote the direction of flow, i.e., how the concentration changes in time. Particularly interesting behavior is related with independence (bottom panel). Starting from two different initial concentrations disorder or order can be reached as a steady state (hysteresis).

V. PHASE TRANSITIONS

As already noticed there is a continuous phase transition for the model with anticonformity I and II for arbitrary values of q. Below a critical value $p = p^*(q)$ the effective potential has two minima and above the critical value it has only one. Consequently, the stationary probability density function $\rho_{st}(c)$ for $p < p^*$ has two maxima and for $p > p^*$ only one at $c = 1/2$ (i.e., there is no majority in the system). Analogous behavior is observed for the model with nonconformity but only for $q \leqslant 5$. In all these cases we can easily calculate the critical value p^* by making a simple observation concerning the behavior of the effective potentials (15)–(17) for $q \leqslant 5$ at $c = 1/2$ (see also Fig. 1 for clarity): For $p < p^*$ potentials $V(c,p,q)$ have the maximum values for $c = 1/2$ and therefore

$$\left.\frac{\partial^2 V(c,p,q)}{\partial c^2}\right|_{c=\frac{1}{2}} < 0. \tag{21}$$

For $p > p^*$ potentials $V(c,p,q)$ have the minimum values for $c = 1/2$ and therefore

$$\left.\frac{\partial^2 V(c,p,q)}{\partial c^2}\right|_{c=\frac{1}{2}} > 0. \tag{22}$$

This means that for $p = p^*$ the maximum changes to the minimum at $c = 1/2$:

$$\left.\frac{\partial^2 V(c,p,q)}{\partial c^2}\right|_{c=\frac{1}{2}} = 0 \Rightarrow \left.\frac{\partial F(c,p,q)}{\partial c}\right|_{c=\frac{1}{2}} = 0. \tag{23}$$

Hence, the critical values are as follows: For anticonformity I,

$$p^*(q) = \frac{q-1}{q+1}, \tag{24}$$

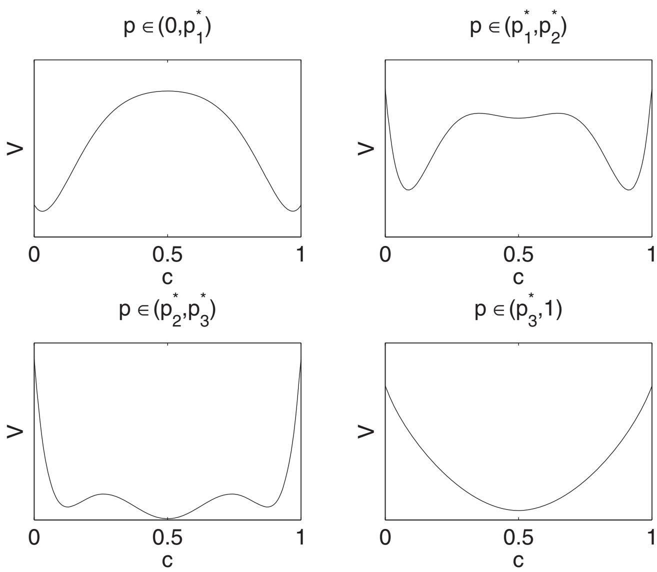

FIG. 9. Schematic plot of a potential for the model with independence and $q > 5$. For $p \in (0,p_1^*)$, potential $V(c,p,q)$ has two minima that correspond to the states with majority. For $p \in (p_1^*,p_2^*)$, the potential $V(c,p,q)$ has three minima and the state without majority is metastable. For $p \in (p_2^*,p_3^*)$, the potential $V(c,p,q)$ has three minima and the states with majority are metastable. Finally, for $p \in (p_3^*,1)$, the potential $V(c,p,q)$ has only one minimum that corresponds to the state without majority. The exact form of the potential is given by Eq. (17).

for anticonformity II,

$$p^*(q) = \frac{q-1}{2q}, \tag{25}$$

and for independence with $q \leqslant 5$,

$$p^*(q) = \frac{q-1}{q-1+2^{q-1}}. \tag{26}$$

As we see, simple calculations allowed us to find the critical points for almost all cases, except for the model with nonconformity for $q \geqslant 6$. In all cases considered above, there is a continuous phase transition between phases with and without majority. However, for the model with independence and $q \geqslant 6$ the phase transition becomes discontinuous, which has been already discussed in Sec. III. This behavior can be also suspected from the form of the effective potential (17), which for $q \geqslant 6$ has the following properties (see also Fig. 9):

For $p \in \langle 0,p_1^*)$, $V(c,p,q)$ has two minima.
For $p = p_1^*$, in $V(c,p,q)$ a third minimum emerges.
For $p \in (p_1^*,p_2^*)$, $V(c,p,q)$ has three minima.
For $p = p_2^*$, $V(c,p,q)$ has three equal minima.
For $p \in (p_2^*,p_3^*)$, $V(c,p,q)$ has three minima.
For $p = p_3^*$, in $V(c,p,q)$ the third minimum disappears.
For $p \in (p_3^*,1\rangle$, $V(c,p,q)$ has one minimum.

As we see, there is an interval $p \in (p_1^*,p_3^*)$ in which potential $V(c,p,q)$ has three minima and therefore the stationary probability density function has three maxima (see Fig. 2). In this region we have a coexistence of two phases—with and without majority. For $p < p_2^*$ the state with majority is stable and the state without majority is metastable and for

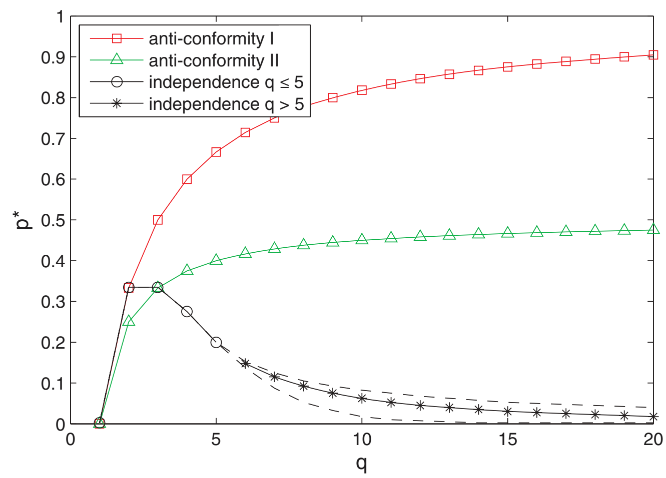

FIG. 10. (Color online) Transition points p^* as a function of q for all three models. Solid lines denote the line of the phase transition and dashed lines denote spinodal lines, i.e., determine the region with metastability. Several differences among models are visible. As seen, both models with anticonformity behave qualitatively the same: the critical value of p increases with q. However, for the model with independence the transition point decreases with p. Moreover, for $q \geqslant 6$ the phase transition changes its type from continuous to discontinuous.

$p > p_2^*$ the state with majority is metastable and that without majority is stable. Consequently, the phase transition appears at $p = p_2^* = p^*$ and $p = p_1^*,p_3^*$ designate spinodal lines [27,28].

Transition points p^* as a functions of q for all three models are presented in Fig. 10. As seen, both models with anticonformity (I and II) behave qualitatively the same: the critical value of p increases with q. However, for the model with independence the transition point decreases with p. Moreover, for $q = 5$ in the case with independence, the phase transition changes its type from continuous to discontinuous. To clarify our results we decided to present the complete phase diagrams for the models with anticonformity and independence in Fig. 11. Because results for both models with anticonformity (I and II) are qualitatively the same we present the phase diagram only for the model with anticonformity II.

The first difference between models with anticonformity and independence—connected with the qualitative dependence between p^* and q—is easy to explain heuristically. It is quite obvious why in the model with independence the critical point p^* decreases when q increases. When q increases it becomes unlikely to choose randomly q parallel spins and therefore the noise term dominates because it is independent of the state of the q lobby. Similarly, it can be understood why in the model with anticonformity the critical point $p*$ increases with q. It should be recalled here that anticonformity takes place only when $q + 1$ parallel spins are chosen randomly, which is more unlikely than choosing q parallel spins. Therefore the anticonformity term declines in importance even more than the conformity term as q increases. The second difference between models—the change of the transition type in the model with independence—is not so easy to understand intuitively. This result has been obtained numerically from the potential (17), but in the next section

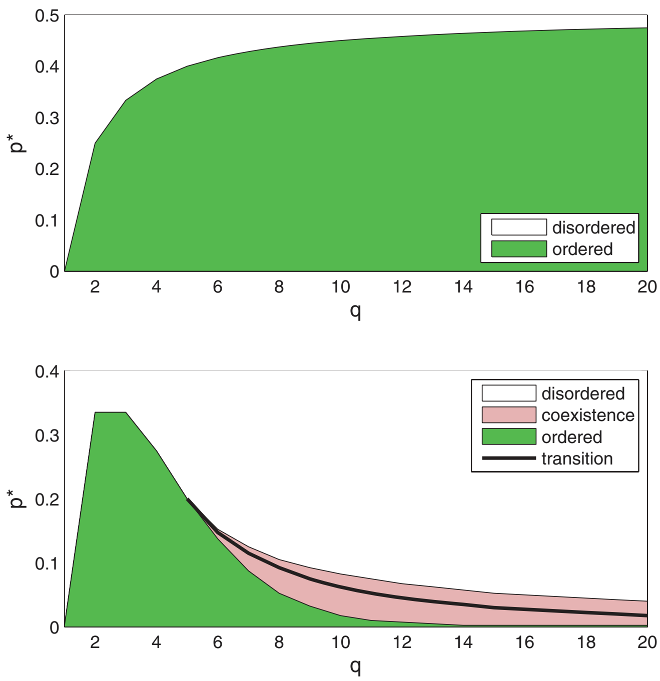

FIG. 11. (Color online) Phase diagrams for the models with anticonfomity II (top panel) and independence (bottom panel). As seen, for the model with anticonformity, the critical value p^* increases with q, and for the model with independence, it decreases with q. For anticonformity (top panel) there is a continuous phase transition (denoted by the solid line) between order (i.e., $c \neq 1/2$ or, equivalently, $m \neq 0$) and disorder (i.e., $c = 1/2$ or, equivalently, $m = 0$). In the model with independence (bottom panel) there is a continuous phase transition only for $q < 5$. At $q = 5$ the phase transition changes its type from continuous to discontinuous. For $q > 5$ an area in which one of two phases (ordered or disordered) is metastable is limited by so-called spinodal lines. This area is labeled as "coexistence," although the real coexistence occurs only on the transition line. However, in the region of metastability both phases can be observed depending on the initial conditions (hysteresis), which can be also seen from the flow diagram in Fig. 8.

we will show how this result could be also derived from an approximate Landau description.

VI. LANDAU DESCRIPTION

Although we were able to calculate critical points for the model with anticonformity and for the model with independence with $q \leqslant 5$ directly from the potentials (15)–(17), it can be instructive to use the classical description proposed by Landau for equilibrium phase transitions [26]. It has been shown that this kind of description can be also obtained as a mean-field approach for the Langevin equation of nonequilibrium systems with two (Z_2) symmetric absorbing states [21,22].

In our paper we have written the master equation as a function of concentration $c = N_\uparrow/N$ of up spins. We have decided to use this quantity for convenience since calculations are simple and equations have compact forms. However, to meet the symmetry requirement [21,22] one should use an order parameter (in this case magnetization) defined as

$$\phi = \frac{N_\uparrow + N_\downarrow}{N} \tag{27}$$

for which potentials (15)–(17) are symmetric under reversal $\phi \to -\phi$.

Following the approach presented in Refs. [21,22], which coincides with the classical approach proposed by Landau, we expand potentials (15)–(17), rewritten as a function of ϕ, into power series and keep only the first three terms of the expansion:

$$V(\phi) = A\phi^2 + B\phi^4 + C\phi^6, \tag{28}$$

where coefficients $A = A(p,q)$, $B = B(p,q)$, and $C = C(p,q)$ depend on the model.

For the model with independence,

$$\begin{aligned} A(p,q) &= -\frac{(1-p)}{2^q}\frac{(q-1)}{2} + \frac{p}{4},\\ B(p,q) &= -\frac{(1-p)}{2^q}\frac{q(q-1)(q-5)}{24},\\ C(p,q) &= -\frac{(1-p)}{2^q}\frac{q(q-1)(q-2)(q-3)(q-9)}{720}. \end{aligned} \tag{29}$$

From Landau theory it is known that for $B(p,q) > 0$ and $C(p,q) > 0$ there is a critical point at which $A(p,q)$ changes sign [26]. For $A < 0$ the potential $V(\phi)$ has two symmetric minima and thus the system is driven to one partially ordered state with $\phi \neq 0$. For $A > 0$ the potential $V(\phi)$ has a minimum at $\phi = 0$ and therefore the system remains in an active disordered state and a magnetization ϕ fluctuates around zero. From Eq. (29) it is easy to calculate that

$$\begin{aligned} A(p,q) = 0 &\to p = p^* = \frac{q-1}{q-1+2^{q-1}},\\ A < 0 &\to p < p^*, \quad \phi \neq 0,\\ A > 0 &\to p > p^*, \quad \phi = 0, \end{aligned} \tag{30}$$

which coincides with the result (26) obtained from the exact version of the potential (17).

As shown within classical Landau theory, for $B(p,q) < 0$ and $C(p,q) > 0$ a discontinuous jump in the order parameter is expected [26]. Again from Eq. (29) it is easy to see that $B(p,q) < 0$ for $q > 5$ (see also Fig. 12). Therefore we expect a discontinuous phase transition for $q > 5$, which also agrees with the results obtained from Eq. (17). It should be mentioned here that a transition for $B \leqslant 0$ could possibly be included in the class of generalized voter models (the so-called unique GV transition) [21]. It has been noticed for a general class of models with two (Z_2) symmetric absorbing states that for $B \leqslant 0$ the location of the potential minimum changes abruptly from $\phi = 0$ to $\phi \pm 1$; i.e., a discontinuous phase transition is observed [21]. In our model the situation is slightly different, because for $p > 0$ there are no absorbing states and below the transition point $|\phi| < 1$. However, still the system jumps from a totally disordered to a partially ordered state; i.e., a discontinuous phase transition is observed.

One should also notice that in the case of a q-voter model with independence for $q > 9$ also $C(p,q)$ becomes negative and than the approximation (28) is no longer valid.

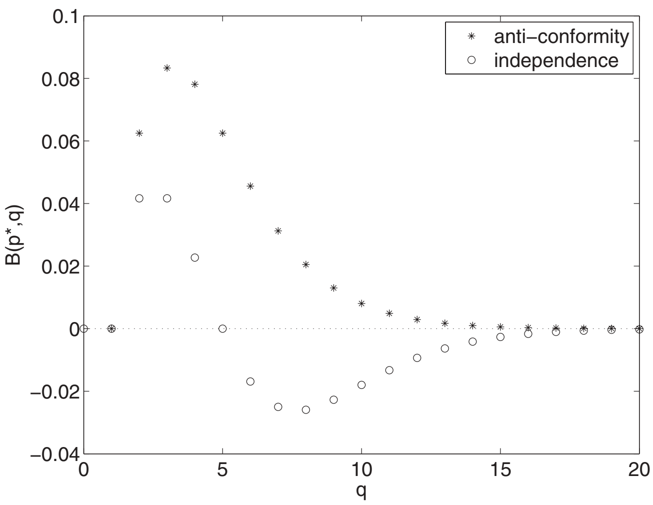

FIG. 12. Coefficient $B(p,q)$ [see the effective potential (28)] for a critical point $p = p^*$ at which $A(p,q)$ changes sign. For the model with independence (denoted by "o") coefficient $B < 0$ for $q > 5$, which suggests a discontinuous phase transition, whereas for the model with anticonformity (denoted by "*") coefficient $B \geqslant 0$ for any value of q and therefore the transition is continuous for arbitrary values of q.

Analogous calculations can be done for the models with anticonformity. Because both models with anticonformity are qualitatively the same we present here results for the model with anticonformity II. In this case,

$$A(p,q) = \frac{2pq - q + 1}{2^q},$$
$$B(p,q) = -\frac{1}{4}\frac{1-p}{2^q}\left[\binom{q-1}{3} - \binom{q-1}{1}\right] + \frac{1}{4}\frac{p}{2^q}\binom{q+1}{3},$$
$$C(p,q) = -\frac{1}{6}\frac{1-p}{2^q}\left[\binom{q-1}{5} - \binom{q-1}{3}\right] + \frac{1}{6}\frac{p}{2^q}\binom{q+1}{5}. \tag{31}$$

Therefore in the case with anticonformity,

$$A(p,q) = 0 \rightarrow p = p^* = \frac{q-1}{2q},$$
$$A < 0 \rightarrow p < p^*, \quad \phi \neq 0, \tag{32}$$
$$A > 0 \rightarrow p > p^*, \quad \phi = 0,$$

which coincides with the result (25). Moreover, for the model with anticonformity (see Fig. 12) coefficient $B(p = p^*,q) \geqslant 0$ for any value of q; i.e., the transition is continuous for arbitrary values of q.

VII. CONCLUSIONS

In this paper we have asked questions about the importance of the type of nonconformity (anticonformity and independence) that is often introduced in models of opinion dynamics (see, e.g., [13,14,19,20]). We realized that the differences between the different types of nonconformity are very important from social point of view [17] but we have expected that they may be irrelevant in terms of microscopic models of opinion dynamics. To check our expectations we have decided to investigate a nonlinear q-voter model on a complete graph, which has been recently introduced as a general model of opinion dynamics [12].

To our surprise, the results for the model with anticonformity are qualitatively different from those for the model with independence. In the first case there is a continuous order–disorder phase transition induced by the level of anticonformity p. The critical value of p grows with the size of the q lobby. On the other hand, for the model with independence the value of the transition point p^* decays with q. Moreover, the phase transition in this case is continuous only for $q \leqslant 5$. For larger values of q there is a discontinuous phase transition and coexistence of ordered (with majority) and disordered (without majority) phases is possible.

We have suggested in the title and the introduction of the paper that both types of nonconformity play the role of noise. However, only independence introduces real random noise, which plays a role similar to temperature. From this point of view the change of the type of transition resembles a similar phenomena in the Potts model (for a review see [29]). In the Potts model there is a first-order phase transition for $q > 4$ and a second-order phase transition for smaller values of q, where q denotes the number of spin states. Of course, in the case of our model q does not denote the number of states, which is always 2, but the size of the group. A similar observation has recently been made by Araujo *et al.* [30] within a model of tactical voting. They have considered q candidates on which citizens vote and proposed a balance function to quantify the degree of indecision in the society due to the coexistence of different opinions. It turned out that for some values of model parameters the model boiled down to the q-state Potts model, although similarly, like in our model, q denoted the number of candidates instead of the number of states. A similar change of the type of transition has been also observed in a general class of systems with two (Z_2) symmetric absorbing states within a Langevin description [21,22]. Moreover, it has been suggested that models with many intermediate states (i.e., the Potts model or a simple three-state model described in Ref. [22]) behave as equivalent two-state models with effective transitions that are nonlinear in the local densities [22], which is the case of a q-voter model or a two-state model of competition between two languages [31]. The theory presented in Refs. [21,22] suggests also that the continuous phase transition that is observed for $q < 5$ could possibly be included in the Ising class, whereas the discontinuous phase transition that is observed for $q > 5$ would fall into the class of generalized voter models.

Concluding the paper we would like to pay attention to one more phenomena that is visible in Fig. 10. For lobby $q = 2$ the results are the same for anticonformity and independence. Therefore it comes as no surprise that the difference between the two types of nonconformity has not been noticed while studying the Sznajd model (i.e., $q = 2$) [14,20].

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Anticonformity or Independence?—Insights from Statistical Physics

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Abstract The aim of this paper is to examine how different types of social influence, introduced on the microscopic (individual) level, manifest on the macroscopic level, i.e. in the society. The inspiration for this task came mainly from two sources—social psychology that recognize two different types of nonconformity (anticonformity and independence) and the observation related to the agent-based modeling that was verbalized in 2002 by Macy and Willer that there was a little effort to provide analysis of how results differ depending on the model designs. To achieve the goal, we propose a generalized model of opinion dynamics, that as a special cases reduces to the linear voter model, Sznajd model, q-voter model and the majority rule. We use the model to examine the differences, that appear at the macroscopic level, under the influence of two types of nonconformity, introduced on the microscopic level. We answer the question if the observed differences are universal or model dependent.

Keywords Agent-based modeling · Opinion dynamics · Phase transitions

1 Introduction

Recently several very interesting reviews on agent-based models (ABM) have appeared [1–5] indicating the rapid growth of interest in using ABM in social sciences. In physics this type of approach is known for years under the name microscopic modeling and is the domain of statistical physics. Therefore, perhaps one should not wonder that physicists had the idea to use methods of statistical physics to analyze social systems. In 1982 Serge Galam et al. published the first paper on *sociophysics* [6]. As Galam mentioned in his personal testimony [7] at the beginning it was a hard opposition to Sociophysics from inside Physics.

Nowadays the field of Sociophysics is widely accepted (see a very recent book by Galam [8] and a review by Castellano, Fortunato and Loreto [9]). On the other hand, as

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has been noted recently, despite the power of ABM in modeling complex social phenomena, widespread acceptance in the highest-level economic and social journals has been slow due to the lack of commonly accepted standards of how to use ABM rigorously [4]. One of the main problems in the field of social simulations, as mentioned by Macy and Willer [1], is *'little effort to provide analysis of how results differ depending on the model designs'*. As have been noted [8, 10, 11], the similar problem is seen in a field of sociophysics. As an example let us consider opinion dynamics models [9]. Among variety of models describing opinion dynamics there is a particularly interesting class of simple models based on the idea of Ising spins (for some comments on the Ising model see Sect. 3). Generally, an individual in ABM (so called agent) can be characterized by:

– several traits of different types, like in e.g. social impact models [12, 13] or CODA model [14],
– a set of traits of the same type, for example a vector of integer variables in Axelrod model [15],
– a single trait that takes continuous values, like an opinion in bounded confidence models [16, 17],
– or a single trait that takes discrete values, e.g. various voter models [18, 19].

1.1 Spinson—A Particularly Simple Agent

In the simplest possible case an agent is characterized by a single variable that takes only one of two values (usually $+1, -1$). This type of agents were introduced in many different models of opinion dynamics, e.g. voter model [18, 19], majority model [20–22], Sznajd model [23] or the general sequential probabilistic model [24]. Analogous type of a variable has been introduced in 1920 by Wilhelm Lenz to describe a phase transition in a magnetic system and therefore this type of an agent is often called *a spin*, at least by physicists. The spin in the Ising model can be imagined as an arrow pointing up or down ($\uparrow$ or $\downarrow$). From the social point of view we deal with a person that has one of two possible opinions, attitudes etc. (e.g. 'yes' or 'no', 'in favor' or 'against', Mac or PC user). Therefore, to avoid such nonsensical statements like 'person up' or 'person down', that may be confusing for people from outside sociophysics, from now on we call this type of an agent **spinson**—as a combination of two words **spin** and **person**. Spinson should be understood as a type of an agent in ABM that is characterized by only one binary trait and is represented in all illustrations that appear in this paper as a combination of an arrow and a man (see Figs. 1, 2, 3 and 7).

1.2 The Goal and the Structure of the Paper

To face the problem posed by Macy and Willer [1], we introduce a generalized voter model. The model, as a special cases, includes other popular sociophysics models like the linear voter model, Sznajd model, q-voter model and the majority rule. We should admit here that this is not the first generalization of sociophysics models. Very interesting idea has been introduced already in 2005 by Galam [24]. He has proposed a general sequential probabilistic frame (GMP), which was aimed to extend a series of earlier opinion dynamics models based on spinsons. In GMP the majority rule is weighted by a function of the majority to minority ratio. In 2008 Lambiotte and Redner have studied a family of models where the propensity for a spinson to align with its local environment depended nonlinearly on the fraction of disagreeing neighbors [25]. Other attempt to the generalization of spinson's models has appeared in [26] under a name *nonlinear q-voter model*. This model became the basis for the model that will be presented in this paper. We would like to strongly emphasize that inventing another new model of opinion dynamics is not a goal of this paper. The main goal is to answer questions related to different types of nonconformity—anticonformity and independence (see Sect. 2):

1. Do differences between two types of nonconformity, that are recognized by social psychologists on the individual (microscopic) level, manifest on the society (macroscopic) level, at least in ABM approach based spinsons?
2. If any differences manifest on the macroscopic level, are they universal, i.e. do not depend on the model designs?

One could argue that to fairly answer at least the second question, one should not only investigate all existing spinson's models but also all that might be potentially invented in the future. It would be hard to disagree with such an objection. However, it would be hard under one condition—if in all considered cases results would be the same (at least qualitatively). Within such a scenario we would not be able to give any conclusive answer. However, there is another possibility—results would significantly depend on the model designs. Within such a scenario we would be able to give the fair answer to the question. Therefore, it seems that a good starting point is to choose a possibly general model in which it is easy to implement various types of social influence. Just because of universality and simplicity we have decided to deal with a generalized q-voter model.

The paper is organized as follows. In the next section we describe the motivation that came from the social psychology and concerns different types of social influence. In Sect. 3 we present some insights from statistical physics, mainly concerning the Ising model, but also differences between continuous and discontinuous phase transitions. In Sect. 4 we describe shortly different types of social response introduced in sociophysics. Next, in Sect. 5 we introduce a basic q-voter model [10] and next show differences between two types of nonconformity within this model (Sect. 6). Finally, in Sect. 7 we introduce a generalized q-voter model and try to answer the question about universality of the obtained results. We conclude the paper by the section 'Summary'. Because the paper is dedicated to a possibly general audience, also from outside physics, we try to avoid calculations or technical issues that are not particularly important for the paper. If any equations appear in this paper they are dedicated for those who would like to repeat our results. Other readers can omit them hopefully without loss of understanding the meaning of the paper. Moreover, to make paper accessible for a broad audience, we describe some basics, that might be well known for some readers, and we support the descriptions of models with illustrations.

2 Inspirations from Social Psychology

Decades of research in a field of social psychology have shown that conformity is ubiquitous. Numerous studies have indicated that there are many various motivations to match or imitate others and many different factors influence the level of conformity [27, 28]. For example it has been show that conformity increases with the number of people serving as the source of social impact. In 1981 Latane has analyzed data from several social experiments and concluded that the level of conformity I seems to grow with a group size N according to the psychosocial power law $I \sim N^{\alpha}$, where $\alpha < 1$ various for different experiment [29]. This means that although conformity grows with the size of the majority, the effect of the N-th person is weaker than that of the $N - 1$-th.

Moreover, Solomon Asch has shown, withing his classical experiment 'with lines', that the presence of a social supporter reduced significantly the level of conformity. The power

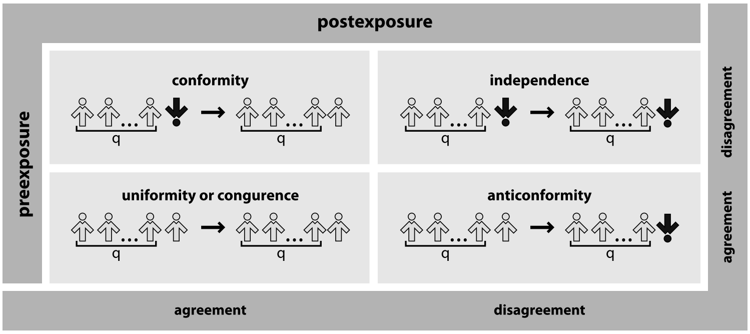

Fig. 1 Possible responses to the social influence (so called diamond model) derived from Willis's scheme [33, 34] and formalized by Nail et al. [35–38]. Here presented within a q-voter model that will be described precisely in Sect. 5. In the q-voter model a group of q spinsons is the source of the social influence only if all q spinsons are parallel (unanimous majority)

of social support was demonstrated in the empirical studies showing that participants were far more independent when they were opposed by a seven person majority and had a partner than when they were opposed by a three-person majority and did not have a partner [30]. Taking into account all results of social experiment, it is not entirely clear how the conformity should be modeled and which factor is the more important—the size of the majority or the unanimity (if any of two).

Although the power of social influence is undeniable, people usually fail to recognize their own susceptibility to the social influence. In series of 5 social experiments it has been shown that people see themselves as *alone in a crowd of sheep*, i.e. see others as more conforming than themselves [31]. Moreover, *although conformity can confer many benefits on an individual, but on the other hand nonconformity can also be advantageous* [28]. Relatively recently it has been shown in the series of three experiments how conformity and nonconformity may be influenced by two fundamental social motives—protecting oneself from harm and seeking for a sexual partner [28]. On the other hand, it is also known that the level of conformity/nonconformity is different in different human cultures, although the origin of those differences remains unclear [32].

Although there are different motives and factors influencing conformity, this kind of behavior manifests always as a match for a certain group. On the other hand, nonconformity can manifest in two different ways. According to [33–38], there are two types of nonconformity (see also Fig. 1):

– Independence—resisting influence. In this case the situation is evaluated independently of the group norm. Truly independent people are oblivious to what is expected [39].
– Anticonformity—rebelling against influence. It appears often as a result of maintaining the uniqueness. Anticonformists are similar to conformers in the sense that both take cognizance of the group norm—conformers agree with the norm, anticonformers disagree.

As mentioned in [28], both types of nonconformity tend to be effective in differentiating people from others. The question, that we have recently posed, has been related to the way in which nonconformity differentiates people. It seems to be quite important, from the psychological point of view, what is the type of nonconformity—independence or anticonformity. However, at the level of the society it is probably very difficult, or even impossible, to distinguish which type of nonconformity is responsible for a particular social phenomenon.

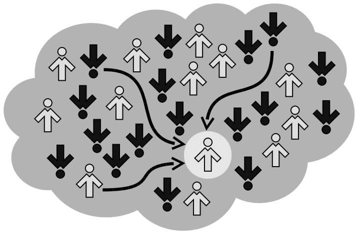

Fig. 2 All models in this paper are considered on a complete graph, i.e. all agents are nearest neighbors. In other words, each agent is connected by direct links with all others. This topology is particularly convenient for the analytical calculations and corresponds to the approach known from statistical physics as a mean field approximation (MFA). Within such a topology the concentration (or equivalently the number) of 'up' spinsons defines completely the state of the system—if all agents are the nearest neighbors there is no sense to talk about a distance or a structure. From the social point of view this can be understood as a community in which each member can equally influence any other member (in biology this is called *panmictic population*). This is implemented as follows—there is a set of N agents and in every time step a randomly chosen group of individuals can influence a voter which is also randomly chosen

Therefore it would be very difficult to answer if the differences between two types of nonconformity are significant in the real societies. However, one could try to answer much simpler question if the differences between two types of nonconformity are significant for the artificial societies described by ABM. Recently we have answered this question within so called q-voter model [10]. In this paper we would like to face the problem within a broader class of models. However, before going further let us recall some facts from the statistical physics, that might be interesting and instructive for those who are dealing with problems related to the social influence.

3 Insights from Statistical Physics

Probably each physicist has heard about the Ising model. This is undoubtedly the most prominent model in a field of statistical physics and probably also the one that was applied the most intensively outside pure physics. The history of the model is very interesting itself, but it has been described already many times so it does not make sense to repeat it here. For those who would like to learn more about the model we recommend one older and several recent reviews [40–43].

3.1 Recipe for the Ising Model

Here, we only recall this informations that might be useful for people dealing with the social influence. We present a model giving a simple recipe describing its main components:

1. **Topology** can be treated in this case as a set of N nodes and M links between nodes, for example: regular lattice, complete graph, complex network etc. In statistical physics we use usually regular lattices due to the crystal structure of the condensed matter. However, for the social system probably complex network or other type of graph is more accurate. In turn of modeling panmictic population (each individual can potentially contact with any other) one can use complete graph, which is also very convenient for analytical calculations (for the explanation of a complete graph see Fig. 2).

2. **Dynamical binary variables** that occupy nodes $S_i = \pm 1,\ i = 1, \ldots, N$. In statistical physics we usually think about spins (in this case arrows pointed up ↑ or down ↓). For social applications one can think about person having one of the two possible attitudes, opinions or behaviors (yes/no, in favor/against, PC/Mac user etc.).
3. **Internal interactions**, i.e. interactions between individuals. In the basic Ising model interactions take place only in the nearest neighborhood (nn) which are determined by direct links. Two spins (or spinsons) are nn if they are directly connected by a link. Because the Ising model has been originally designed to describe magnetic systems, usually one of two types of internal interactions are considered:
   (a) **Ferromagnetic interactions** play the similar role to the **conformity**. Due to these interactions spins flip to mimic the majority in the neighborhood. For example if considered spin is ↓ and it has three neighbors that are ↑ and one ↓ it flips.
   (b) On the other hand, **antiferromagnetic interactions** plays in a sense a role of **anti-conformity**—spin takes a state of the minority.

   In the Ising model ferromagnetic and antiferromagnetic interactions are described by the following Hamiltonian:

$$H = -\sum_{\langle i,j \rangle} J_{i,j} S_i S_j, \tag{1}$$

   where $\langle i, j \rangle$ denotes that i and j are nearest neighbors and $J_{i,j}$ describes interactions between two spins S_i and S_j. If $J_{ij} > 0$ interactions are ferromagnetic and if $J_{ij} < 0$ antiferromagnetic. It is worth to mention that mixing ferro and antiferro interactions was introduced in the case of social applications by Galam in [44]. We would like to reassure those who are not familiar with the mathematics or physics, that this is not necessary to understand precisely Eq. (1) in order to follow the paper and it is presented mainly to show that in the statistical physics interactions are represented usually by the Hamiltonian.
4. **External interactions**, i.e. interactions of a system with some external force. Magnetic system can for example interact with an external magnetic field. From the social point of view it might be information, advertisement in mass media or a strong leader. In the Ising model interactions with an external field are described by the term $-h\sum_i S_i$ and therefore the full Hamiltonian for the basic Ising model:

$$H = -\sum_{\langle i,j \rangle} J_{i,j} S_i S_j - h \sum_i S_i. \tag{2}$$

   It should be mention here that the above Hamiltonian was used for social application for the first time in [45]. The extension by adding local symmetry breaking fields was done in [45, 46]

   There is another type of an external influence that is not described by the above Hamiltonian—the one that introduces temperature. In physics we think usually about interactions with thermostat that provides a certain temperature in the system.

   – If the temperature is low interactions described by the Hamiltonian play a main role.
   – As temperature increases spins start to behave more chaotic. The higher is the temperature the more *nervous* are spins neglecting all interactions described by the Hamiltonian. In the very high temperature spins are flipping completely randomly, independently of the interactions with the neighborhood.

   From this point of view the **temperature** plays a role of the **independence**. With the temperature independent behavior of spins increases. As a result of the competition between interactions (that dominate in lower temperatures) and the temperature (which introduces randomness) a continuous phase transition appears. Above a certain critical temperature T_c the system is disordered and below ordered. This means that, for example in a case of ferromagnetic interactions (conformity), there is some majority in the system.

3.2 A Few Words on Phase Transitions

As we have already mentioned a certain type of a phase transition, that is called **continuous**, appears in the Ising model due to the temperature. This is a very different type of the phase transition that is observed for example between ice and liquid [47]. At the temperature below zero Celsius degrees water is usually in a solid state, i.e. ice. However, if one cools down water gently, it is possible to reach the negative temperature and keep water as liquid. In such a case we speak about supercooled water which is a metastable state—small disturbance can bring the system to the stable state, i.e. ice. Moreover, at the transition point water coexists with an ice—therefore we can have ice cubes in our drink. This type of transition is called **discontinuous**. If the transition between ice and water would be continuous, we could neither prepare drinks on rocks nor observe floes on the lake.

In the case of a continuous phase transition there is no phase coexistence. For a given conditions (for example for a given temperature) we have only one of possible phases (for example paramagnetic or ferromagnetic phase). In the case of continuous phase transitions *an order parameter* describing the state of the system (e.g. magnetization, concentration, opinion etc.) changes continuously. For example with an increasing temperature magnetization continuously decreases and achieves zero (no magnetization, complete disorder) at the critical temperature.

As we have written above, the temperature plays a similar role as independence. From this point of view:

- Continuous phase transition means that there is a complete consensus (all members of the society have the same opinion) if the level of independence is zero and it continuously decreases with an increasing independence up to a critical point. At a critical level of independence consensus achieves zero and above the critical point there is no majority in the society (status-quo or stalemate situation).
- In the case of discontinuous phase transition there is a 'jump' of an order parameter. If the phase transition between consensus and status-quo would be discontinuous than almost fully ordered system (large majority in the society) could turn to a stalemate system at the transition point. Moreover, unambiguous prediction of the state near the transition point would be impossible due to the metastability and phase coexistence.

3.3 Spin Models Versus Social ABM Models

As noted by Stauffer the pioneer ABM models of segregation, proposed by Thomas Schelling [48] over 40 years ago, are strikingly similar to the Ising model with Kawasaki dynamics [49]. After a couple of decades, the idea of binary states, that can be easily linked with the famous Ising model, is still present in many social papers. For example a number of innovation diffusion models represent adoption behavior by means of a single dichotomous variable that represents agents' state—agents are either in a 'potential adopter' or an 'adopter' state [5, 50–52].

Of course, in many cases limitation to binary variables may oversimplify the problem, not only in sociology but also in physics. Therefore statistical physics deliver many other microscopic lattice models that use multi-states variables. It seems that some ABM models may be linked to them, even if the similarity is only incidental. For example, Moldovan and Goldenberg [53] have modeled the resistance to innovations by introducing consumers that may be in one of three states (uninformed, adopters, and resisters), which reminds the 3-states Potts model (for review on Potts model see [54]). Multi-state discrete variables, analogous to Potts spins, have been used also by Deffuant et al. [55]. They have used a fixed state transition scheme based on interest (no, maybe, yes) and information states (not-concerned, information request, no adoption, pre-adoption, adoption) to describe adoption decisions. Thiriot and Kant have also used multiple discrete states to model diffusion of innovation [56].

The surprising similarity is evident between the XY spin model [57, 58] and a recent model by Flache and Macy [59]. In both models agents/spins have continuous states and the power of interaction increases with the similarity between agents. The idea of continuous variables has been also used in famous models of opinion dynamics based on bounded confidence [17, 60].

Among many interesting spin models there is one, which seems to be particularly interesting for social application, yet it is somehow forgotten. By this we mean so called Ashkin-Teller model [61], which uses a vector of two traits to describe a state of a single spin. Exactly the same idea has been used in [62] to describe political attitudes (one trait was connected with the attitude to the personal freedom and second with the attitude to the economic freedom). Similar but more general idea appears also in the famous Axelrod's model for cultural dissemination [15].

4 Social Influence in Sociophysics Models

Almost all models of opinion dynamics based on spinsons—voter, majority and Sznajd—have been designed to describe opinion dynamics under the same type of the social influence, i.e. conformity. Therefore, it is not surprising that in all these models complete consensus (all spinsons parallel) is a steady state. Obviously, in real social systems complete unanimity is not reached or if ever reached never stays forever. If one follows public opinion records, immediately realizes that it permanently changes. To make models of opinion dynamics more realistic, several modifications have been proposed:

- In 1991 Serge Galam and Serge Moscovici proposed a model with *non-social state*, i.e. the state in which an individual is not subjected to the environment [45]. This idea, which is perfectly consistent with the concept of independence, has been developed by Galam in [46]. Interplay between independent and biased choices has been introduced by an exchange amplitude I that measured a degree of interactions between individuals.
- In 2003 Mauro Mobilia has introduced *zealot* [63]—a biased individual who favors one opinion. Zealot is allowed to change his state from -1 to $+1$ (with rate $r > 0$) without regard to his neighbors, with whom he nevertheless interacts. Therefore, zealot represents a person who acts independently with probability r, which reminds the idea proposed earlier by Galam [45, 46].
- In 2004 Serge Galam has introduced *contrarians* [64]—with a certain probability an agent adopts the choice opposite to the prevailing choice of the others, whatever this choice is. Keeping the naming from the diamond model (see Fig. 1) this type of the social response corresponds to anticonformity. This type of social behavior have been introduced later in the Sznajd model by Schneider [65] and in a modified version by Lama et al. [66].
- In 2007 Galam and Jacobs have introduced *inflexibles* [67]—inflexible agents keep their opinion always unchanged. Therefore, from the psychological point of view inflexibles represent a special case of independence.

– In 2011 independence has been introduced to the Sznajd model [68]—with probability p each individual in the system acts completely independently from the neighborhood and may randomly change its state.

It was shown that the presence of zealots, contrarians, inflexibles or independence significantly changes not only the time evolution of the system but also steady states. In particular, it was shown that for a low concentration of contrarians or a low level of independence a new mixed phase is stabilized, with a coexistence of both opinions, i.e. minority persists. Moreover, there is a continuous phase transition into a new disordered phase with no dominating opinion [64, 66, 68].

5 Anticonformity and Independence Within the q-Voter Model

Voter model [18, 19] is one of the most recognized in a field of non-equilibrium statistical physics. It can be treated not only as a toy model of an Ising spin's system but also caricature of opinion dynamics. In the voter model, as in the Ising model, individuals occupy the nodes of a graph. In the simplest version of the model, each individual can be in one of two equivalent states and simply adopts a state of one of its neighbors that is randomly chosen in each update event. This is the simplest way to introduce conformity that one can imagine and probably oversimplified. As we have already written in Sect. 2, there are numerous factors that influence conformity and it is not entirely clear how it should be modeled. Probably for this reason several models has been already proposed [8, 9], among them Sznajd model [23], majority model [20–22] or recently q-voter model [26]. The latter model is particularly interesting because as special cases reduces to the linear voter or the Sznajd model. In this model q, randomly picked, neighbors influence a voter to change opinion. If all q neighbors agree, the voter takes their opinion; if they do not have a unanimous opinion, still a voter can flip with probability ϵ. For $q = 2$ and $\epsilon = 0$ the model reduces to the Sznajd model on a complete graph [75] and for $q = 1$ to the linear voter model. Moreover, the case of $\epsilon = 0$ can be justified by results obtained in the social experiments by Asch—unanimity is the key! [30] (see Sect. 2).

In [10] we have investigated q-voter model with $\epsilon = 0$ and two types of nonconformity—anticonformity and independence. We have chosen a topology of a complete graph, as a particularly convenient for analytical calculations (see Fig. 2). We would like to stress here that we do not claim that the topology of a complete graph is the most suitable for describing social systems. It is well known that society are much better described by complex networks e.g. small-world or scale-free Barabasi-Albert networks (for reviews see [69–71]). We have chosen the topology of a complete graph mainly because it allows for the analytical treatment and in fact corresponds to the method known from statistical physics as a mean field approximation [75].

Let us recall here briefly the model itself and results that have been obtained. Within the q-voter model we consider a set of N spinsons. At each elementary time step q spinsons $S_1, \ldots, S_q$ are picked at random and form a group of influence, lets call it q-lobby. Then the next spinson, on which the group can influence is randomly chosen, we call it voter. In the model proposed in [10] conformity and anticonformity take place only if the q-lobby is homogeneous, i.e. all q spinsons are parallel. In a case of conformity (that takes place with probability $1 - p$) voter takes the same decision as the q-lobby, whereas in a case of anticonformity (that takes place with probability p) the opposite opinion to the group. In a case of independent behavior (that takes place also with probability p), voter does not follow the group, but acts independently—with probability $1/2$ it flips to the opposite direction, i.e.

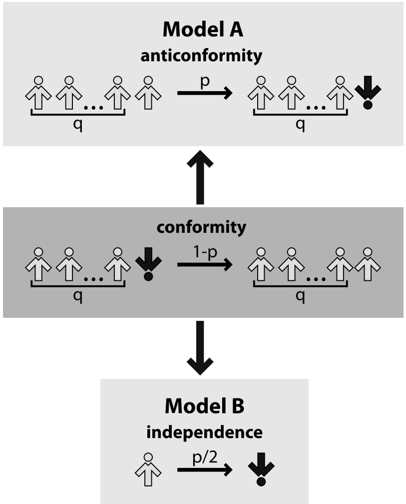

Fig. 3 Illustration of the q-voter model with anticonformity (Model A) and independence (Model B). At each elementary time step q spinsons are picked at random and form a group of influence (q-lobby). Then the voter, on which the group can influence, is randomly chosen. With probability p voter behaves like anticonformist (in a case of Model A) or independent (in Model B) and with probability $1-p$ like conformist. In this model conformity and anticonformity take place only if the q-lobby is homogeneous, i.e. all q individuals are in the same state. In the case of conformity the voter takes the same decision as the q-lobby, whereas in a case of anticonformity the voter takes the opposite opinion to that of the group. In a case of independent behavior, the voter does not follow the group, but acts independently—with probability $1/2$ it flips to the opposite direction

$S_{q+1} \to -S_{q+1}$ (see Fig. 3). In [68] we have proposed a more general type of nonconformity in which spinson flips with probability $f \in [0,1]$, but it has been shown that there is a scaling relation between p and f and therefore only one of these parameters can be chosen as an independent.

The precise algorithms in a case with anticonformity (Model A in Fig. 3) and independence (Model B in Fig. 3) are given below.

5.1 Algorithm in a Case with Anticonformity

1. Initialization. For each node of a graph $i = 1, \dots, N$ choose a random number $pr_i \in [0,1]$. If $pr_i < c_0$ then spinson $S_i = 1(\uparrow)$, otherwise $S_i = -1(\downarrow)$. With this procedure we set initial state of the system in which the concentration of $\uparrow$-spinsons is equal c_0. After initialization goto 2.
2. Choose randomly q spinsons (q-lobby) $S_1, \dots, S_q$ and goto 3.
3. If q-lobby is homogeneous, i.e. $S_1 = S_2 = \cdots = S_q$ than goto 4 otherwise goto 2.
4. Choose randomly a voter S_{q+1} and goto 5.
5. Choose a random number $pr \in [0,1]$ and goto 6.
6. If $pr < p$ then a voter takes a state opposite to the state of the q-lobby, i.e. $S_{q+1} = -S_q$, otherwise goto 7.
7. A voter takes a state of a q-lobby, i.e. $S_{q+1} = S_q = \cdots = S_1$, goto 2.

Point 6 of the above algorithm means that with probability p voter behaves like anticonformist, whereas point 7 means that with probability $1-p$ voter conform unanimous majority of the q-lobby.

5.2 Algorithm in a Case with Independence

1. Initialization. For each node of a graph $i = 1, \dots, N$ choose a random number $pr_i \in [0, 1]$. If $r_i < c_0$ then spinson $S_i = 1(\uparrow)$, otherwise $S_i = -1(\downarrow)$. With this procedure we set initial state of the system in which the concentration of $\uparrow$-spinsons is equal c_0. After initialization goto 2.
2. Choose randomly a voter S_{q+1} and goto 3.
3. Choose a random number $pr \in [0, 1]$ and goto 4.
4. If $pr < p$ then $S_{q+1} \rightarrow -S_{q+1}$ with probability 1/2 and goto 2, otherwise goto 5.
5. Choose randomly a group of q spinsons (q-lobby) $S_1, \dots, S_q$ and goto 6.
6. If q-lobby is homogeneous than a voter takes a state of a q-lobby, i.e. $S_{q+1} = S_q$, goto 2.

Point 4 of above algorithm means that with probability p voter behaves independently, i.e. flips to the opposite direction with probability $1/2$, whereas point 6 means that with probability $1 - p$ voter conform unanimous majority of the q-lobby.

5.3 Anticonformity vs. Deviance

Before moving on to discuss the results we would like to explain the naming issues that are used in this paper. For some readers *anticonformity*, as introduced by Willis [33], may seem very obscure term and they would prefer the term deviance, which is widely recognized in sociology. Let us now explain why we have decided to use terms derived from social psychology rather than sociology. Going back to Merton's typology on deviance [72, 73], deviance is any behavior that violates social norms. From this point of view the concept of the social deviance is extremely complex—norms can be different in different cultures and they evolve in time. As a matter of fact, it can be hard to define if a certain behavior is already a social norm. Therefore in our paper we do not introduce the concept of the social norm. However, if we would like to define the norm we would probably decide to use some macroscopic variable like the average opinion (magnetization). In such a case it would be still possible to introduce nonconformity. For example in paper [74] we have introduced a kind of social deviance that could be recognized as the innovation [72, 73]. However, in this paper we do not consider this type of social influence. In our case individual's choice can be influence only by the contact with a selected group. In each time step the same individual can contact with different group. This reminds changing an opinion during a conversation over a lunch in the cafeteria rather than adjust to the social norm. The idea of introduced by Willis can be understood as micro interactions between individuals, similarly as spins are interacting with each other in the Ising model. On the other, hand the idea of deviance can be understood as the interaction of an individual with some macroscopic variable called the social norm. Therefore, we believe that the idea of anticonformity and independence as introduced by Willis is much more suitable in the case of our model than the idea of deviance.

6 Results for the q-Voter Model with Nonconformity

As we have written, our aim is to check if two different types of nonconformity, introduced on the microscopic level, lead to different results on the macroscopic level. Therefore we have to choose and investigate some macroscopic quantity that describes the state of the system. Because, we deal with a complete graph (see Fig. 2) there is no need to consider quantities related to the structure. In this case there are two natural quantities that fully describe the state of the system:

1. concentration of $\uparrow$-spinsons:

$$c = \frac{N_\uparrow}{N} \rightarrow c \in [0, 1], \tag{3}$$

   where $N_\uparrow$ is the number of $\uparrow$-spinsons and N is the total number of spinsons, i.e. $N = N_\uparrow + N_\downarrow$,
2. or magnetization which a good measure of a public opinion:

$$m = \frac{N_\uparrow - N_\downarrow}{N} \rightarrow m \in [-1, 1]. \tag{4}$$

Because there is a simple relation between above quantities:

$$m = 1 - 2c, \tag{5}$$

one can freely choose one of them depending on preferences. We have decided to investigate the behavior of concentration c for convenience—calculations are easier at such a choice.

6.1 The Stationary Value of the Concentration

Initially concentration of $\uparrow$-spinsons is c_0 and can take any value between 0 and 1. Due to the social interactions, described by Algorithms 5.1 and 5.2, it changes in the subsequent time steps. Eventually the system reaches certain steady state that depends on model's parameters:

- For the probability of nonconformity $p = 0$ the system reaches a steady state in which all spinsons are 'up' or all spinsons are 'down' (complete order), analogously to the Ising model at temperature $T = 0$.

  In such a case the stationary value of the concentration $c = 1$ or $c = 0$, depending on the initial conditions. If initially the concentration of $\uparrow$-spinsons is the same as the concentration of $\downarrow$-spinsons, which corresponds to $c_0 = 0.5$, both steady states $c = 1$ and $c = 0$ are equally probable.
- For the probability of nonconformity $p > 0$ the situation is more complicated. The system evolves and eventually reaches the stationary state but, in a case of a finite system, there is no single value of a stationary concentration (see Fig. 4).

  As seen from the bottom panel of Fig. 4 for the large value of nonconformity p there is no majority in the system and c fluctuates around 0.5. For smaller values of p there is a majority in the system and spontaneous transitions between two states appear.

  Spontaneous transitions between two states may look intriguing—for a relatively long time $\uparrow$-spinsons are in the majority and then suddenly, without any reason, there is a rapid change and $\downarrow$-spinsons become the majority. These sudden transitions are related to the fluctuations and therefore are less probable in larger systems. Resistance time in a given state increases, and fluctuations decrease with the size of the society N. For the infinite system spontaneous transitions are not observed and the concentration reaches one of the stationary values with the probability depending on the initial state.

6.2 Probability Density Function of the Concentration

Probability of a state with a given concentration c is described by the probability density function $\rho(c, t)$, where t denotes time. Because initially concentration of $\uparrow$-spinsons is c_0, at time $t = 0$ the density $\rho(c, t)$ consists of a single peak at $c = c_0$ and is equal zero elsewhere. Then the system evolves according to the one of above algorithms, which means

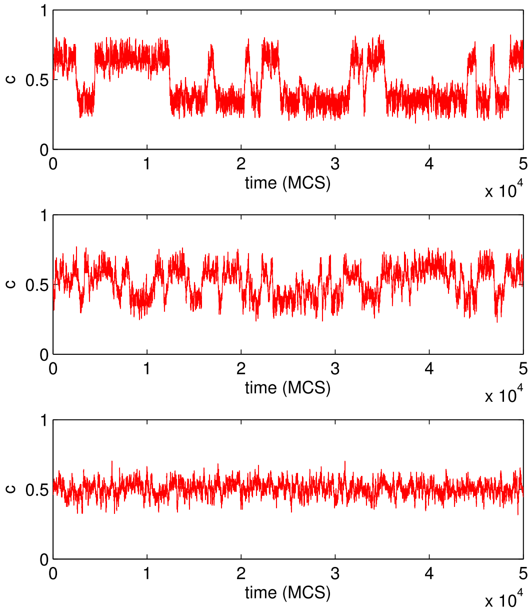

Fig. 4 Sample trajectories showing the change of the concentration in time for the model of $N = 100$ spinsons with anticonformity and q-lobby of size $q = 7$. The level of anticonformity increases from *upper* to *bottom row* and $p = 0.35, 0.4, 0.5$ respectively

that the state of the system, and simultaneously $\rho(c,t)$, changes. The time evolution of $\rho(c,t)$ can be obtained from the master equation, which is kind of a 'gain and loss' formula [19]. Because in a single time step Δ_t, three events are possible—the concentration c of $\uparrow$-spinsons increases or decreases by $\Delta_N = 1/N$ or remains constant:

$$\begin{aligned}
\gamma^+(c) &= \text{Prob}\{c \to c + \Delta_N\} \\
\gamma^-(c) &= \text{Prob}\{c \to c - \Delta_N\} \\
\gamma^0(c) &= \text{Prob}\{c \to c\} = 1 - \gamma^+(c) - \gamma^-(c),
\end{aligned} \tag{6}$$

the master equation takes form:

$$\begin{aligned}
\rho(c, t + \Delta_t) = {} & \gamma^+(c - \Delta_N)\rho(c - \Delta_N, t) \\
& + \gamma^-(c + \Delta_N)\rho(c + \Delta_N, t) \\
& + \big[1 - \gamma^+(c) - \gamma^-(c)\big]\rho(c, t).
\end{aligned} \tag{7}$$

Above equation may look complicated for some readers, but as we have already written it is a kind of a 'gain and loss' equation—some events increases the probability that the concentration of $\uparrow$-spinsons increases (these events take place with probability γ^+), other events decreases this probability (these events take place with probability γ^-). Therefore the above equation is simply a recipe for calculating the state of the system at time t. For the infinite system probabilities $\gamma^+(c), \gamma^-(c), \gamma^0(c)$ take particularly simple form that were already presented in [10]:

(A) In the case of anticonformity (Model A in Fig. 3):

$$\begin{aligned}
\gamma^+ &= (1-p)(1-c)c^q + p(1-c)^{q+1} \\
\gamma^- &= (1-p)c(1-c)^q + pc^{q+1},
\end{aligned} \tag{8}$$

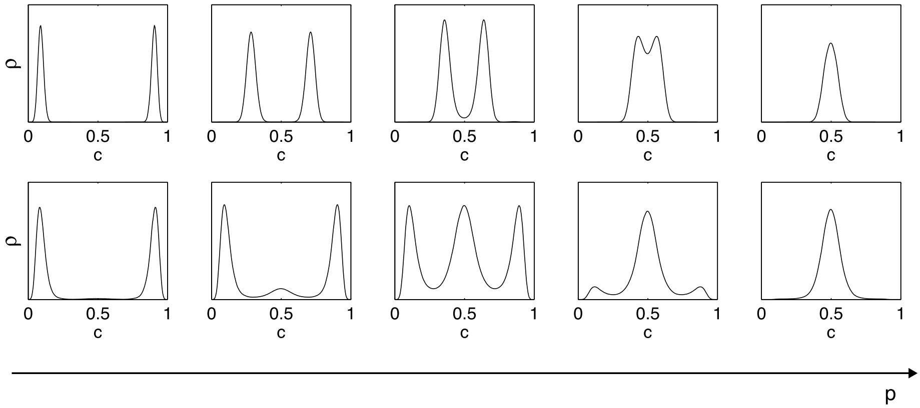

Fig. 5 Stationary probability density function of the concentration of ↑-spinsons for the q-voter model with anticonformity (*upper row*) and independence (*bottom row*) for a system of $N = 200$ individuals and a lobby size of $q = 7$

(B) whereas in the case with independence (Model B in Fig. 3):

$$\begin{aligned}\gamma^+ &= (1-p)(1-c)c^q + p(1-c)/2 \\ \gamma^- &= (1-p)c(1-c)^q + pc/2.\end{aligned} \tag{9}$$

It is also possible to give formulas in a case of a finite system, but since they are longer and were already presented in [10], we have decided to not repeat them here. Generally, this is not an easy task to solve analytically equation (7), but it can be done relatively easy numerically. Solving the equation, we see that the state of the system changes in time and eventually reaches a certain steady state in which $\rho(c,t) = \rho(c)$ does not change anymore.

As already written:

– If the probability of nonconformity $p = 0$ then eventually system reaches a steady state in which all spinsons are 'up' or all spinsons are 'down'.
  In such a case stationary probability density function $\rho(c)$ consists of two peaks at $c = 0$ and $c = 1$ and is equal zero elsewhere.
– If the level of nonconformity is very large $p \to 1$ there is no majority in the system, i.e. $c = 0.5$ in the steady state (disorder). In such a case $\rho(c)$ consists of a single peak at $c = 1/2$ and is equal zero elsewhere.

In Fig. 5 we have illustrated how $\rho(c)$ changes with the level of nonconformity p for the system of $N = 200$ spinsons and parameter $q = 7$. As expected, for small values of p the system is polarized and for large values of p there is no majority in the system.

The transition between the state with and without majority is qualitatively different in the case of anticonformity than in the case of independence.

(A) For anticonformity with increasing p maxima become lower and approach each other. Eventually they form a single maximum at $c = 0.5$ for $p = p^*$. This is a typical behavior for a continuous phase transition [47]. The critical value p^* increases with q and has been found analytically in [10] as $p^*(q) = (q-1)/2q$.
(B) In the case with independence, for $p = p^*(q)$ the third maximum appears at $c = 0.5$ (no majority). This maximum increases with p, while the remaining two maxima decrease.

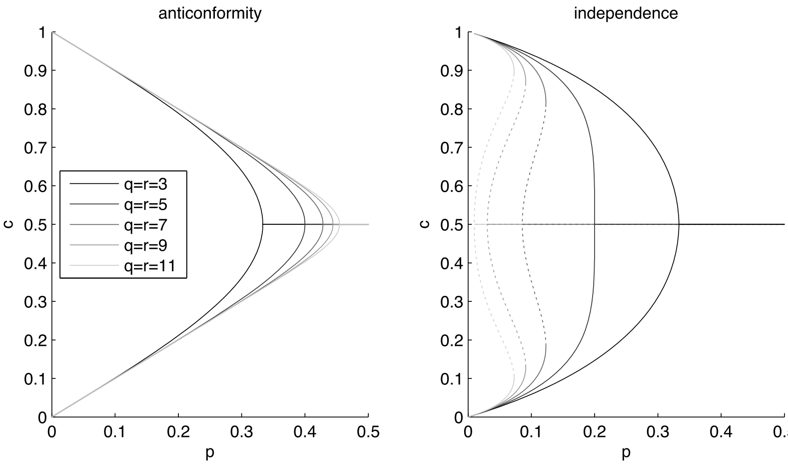

Fig. 6 Dependencies between steady values of concentration c and the level of the nonconformity p for the model with anticonformity (*left panel*) and independence (*right panel*) in the case of the q-voter model. *Solid lines* correspond to the stable steady states that are eventually reached. For the initial value of concentration $c_0 > 0.5$ the *upper branch* is reached, whereas for $c_0 < 0.5$ the *lower branch* is reached. *Dotted lines*, that are visible in a case with independence, denote unstable steady states—if the concentration is equal to the concentration denoted by the *dotted line* the system will not evolve. However, if the concentration is above or below the *dotted line*, the system will evolve towards stationary concentration denoted by the *solid lines*. It is seen that the value of the transition point p^* between the phase with majority (i.e. $c \neq 0.5$) and without majority (i.e. $c = 0.5$) increases for the model with anticonformity and decreases for the model with independence

This is a typical behavior for a discontinuous phase transition for which we can observe the phase coexistence [47]. The critical value $p^*(q)$ decreases with q and has been found analytically in [10] as $p^*(q) = (q-1)/(q-1+2^{q-1})$.

6.3 Results for the Large System

As we have already mentioned, with the increasing system size N fluctuations decrease. This can be seen also from $\rho(c)$. With increasing N peaks in Fig. 5 become more narrow and higher but do not shift in respect to c. Therefore, the stationary concentrations can be easily derived from considering the infinite system. In the stationary state the probability of growth γ^+ should be equal to the probability of loss γ^-:

$$\gamma^+ - \gamma^- = 0. \tag{10}$$

To calculate stationary values of concentration we simply solve the above equation. More detailed calculations can be found in [10] and here we present results only as a figure—dependencies between steady values of concentration c and the level of the nonconformity p for the q-voter model with anticonformity (Model A in Fig. 3) and independence (Model B in Fig. 3) are presented in Fig. 6.

Two main differences are seen between Models A and B:

- First of all, the value of the transition point p^* between the phase with the majority (i.e. $c \neq 0.5$) and without majority (i.e. $c = 0.5$) increases for the model with anticonformity and decreases for the model with independence. This behavior can be easily explained recalling that both conformity and anticonformity depend on the state of the q-lobby,

whereas independence does not. When q increases it becomes unlikely to choose randomly q parallel spinsons and therefore the independence term dominates. On the other hand, anticonformity takes place only when $q + 1$ parallel spinsons are chosen randomly. Therefore the anticonformity term decline in importance more than conformity and p^* increases with q.
– The second difference between models is related to the type of the phase transition. There is a continuous phase transition for an arbitrary value of $q \geq 2$ in a case of anticonformity, whereas in a case with independence the transition becomes discontinuous for $q \geq 6$ (this has been already seen in Fig. 5). Unstable fixed point, which is represented by the dotted line (on right panel in Fig. 6), means that the system cannot escape from it without external fluctuations but simultaneously never reaches this point from the outside. Therefore, concentration evolves only towards stable values, denoted by solid lines, and it jumps between two almost fully ordered states. This is very different from the behavior observed in the case with anticonformity (left panel in Fig. 6) where the concentration changes continuously. From the social point of view it means that, in the case with independence, very dramatic change of public opinion can take place with the small change of the independence level (for $q \leq 6$). In the case with anticonformity such a rapid change is not observed. From this point of view independence is more 'dangerous' for the social system than anticonformity, in a sense that it can cause more unexpected changes in the society.

One should remember that the differences between anticonformity and independence, that are described above, are observed merely within the q-voter model presented in Fig. 3. Therefore at least two questions naturally arise:

1. Are these differences observed within other models?
2. What is observed in real social systems?

The second question is obviously more important that the first one. Simultaneously, to be really fair with answering the second question, one should design and conduct adequate social experiment which is a very difficult task. Therefore, in this paper we will try at least to answer the first question. As already mentioned in the Introduction, we will not consider all possible models of opinion dynamics. Instead, we will consider a generalized q-voter model that as a special cases includes a nonlinear q-voter model, a certain type of a majority rule and also other cases that have not been considered up till now in the literature.

7 Generalized q-Voter Model

Again we consider a set of N spinsons and at each elementary time step we choose randomly q-lobby, i.e. a group of q spinsons $S_1, \ldots, S_q$. Then we choose randomly a voter S_{q+1} on which the group can influence. In the q-voter model conformity and anticonformity take place only if the q-lobby is homogeneous, i.e. all q spinsons are parallel.

In the generalized q-voter model conformity takes place if at least r spinsons among q are parallel, with the assumption that $r \in [\lceil q/2 \rceil, q]$, where ceiling $\lceil x \rceil$ is the smallest integer not less than x. This assumption is made to make the model reasonable from the social point of view. The generalized model includes, as a special cases, the basic q-voter model and the majority rule:

– For $r = q$ the model reduces to the q-voter model presented in the previous section, i.e. unanimous majority is needed.

– For $r = \lceil q/2 \rceil$ we deal with a kind of the majority rule. In this case the majority that consists of at least half of the q-lobby is enough for the social influence. This is an idea introduced by Galam [20], although the algorithm is different since in our model only one voter changes the state instead of a whole group.

Now we are ready to introduce two types of nonconformity—anticonformity and independence:

(A′) The most natural way is to introduce anticonformity analogously to the conformity, i.e. it takes place if at least r spinsons among q are parallel. However, this is only one of the possibilities—we will call this kind of nonconformity r-anticonformity.
(A) Another possibility is to introduce anticonformity exactly in a way it was done in the q-voter model, i.e. anticonformity takes place only if all q spinsons are parallel. This assumption could be justified by the statement that the main source of anticonformity is the asserting of uniqueness (see Sect. 2).
(B) This is quite obvious how to introduce independence—spinson changes its state independently of a q-lobby, i.e. analogously as in a q-voter model.

Because it is not entirely clear how the anticonformity should be introduced we will investigate both types (see the illustration of the model in Fig. 7).

In the q-voter model there are only two parameters—the probability of nonconformity p and the size of the group q. In the generalized q-voter model additionally there is a third parameter—the threshold r. Therefore analysis is a bit more complicated but nevertheless we are able, as in a case of the q-voter model, find the stationary behavior of the system. Again stationary values of concentration c can be derived relatively easily for the infinite system.

The probability that the number of ↑-spinsons increases in a single time step is given by:

$$\gamma^+ = (1-p)\alpha_\uparrow + p\beta_\uparrow, \tag{11}$$

where $\alpha_\uparrow$ denotes the probability that the number of ↑-spinsons increases due to the conformity and $\beta_\uparrow$ denotes the probability that the number of ↑-spinsons increases due to the one of two possible types of nonconformity (anti-conformity or independence). Analogously, the probability that the number of ↓-spinsons increases (which is equivalent to the probability that the number of ↑-spinsons decreases) in a single time step is given by:

$$\gamma^- = (1-p)\alpha_\downarrow + p\beta_\downarrow, \tag{12}$$

where $\alpha_\downarrow$ denotes the probability that the number of ↓-spinsons increases due to the conformity and $\beta_\downarrow$ denotes the probability that the number of ↓-spinsons increases due to the one of two possible types of nonconformity. Of course there is also nonzero probability that nothing changes:

$$\gamma^0 = 1 - \gamma^+ - \gamma^-. \tag{13}$$

Probabilities that the number of ↑-spinsons increases or decreases due to the conformity can be calculated as:

$$\alpha_\uparrow = \sum_{i=r}^{q} \binom{q}{i} (c)^i (1-c)^{q-i+1} \tag{14}$$

$$\alpha_\downarrow = \sum_{i=r}^{q} \binom{q}{i} (1-c)^i (c)^{q-i+1}, \tag{15}$$

whereases probabilities that the number of ↑-spinsons increases or decreases due to the nonconformity depends on a type of nonconformity:

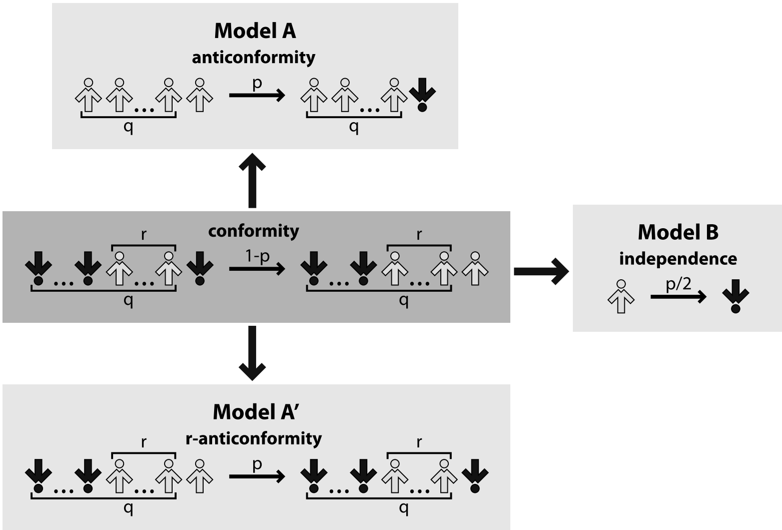

Fig. 7 Illustration of the generalized q-voter model with the threshold r in the case of anticonformity (Models A and A$'$) and independence (Model B). At each elementary time step q spinsons are picked at random and form a group of influence (q-lobby). Then the voter, on which the group can influence, is randomly chosen. With probability p voter behaves like anticonformist or independent and with the probability $1-p$ like conformist. In this model conformity takes place if at least r spinsons in the q-lobby are in the same state. To make the model as general as possible two types of anticonformity are proposed:
(A) The first type of anticonformity (Model A) is identical as in the q-voter model—it takes place only if the q-lobby is homogeneous.
(A$'$) The second type, so called r-anticonformity (Model A$'$), acts in the same way as conformity—it takes place if at least r spinsons in the q-lobby are in the same state.
(B) In a case of independent behavior, the voter does not follow the group, but acts independently—with probability $1/2$ it flips to the opposite direction.
To make the model reasonable from the social point of view we will consider $r \in [\lceil q/2 \rceil, q]$, where ceiling $\lceil x \rceil$ is the smallest integer not less than x. For $r = q$ the model reduces to the q-voter model presented in Fig. 3. In such a case Model A is equivalent with A$'$. For $r = \lceil q/2 \rceil$ we deal with a kind of the majority rule—at least half of the group has to be in the same state to influence the voter

(A$'$) In the case of r-anticonformity (see Model A$'$ in Fig. 7):

$$\beta_\uparrow = \sum_{i=r}^{q} \binom{q}{i} (1-c)^{i+1} (c)^{q-i}, \tag{16}$$

$$\beta_\downarrow = \sum_{i=r}^{q} \binom{q}{i} (c)^{i+1} (1-c)^{q-i}. \tag{17}$$

(A) In the case of anticonformity like in the original q-voter model (see Model A in Fig. 7):

$$\beta_\uparrow = (1-c)^{q+1}, \tag{18}$$

$$\beta_\downarrow = c^{q+1}. \tag{19}$$

(B) In the case of independence (see Model B in Fig. 7):

$$\beta_\uparrow = \frac{1-c}{2}, \tag{20}$$

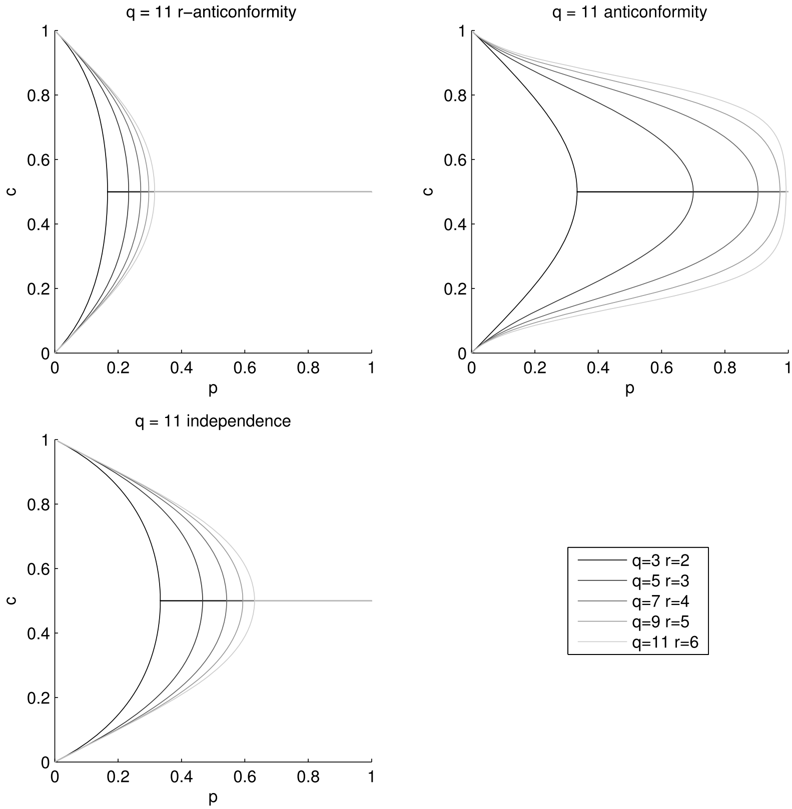

Fig. 8 Dependencies between steady values of concentration c and the level of the nonconformity p for the model with anticonformity, r-anticonformity and independence in the case of the majority rule, i.e. $r=\lceil q/2\rceil$. As seen, in contrast to the q-voter model (i.e. $r=q$), results are qualitatively the same for all types of nonconformity. In all three cases there is a continuous phase transition between states with and without majority and the value of the transition point increases with q. This means that within majority rule we do not observe, on the macroscopic scale, any qualitative differences between various types of nonconformity

$$\beta_{\downarrow}=\frac{c}{2}. \tag{21}$$

Again we find the stationary values of concentration from the condition $\gamma^{+}-\gamma^{-}=0$.

7.1 The Majority Case

We begin with presenting results for the majority case, i.e. for $r=\lceil q/2\rceil$ (see Fig. 8). It is seen that dependencies between c and p for various types of nonconformity do not differ as much as they did in the case of the q-voter model (see Fig. 6). In all three cases the continuous phase transition between states with majority and without majority is observed. Moreover, in all three cases the value of the transition point increases with the size of the q-lobby.

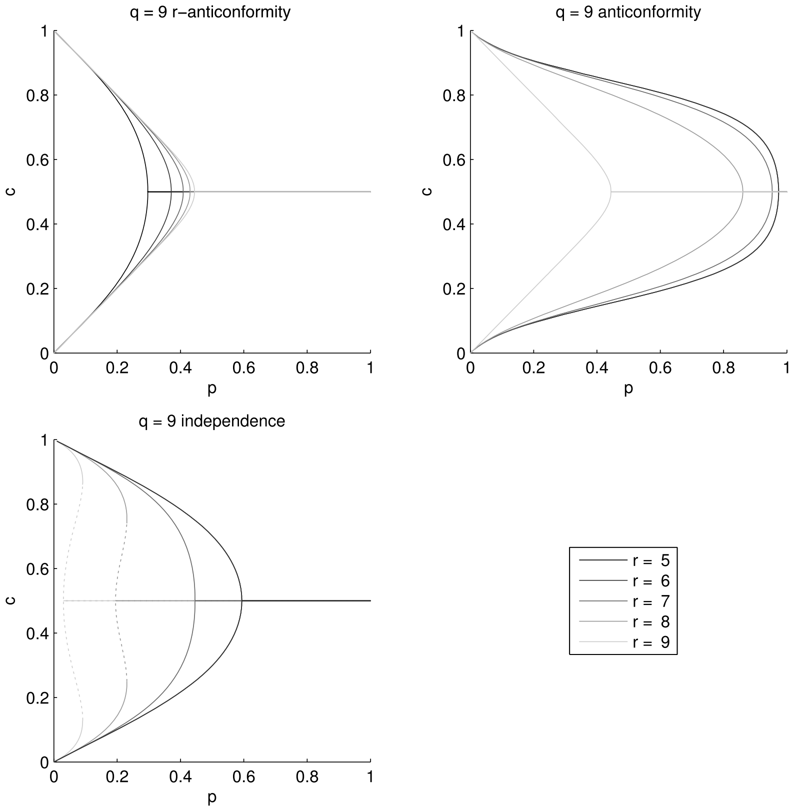

Fig. 9 Dependencies between steady values of concentration c and the level of nonconformity p for the q-voter model with the threshold r for three types of nonconformity: anticonformity, r-anticonformity and independence. In all cases $q = 9$ and the threshold $r \in [\lceil q/2 \rceil, q]$. Although in all three cases there is a phase transition, there are qualitative differences between three types of nonconformity. As usually, *solid lines* correspond to the stable steady states and *dotted lines*, that are visible in a case with independence, denote unstable steady states.
(A′) In the case with r-anticonformity the phase transition is continuous and p^* decreases with r.
(A) In the case with anticonformity the phase transition is continuous and the critical point p^* increases with r.
(B) In the case of independence the phase transition changes its character from continuous to discontinuous for $q \geq 6$ and $r \geq r^*(q)$ that grows almost linearly with q. For example: $r^*(q = 6) = 6, r^*(q = 9) = 8, r^*(q = 12) = 10, r^*(q = 20) = 14$ and approximately satisfies the equation $q = 1.8r^* - 4.9$

7.2 The General Case

Now we move to the general case with $q > 0$ and $r \in [\lceil q/2 \rceil, q]$. Dependencies between c and p for a fixed value of $q = 9$ and different values of r are presented in Fig. 9. In this case results for three types of nonconformity are qualitatively different. As usually, there is an order-disorder phase transition at $p = p^*$. However, the type of the phase transition is different for independence than in the cases with anticonformity. For a critical value of $r = r^*(q)$, that scales almost linearly with q, the transition changes its type from continuous to discontinuous. We should stress here that such a behavior is observed only for the size of the q-lobby $q \geq 6$. For $q = 6$ there is a discontinuous phase transition only for the threshold

$r = 6$ (i.e. the case of the q-voter model). For the size of the lobby $q = 7$ still a discontinuous phase transition is observed only for $r = q = 7$. For $q = 9$ a discontinuous phase transition is observed for $r = 8$ and $r = 9$, i.e. $r^*(9) = 8$ and for $q = 12$ a discontinuous phase transition is observed for $r = 10, 11, 12$, i.e. $r^*(12) = 10$. It is also seen in Fig. 9 that the value of the phase transition point p^* between state with majority (i.e. $c \neq 0.5$) and without majority ($c = 0.5$) increases for r-anticonformity and decreases for anticonformity and independence. This can be understood also heuristically and we will return to this issue later.

8 The Phase Diagrams for the Generalized q-Voter Model

In Figs. 6, 7, 8 and 9 we have presented dependencies between stationary values of concentration and the level of nonconformity. As has been already noted in [28] both types of nonconformity, anticonformity and independence, tend to be effective in differentiating people from others. Indeed, within our model, all three types of nonconformity lead to the phase transition between consensus and stalemate (in a sense that there is no majority in the system).

However, the type of the phase transition and the dependence between the critical value p^* and parameters q, r depends strongly on the type of nonconformity. Therefore, to summarize all results we construct and discuss the phase diagrams for all types of nonconformity. The critical value of nonconformity, below which there is a majority in the system, can be calculated as:

$$p^* = \frac{\alpha_\uparrow - \alpha_\downarrow}{\alpha_\uparrow - \alpha_\downarrow - \beta_\uparrow + \beta_\downarrow}, \tag{22}$$

where $\alpha_\uparrow, \alpha_\downarrow$ are given by Eq. (15) and $\beta_\uparrow, \beta_\downarrow$ are given by Eqs. (19), (17), (21) respectively to the type of nonconformity. Intentionally, we do not present here any calculations, since the paper is intended to be available for a wide range of readers. However, the procedure is the same as in the case of the q-voter model, for which detailed calculations can be found in [10].

8.1 The Special Cases—Unanimity and Majority

Let us start with two special cases—'unanimity' (i.e. $r = q$) and 'majority' (i.e. $\lceil q/2 \rceil$). These two cases are particularly simple because r is uniquely determined by q and therefore the critical value of nonconformity p^* depends only on the single parameter q. Phase diagrams for these cases are presented in Fig. 10. As already seen in Figs. 6 and 8:

- In the case of the majority the critical value p^* grows with q for all three types of nonconformity. The phase transition is continuous for arbitrary value of $q \geq 2$.
- In contrast, in the case of the 'unanimity' ($r = q$) the critical value of nonconformity p^* increases for anticonformity and decreases for independence.

  Moreover, in the case of independence there is a discontinuous phase transition for $q \geq 6$, which is denoted by a region of a coexistence. In the coexistence region one of two phases (with or without majority) is metastable and the second one is stable.

  The system can reach metastable state but larger fluctuations can easily derive the system from this state. Therefore, the probability of reaching such a state is lower than probability of reaching the stable state. At the transition point, denoted in Fig. 10 by the dashed line—both states are equally probable, i.e. both states are stable.

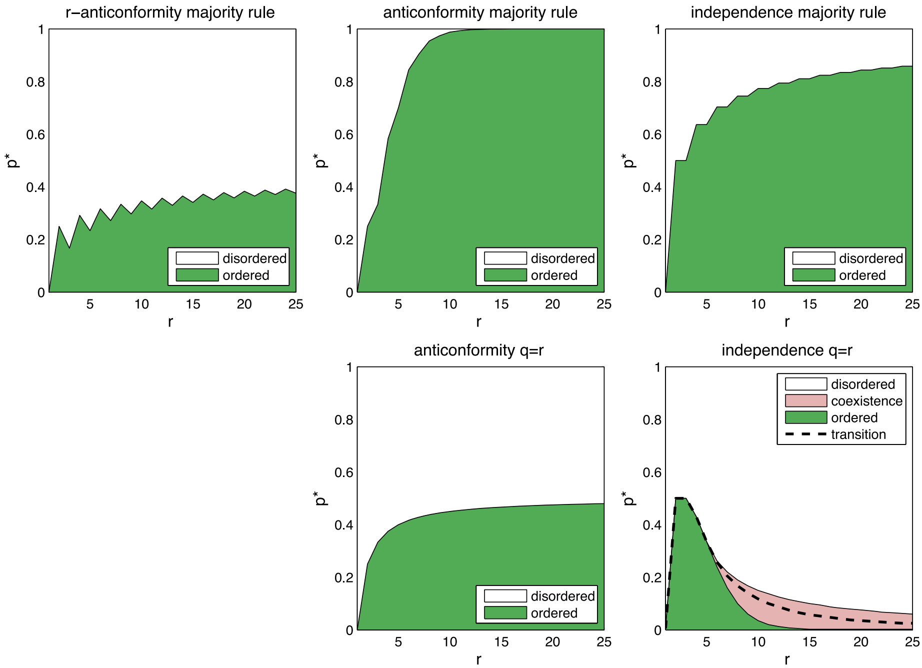

Fig. 10 Phase diagrams for the model with the majority rule, i.e. $r = \lceil q/2 \rceil$ (*upper panels*) and the q-voter model, i.e. $r = q$ (*bottom panels*).
(A′) For r-anticonformity the majority is equivalent to unanimity. There is a continuous phase transition and the critical value p^* grows with q. In this case the majority is equivalent to unanimity.
(A) For anticonformity in both cases there is a continuous phase transition and the critical value p^* increases with q.
(B) For independence, in the case of the majority the critical value p^* increases with q, whereas in the case of the 'unanimity' ($r = q$) decreases. Moreover, in the case of independence there is a discontinuous phase transition for $q \geq 6$, which is denoted by a region of a coexistence. In the coexistence region one of two phases (with or without majority) is metastable and the second one is stable. The system can reach metastable state but larger fluctuations can easily derive the system from this state. Therefore, the probability of reaching such a state is lower than probability of reaching the stable state. At the transition point, denoted in Fig. 10 by the *dashed line*—both states are equally probable, i.e. both states are stable

Results presented up till now have shown that:

- The difference between anticonformity and independence, that is introduced on the microscopic (psychological) level, can influence the behavior on the macroscopic (society) scale.
- Within the presented model, the macroscopic behavior of the system is much richer if we assume that the social influence takes place in the case of unanimous instead absolute majority.

8.2 The General Case

Now we construct the phase diagram in the general case in which $r \in [\lceil q/2 \rceil, q]$. In such a case the critical value p^* depends on two parameters q and r. Therefore, we fist present the phase diagram for a chosen fixed values of $q = 15$ and $q = 20$ (see Fig. 11). We have chosen

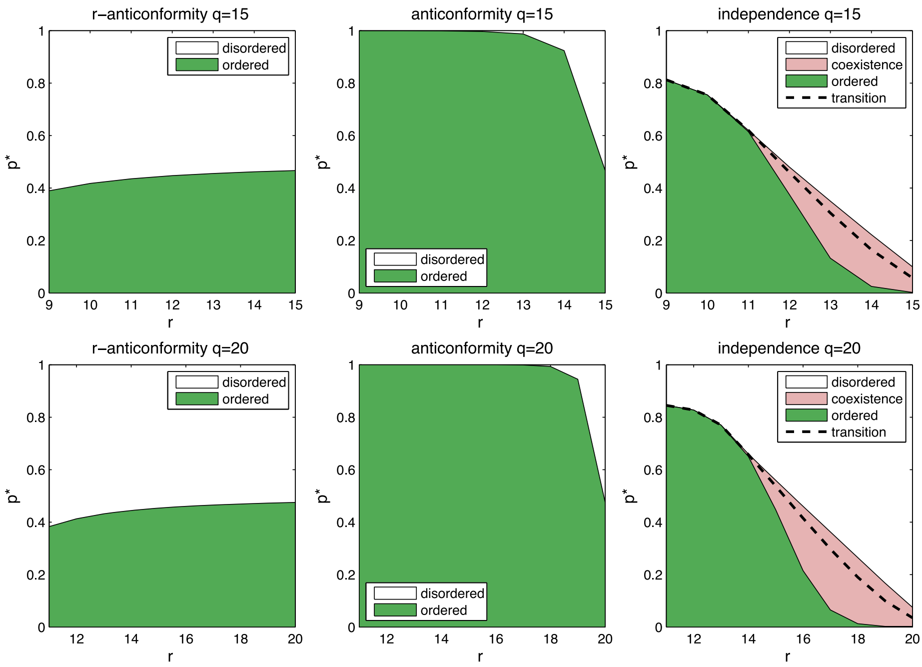

Fig. 11 Phase diagrams for the generalized q-voter model with a threshold r for the fixed values of $q = 15$ (*upper panels*) and $q = 20$ (*bottom panels*). Qualitative differences are seen between three types of nonconformity:
(A′) There is a continuous phase transition in the case of r-anticonformity (Model A′ in Fig. 7) and the critical value of p decreases with r.
(A) For anticonformity (Model A in Fig. 7) the phase transition is also continuous but the critical value of p increases with r.
(B) In the case with independence (Model B in Fig. 7) the critical value of p decreases with r and the transition changes its character from continuous to discontinuous

these values of q just as an example—the complete 3-dimensional phase diagram is far less legible. Qualitative differences are seen between three types of nonconformity:

(A′) There is a continuous phase transition in the case of r-anticonformity (Model A′ in Fig. 7) and the critical value of p decreases with r.
(A) For anticonformity (Model A in Fig. 7) the phase transition is also continuous but the critical value of p increases with r.
(B) In the case with independence (Model B in Fig. 7) the critical value of p decreases with r and the transition changes its character from continuous to discontinuous.

The dependence between a critical value of p and parameter r can be not only calculated analytically or numerically but also heuristically explained:

(A′) For a given size of the lobby q r-anticonformity decreases with r faster than conformity. While conformity takes place if r spinsons inside the q-lobby are parallel, r-anticonformity demands $r + 1$ parallel spinsons.
(A) Anticonformity takes place only if $q + 1$ spinsons are parallel, i.e. does not depend on r. On the other hand conformity is decreasing with increasing r because when r grows it becomes unlikely to find r parallel spinsons inside the q-lobby. Therefore, p^* should decrease with r, what is indeed observed.

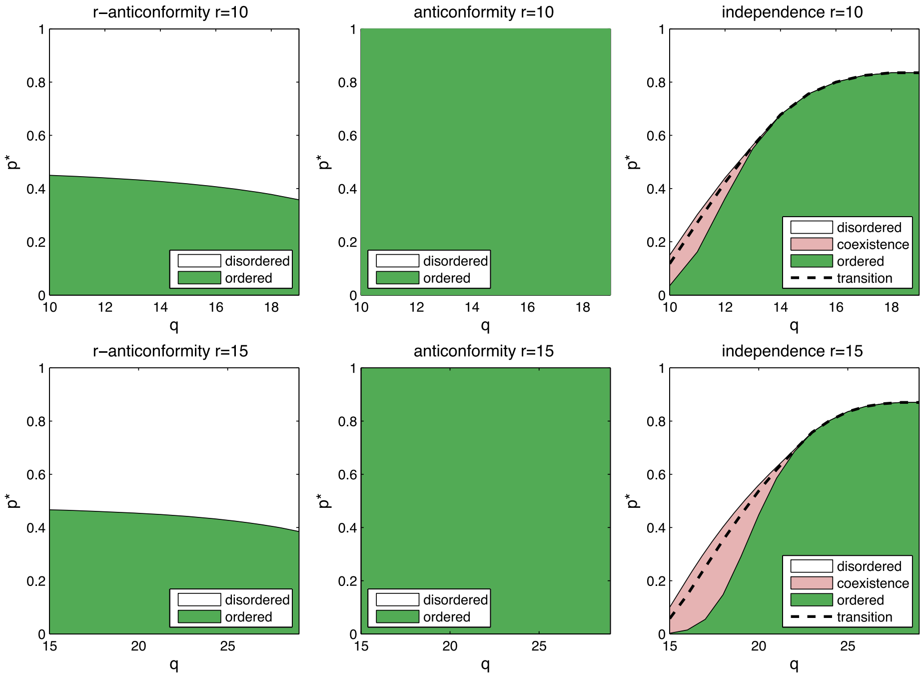

Fig. 12 Phase diagrams for the generalized q-voter model with a threshold r for the fixed values of $r = 15$ (*upper panels*) and $r = 10$ (*bottom panels*). Qualitative differences are seen between three types of nonconformity:
(A′) In the case of r-anticonformity there is a continuous order-disorder transition and the critical value of p decreases with q.
(A) For anticonformity there is no phase transition—the system is ordered for arbitrary value of q.
(B) Finally, in the case with nonconformity the critical value of p increases with q and the transition changes its character from discontinuous to continuous

(B) It is also easy to understand why p^* decreases with r in the case of independence—this type of nonconformity does not depend on the state of q-lobby. On the other hand, as explained above, conformity is decreasing with increasing r.

Now we investigate the dependence $p^*(q)$ for a fixed value of r. In Fig. 12 the phase diagram is presented for a sample values of $r = 10$ and $r = 15$. Again, qualitative differences are seen between three types of nonconformity:

(A′) In the case of r-anticonformity there is a continuous order-disorder transition and the critical value of p decreases with q.
(A) For anticonformity there is no phase transition—the system is ordered for arbitrary value of q.
(B) Finally, in the case with nonconformity the critical value of p increases with q and the transition changes its character from discontinuous to continuous.

Again, we can try to understand heuristically dependencies between critical values of p and parameter q:

(A′) For a given r conformity is increasing with q, since it is easier to find r parallel spinsons in the q-lobby if q is larger. Obviously r-anticonformity, similarly to conformity, becomes more probable with increasing q. The question is which force, conformity or r-anticonformity, grows faster with q. It can be easily checked that r-anticonformity gain more with q because it takes place if at least $r+1$ spinsons are parallel among $q+1$, whereas conformity takes place if at least r spinsons are parallel among q. Therefore p^* decreases with q in this cases.

(A) Far less intuitive is the case with anticonformity. It is obvious that anticonformity decreases with q because it takes place only if $q+1$ spinsons are parallel. Therefore we have a competition of two forces—conformity increases with q and anticonformity decreases. It occurs that this competition results in constant value of $p^*=1$—i.e. for any value of q there is a majority in the system.

(B) Because for a given r conformity is increasing with q it is easy to understand why p^* increases with q in the case of independence—this type of nonconformity does not depend on the state of q-lobby.

9 Summary

The main goal of this paper was to examine how different types of nonconformity, introduced on the microscopic level, manifest at the level of the society (i.e. macroscopic). Moreover, we wanted to check if results would be universal or rather model-dependent. To achieve the goal we have introduced a generalized q-voter model. There are three parameters in the model:

1. q—the size of the q-lobby, i.e. the size of a group of influence,
2. p—the level (probability) of nonconformity that can be one of two types—anticonformity or independence,
3. r—the threshold needed for social influence (e.g. conformity takes place if at least r spinsons among q are parallel).

All these parameters are important, but each for a different reason:

(q) By varying q, we can move from a linear voter model to a nonlinear voter models of different orders (including Sznajd model for $q=2$).

(p) p introduced a noise to the system—both types of nonconformity destroy order, although each in a slightly different way. By varying p we move from the complete consensus to the status-quo situation.

(r) The parameter r allows for the unification of several opinion dynamics models—e.g. for $r=\lceil q/2 \rceil$ we deal with the majority model and for $r=q$ with the original q-voter model. From this point of view the threshold r is not so important from the social point of view, but is used rather to understand how details of the model (in this case the way of introducing conformity) affect results. On the other hand it may be also connected with the level of majority that is needed to persuade.

We would like to emphasize that we are aware of the fact that the parameter space for agent-based models is much richer than the 3-parameter space used in this paper. However, our aim was not to propose a possibly general model of opinion formation. We wanted to check if the differences between independence and anticonformity, that were shown within the q-voter model [10], would be visible if the conformity acted not only in the case of unanimity but for example in the case of absolute majority or in other cases—and hence the idea of introducing the threshold r into the q-voter model.

Our calculations have shown that results depend significantly on parameters. In particular:

– For the majority rule, that corresponds to $r = \lceil q/2 \rceil$, there is a continuous phase transition at $p = p^*$ between state with majority and status-quo for both types of nonconformity for any value of $q > 1$. Moreover, for both types of nonconformity p^* increases with the size of the q-lobby. Therefore, differences between anticonformity and independence are qualitatively indistinguishable on the macroscopic level under the majority rule.
– In the case of a q-voter model, that corresponds to $r = q$, there are significant differences between two types of nonconformity. In the case of anticonformity there is a continuous order–disorder phase transition at $p = p^*$ and the value of p^* increases with the size of the q-lobby. On the other hand, for the model with independence the value of the transition point p^* decays with q. Moreover, the phase transition in this case is continuous only for $q \leq 5$. For larger values of q there is a discontinuous phase transition—and coexistence of ordered (with majority) and disordered (without majority) phase is possible.
– In the general case in which r and q are two independent parameters, the differences between anticonformity and independence depends on r and q. Similarly as in the case of the q-voter model there is a continuous phase transition in the case of anticonformity, whereas in the case of independence the change of the transition's type may appear dependently on parameters r and q.

Above results suggests that differences between anticonformity and independence might be significant or indistinguishable on the macroscopic level, depending on parameters of the model. Therefore, we are able to answer fairly questions posed in the Introduction:

1. Differences between two types of nonconformity, that are recognized by social psychologists on the individual (microscopic) level, can manifest on the society (macroscopic) level within some models. For example there are significant differences between anticonformity and independence in the q-voter model in which unanimous majority is required for the social influence. Analogous results are obtained in the case when the threshold r, needed for a social influence, is high enough, which can be understood as an almost unanimous majority.
2. Differences between anticonformity and independence, that manifest on the macroscopic level, are not universal. They depend on the model designs. For example in a case of majority rule two types of nonconformity are qualitatively indistinguishable.

Above answers, although complete the goal of the paper, raise further important questions related to the value and validity of opinion dynamics models. Differences on the macroscopic level, that were shown in this paper, may indicate which microscopic model is correct, and which one should be rejected. However, to judge which macroscopic behavior is adequate in the real social system might be very hard or maybe even impossible. It would be extremely valuable if one could design and conduct the social experiment which could show differences between anticonformity and independence on the macroscopic level. So far, we can only conclude that the social behavior, even in seemingly simple world of spinsons, is surprisingly complex.

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A nonlinear q-voter model with deadlocks on the Watts–Strogatz graph

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Abstract. We study the nonlinear q-voter model with deadlocks on a Watts–Strogatz graph characterized by two parameters k and β. Using Monte Carlo simulations, we obtain a so-called exit probability and an exit time. We determine how network properties, such as randomness or density of links, influence the exit properties of a model. In particular we show that the exit probability, which is the probability that the system ends up with all spins up, starting with the p fraction of up-spins, has the general form $E(p) = p^\alpha/(p^\alpha + (1-p)^\alpha)$. Moreover, using the finite-size scaling technique we show that the exit probability exponent α depends both on the parameter q as well as the network structure, i.e. k and β.

Keywords: interacting agent models

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Contents

1. Introduction

Describing opinion dynamics has inspired many physicists to build models that could not be justified by physical phenomena (for a review of opinion dynamic models see [1, 2]). Such models are usually more caricatures than precise portraits of real social systems. However, far-reaching simplifications should not be regarded as a defect of these models. Simplicity allows not only for in-depth analysis but also for analytical treatment. First of all it allows us to describe some universal features or even to determine the most important factors that influence a given social phenomenon.

Certainly, the main challenge that persists with opinion dynamics models is the describing of complex social systems in terms of a relatively simple approach. On the other hand, such models are themselves interesting from a theoretical point of view [3]. Therefore they might be also treated as small building blocks which make a contribution to the construction of still emerging non-equilibrium statistical physics. A good example of such an interesting model is the nonlinear q-voter model introduced in [4] along with its modifications proposed in [5, 6]. The precise definition of the model will be given in the next section. For now, what is important is the fact that in the q-voter model only a unanimous group of q neighbors can influence a voter. Hence this model is a simple generalization of the linear voter model [3]. In this paper we will investigate a special case of a model, which we call the q-voter model with deadlocks, considered already in [5] for a one dimensional lattice. We examine the role of a topology in such a model and show that increasing randomness and density of a network helps us to reach a consensus.

The second motivation for this paper came from a recent controversy on the exit probability $E(p)$ (i.e. the probability that the system ends up with all spins up starting with the p fraction of up-spins) for the q-voter model with deadlocks. It has been shown independently in five papers [5, 7–10] that for $q=2$, which corresponds to the Sznajd model, the exit probability can be described by the following formula:

$$E(p) = \frac{p^2}{p^2 + (1-p)^2}. \tag{1}$$

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However, in [11] it has been suggested that the above result is valid only for finite-size systems and should approach a step-like function for the infinite system. It should be stressed that the suggestion that appeared in [11] was taken seriously and considered by others [5, 10] because the formula (1) was obtained by some approximation (different variants of the mean-field approach). The difficulties of finding the exact solution arise from the fact that the average magnetization in the q-voter model is not conserved. In [7, 8] an approximate solution was constructed by truncating the hierarchy of the rate equations of higher-order correlation functions by a decoupling scheme, known as Kirkwood approximation [12]. On the other hand, results obtained in [5, 7–9] suggest that there is no finite size dependence for $E(p)$ in the case of the one-dimensional lattice. Recent results obtained by Timpanaro and Prado [10] for large lattices confirm the result given by (1) and indicate that the step-like function corresponds to the complete graph. It would seem, therefore, that the ambiguity associated with the exit probability for the q-voter model is explained. However, this is true only for $q=2$ on the one-dimensional lattice.

For a higher value of q and different network structures the problem is still open. It has been suggested in [5] that the exit probability on a one-dimensional lattice for some arbitrary value of q is given by the formula:

$$E(p)=\frac{p^q}{p^q+(1-p)^q}. \tag{2}$$

However, recent results [10] suggest that this might be valid only for small lattices in case of $q>2$. We will examine this problem in this paper for different values of q and different network structures.

2. Model

The original q-voter model introduced in [4] on the one dimensional lattice has been defined as follows:

- Each i-th site of a graph of a size N is occupied by a voter $S_i=\pm 1$
- Initially there is a probability p of finding a voter in a state $+1$ and a probability $1-p$ of finding a voter in a state -1
- The system evolves according to the following algorithm:

  (a) At each elementary time step t, choose one spin S_i, located at site i, at random

  (b) Choose q neighbors (q-panel) of site i

  (c) If all q neighbors are in the same state then S_i takes the same state as its neighbors

  (d) Otherwise, if the q neighbors are not unanimous then take $S_i\to -S_i$ with probability ϵ

  (e) Time is updated $t\to t+\dfrac{1}{N}$

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In [5] it was proposed to study a one-dimensional model with $\epsilon = 0$, which for $q = 2$ corresponds to the Sznajd model. It should be noted that for $\epsilon = 0$ the evolution of the system is hampered due to the existence of deadlock configurations. Deadlocks should be understood as configurations in which there is no possibility of an evolution due to the lack of a unanimous q-panel. In the case of a one dimensional lattice and $q = 2$ there is only one deadlock configuration—an antiferromagnetic state $+-+-+\ldots$. For $q = 3$ there are already many more, e.g. $+-+-+-$, ..., $++-++-++\ldots$ or $++-+-++-+\ldots$, etc. However, if initially there is at least one q-panel, evolution will reach one of two final absorbing states. The nonlinear q-voter model with deadlocks has been found to be interesting for several reasons:

- For $q = 2$ it reduces to the Sznajd model for opinion dynamics and in this case the analytical formula for an exit probability has been found independently by [7–9, 11]
- The exit probability does not depend on a system size as reported by [5, 7–9]. This result should be treated with caution, taking into account recent results obtained by [10] for large lattices. It seems that additional simulations are needed to explain this contradiction.
- In a case of random noise, a system undergoes a phase transition which changes its type from continuous to discontinuous at $q = 5$ [6].

To investigate the role of the network topology we have decided to use the model introduced by [14], mainly because it allows us to study various structures—from regular lattices with different sizes of neighborhood, through small-world networks to random graphs. The Watts–Strogatz (WS) algorithm, that we have used, is defined as follows:

- Start with a 1D lattice of size N with periodic boundary conditions in which each node is connected to its $2k$ neighbors
- Then with probability β replace each edge by a randomly chosen edge

For $\beta = 0$ we deal with regular lattices—e.g. for $k = 1$ we have a simple one-dimensional lattice with interactions only to the nearest neighbors, and for $k = (N-1)/2$ we have a complete graph. With increasing β we increase the randomness of the network going through the small-world (for $\beta = 0.01 - 0.1$) to the random graph for $\beta = 1$. Summarizing, we have one parameter q that defines the model itself and two parameters k and β that describe the network properties.

3. Results

3.1. Exit probability on a complete graph

An important property of a system with absorbing states is the so-called exit probability [3]. In our case the exit probability $E(p)$ should be understood as a probability of the absorbing state with all spins '+1' as a function of the initial probability p of finding

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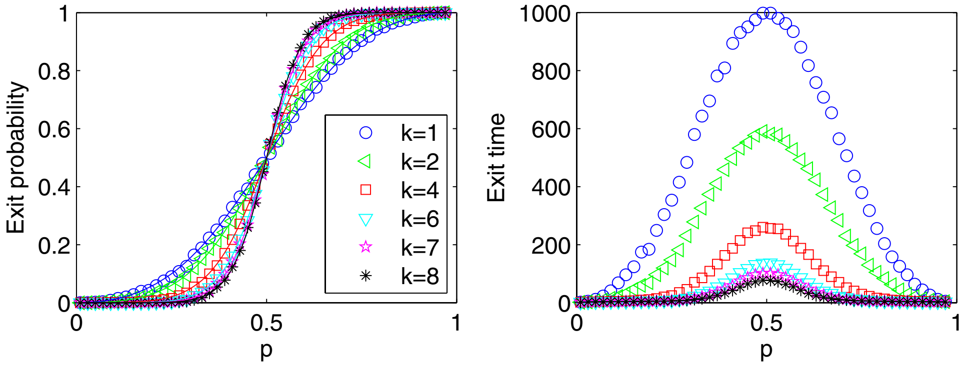

Figure 1. Exit probability (left panel) and exit time (right panel) as a function of the initial probability p of spin $+1$ for $q=2$ (which corresponds to the Sznajd model) and $\beta=0$ (regular graph) for several sizes of the neighborhood given by k. The system size $N=100$ and results were averaged over 10^4 samples. Solid lines on the left panel correspond to the analytical formula given by equation (7). The steepness of the exit probability slope increases with k, and exit time significantly decreases with k.

a spin in a state $+1$. For a complete graph, which corresponds also to the mean-field treatment, the evolution of the probability of 'up'-spins is given by:

$$p(t+\Delta t) = p(t) + p^q(t)\,(1-p(t)\,) - (1-p(t)\,)^q\, p(t) \tag{3}$$

Fixed points can be easily found from the condition $p(t+\Delta t) = p(t) = p^*$, i.e.:

$$(p^*)^q\,(1-p^*) - (1-p^*)^q\, p^* = p^*(1-p^*)\,[\,(p^*)^{q-1} - (1-p^*)^{q-1}\,] = 0. \tag{4}$$

As can be seen there are three fixed points $p^* = 0,\ 1/2,\ 1$. This can be easily checked by calculating the following derivative:

$$\frac{\mathrm{d}}{\mathrm{d}p}(p + p^q(1-p) - (1-p)^q\, p)\,|_{p=p^*}, \tag{5}$$

where $p^*=0$ and $p^*=1$ are stable, but $p^*=1/2$ is an unstable fixed point. Therefore on a complete graph for $p<1/2$ the system eventually reaches the absorbing state $p^*=0$, and for $p>1/2$ the system reaches $p^*=1$. This means that the exit probability for the q-voter model on the complete graph with arbitrary value q is a step-like function:

$$E(p) = \begin{cases} 0 & \text{for} \quad p < 1/2 \\ 1 & \text{for} \quad p > 1/2 \end{cases} \tag{6}$$

Timpanaro and Prado have recently proposed a much more rigorous approach to show that the exit probability is a step-like function for a complete graph [10]. Their and our results confirm that the results obtained by [15] within a unifying frame (GUF) coincides with the mean-field approach and may not be true for arbitrary topology.

3.2. Results on regular graphs

For $\beta=0$, a broad class of WS networks reduces to regular graphs with various sizes of neighborhood given by k. In this section we examine the role of k which from a social

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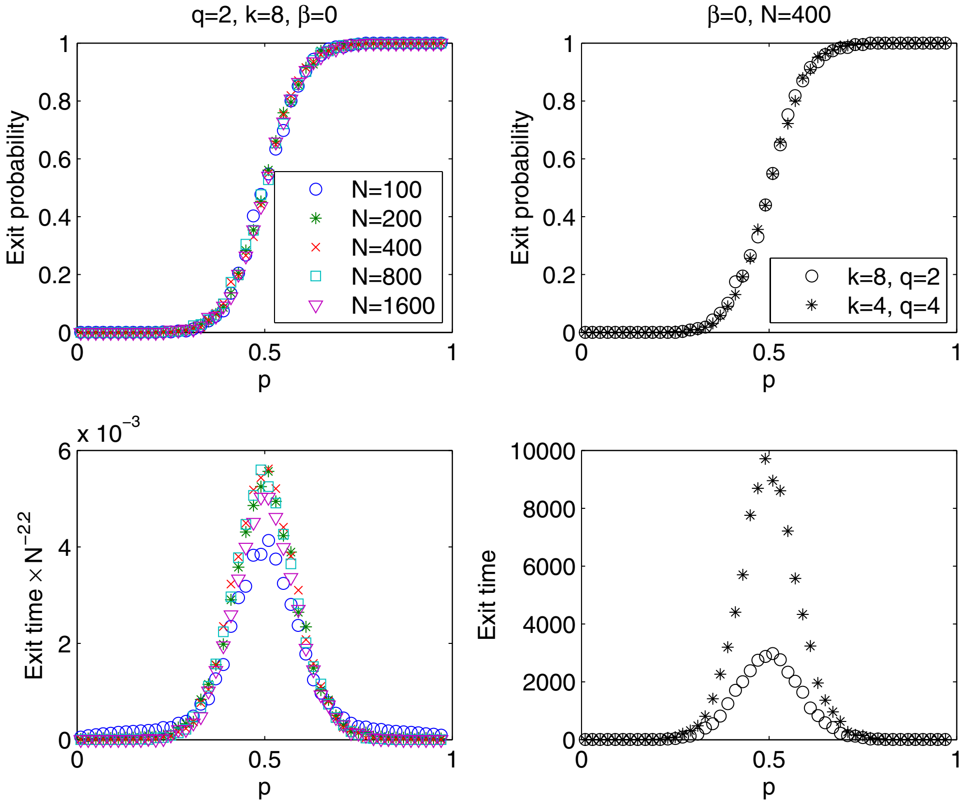

Figure 2. Exit probability (top panels) and exit time (bottom panels) as a function of the initial probability p of spin $+1$. In the left panels results for $q=2$, $k=4$, $\beta=0$ and several system sizes $N=100$, 200, 400, 800, 1600 are presented. In the right panels a comparison between the results for $q=2$, $k=8$ with $q=4$, $k=4$ are presented.

point of view measures the density of the social group. In particular we would like to answer the following questions:

(a) How does parameter k influence the exit probability? Results on a complete graph suggest that exit probability should become steeper with increasing k.

(b) How does k influence exit time, i.e. is a consensus reached faster for smaller or larger values of k?

(c) How do the results scale with the system size for $k>1$? For $k=1$ no finite size dependence has been noted in one-dimensional voter, Sznajd and q-voter models [5, 7–9], as well as several Ising-like models with so-called inflow dynamics [13].

In agreement with our predictions, it is seen in figure 1 that the steepness of the exit probability slope increases with k. This was expected because we have found that for $k=(N-1)/2$ (complete graph) $E(p)$ is a step-like function. It is also the case that for $q=2$ and arbitrary value of k simulation data can be fitted by (see the left panel in figure 1):

$$E(p)=\frac{p^{\alpha}}{p^{\alpha}+(1-p)^{\alpha}}, \tag{7}$$

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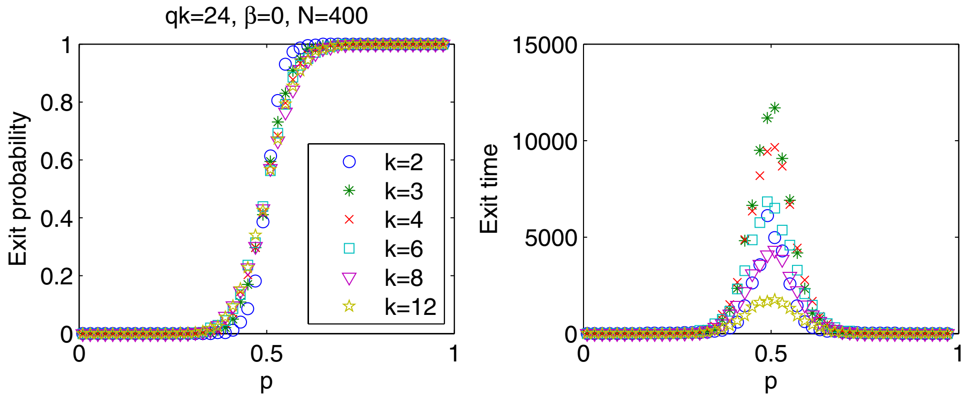

Figure 3. Exit probability (left panel) and exit time (right panel) as a function of the initial probability p of spin $+1$ for several values of q and k, such as $kq = 24$ for $\beta = 0$ (regular graph). The system size $N = 400$ and results were averaged over 10^3 samples. It is seen that the exit probability in all cases is almost identical and therefore one could conclude that it depends mainly on kq, although this is not an exact dependence. Exit time generally decreases with k, as has been already shown in figure 1.

where $\alpha = k/2 + 3/2$. Parameter k also influences exit time, which should be understood as the time needed to reach an absorbing state [3]. As seen the exit time significantly decreases with k, which means that consensus is reached faster in more dense societies.

One of the most interesting questions is related to the finite-size effects. All previous results [5, 7–10] show that the exit probability does not depend on the system size for $q = 2$ on a one-dimensional lattice. Interestingly, analogous results have been very recently obtained for a larger class of Ising-like models—no finite size dependence in the exit probability has been found [13]. In figure 2 we present the exit probability and exit time as a function of the initial probability p of spin $+1$ for $q = 2$, $k = 4$, $\beta = 0$ and several system sizes $N = 100$, 200, 400, 800, 1600. It is seen that the exit probability does not depend on the system size. It is also seen that the exit time (EP) nearly scales with the system size as L^{-2}, though the scaling relation is not exact, in contrast to the voter and Sznajd models on a one-dimensional lattice with $k = 1$ [8]. Interestingly, a comparison between the results for $q = 2$, $k = 8$ and $q = 4$, $k = 4$ that are presented in the right panels of figure 2 suggests that the exit probability depends on kq, instead of two parameters k and q. This conjecture has been examined for other values of k and q (see figure 3).It can be seen in figure 3 that the relation $EP\ (q, k) = EP\ (qk)$ is roughly valid.On the other hand, the exit time (or in other words the consensus time) clearly decreases with k. One might expect that the exit time for different system sizes could be somehow rescaled by k, for example $\tau(k, N) = \tau(k/N)$. To check this expectation we have conducted simulations for several values of q and ten pairs (k, N), for $k/N = 100$ and $N = 100$, 200, ..., 1000. Unfortunately, no simple relation has been found.

3.3. Results on a WS graph

We now investigate the model on a WS graph for $\beta = 0.01$ and different values of k and q. Similarly as for $\beta = 0$, the steepness of the exit time increases both with q and k, and the exit time strongly decreases with k (see figure 4). However, contrary to EP for $\beta = 0$,

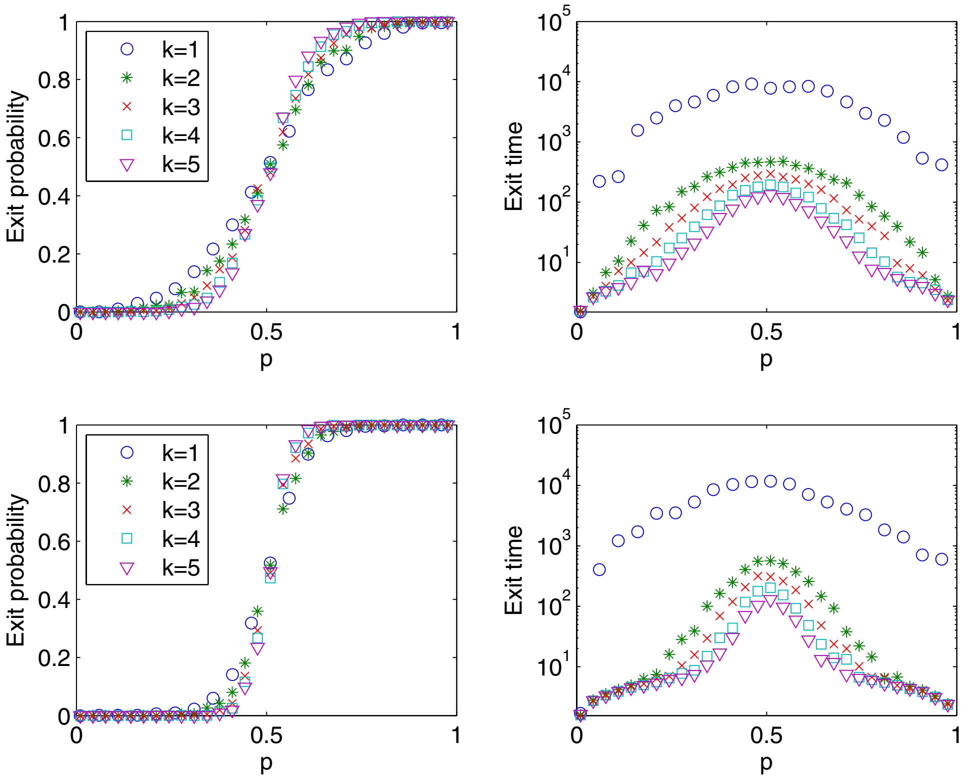

Figure 4. Exit probability (left panels) and exit time (right panels) as a function of the initial probability p of spin $+1$ for $q=2$ (upper panels) and $q=4$ (bottom panels) for $\beta=0.01$ (small-world). The system size $N=100$ and results were averaged over 10^3 samples.

finite-size effects are clearly seen for any value of q (see figure 5). Using the finite-size technique we were able to determine the scaling exponent ν. As usual, we choose the scaling variable $x=(p-p^*)N^{1/\nu}$ [16], where ν is the critical exponent describing the correlation length and p^* is the critical value, i.e. in our case $p^*=0.5$. In figure 5 we present results for $q=2$ and two values of k. Data for different system sizes $N=100$, 200, 400, 800, 1600 collapse into a single curve, though the scaling exponent is not universal and depends on the network structure.

Now we are ready to examine the role of the second parameter, which describes the level of randomness i.e. β.

In figure 6 we present results for $k=4$, $q=4$ and two system sizes $N=100$, 800 for several values of β. First of all, it is seen that the steepness of the exit probability slope increases and simultaneously the exit time decreases with β. However, it is seen that the EP for the small system $N=100$ differs from the one for the larger system ($N=800$). The dependence between the system size N and EP is much weaker for $q=2$ than for larger values of q. This is an interesting result taking into account results obtained recently by [10]. For regular lattices, i.e. $\beta=0$, the dependence between the exit probability and the system size was very difficult or even impossible to observe. Only results on the very large system sizes (10^5–10^7), that required redefinition of the simulation algorithm as proposed in [10], have shown a small dependence on the system size for $q>2$. For the system sizes investigated in this paper, i.e. $L=100$, 200, 400, 800,

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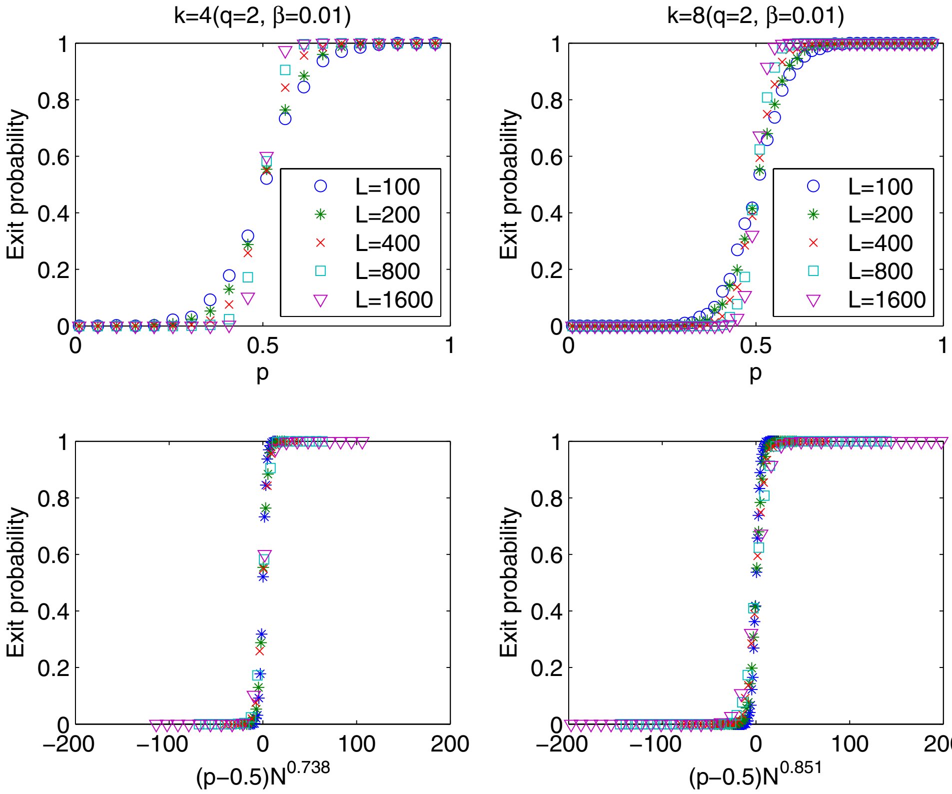

Figure 5. Exit probability as a function of the initial probability p of spin $+1$ for $q=2$ and $\beta=0.01$ for several lattice sizes. Original results are presented in upper and rescaled results in bottom panels. It is seen that the scaling exponent depends on parameter k. As will be discussed further in the paper, the scaling exponent depends also on q and β.

1600, this dependence was almost invisible. However, for $\beta>0$ the difference between results for $q=2$ and $q>2$ are very clear (see upper panel in figure 6).

The second interesting result is related to the exit (consensus) time. Results are presented on a semi-log scale (bottom panel in figure 6) to allow for the comparison between different values of β. The exit time clearly decreases with increasing randomness of the network. A similar result has been found in the case of a linear voter model, which corresponds to $q=1$—*for networks of finite size the ordering process takes a time shorter than on a regular lattice of the same size* [17]. Certainly shorter paths help to reach consensus faster, although our research does not allow us to determine if this is the only property of the network which helps to achieve a consensus. Probably the effect of the network structure is more complex.

4. Discussion

We have investigated a special case of a q-voter model (with $\epsilon=0$) on a WS network described by parameters k and β. We have shown that the exit probability is the

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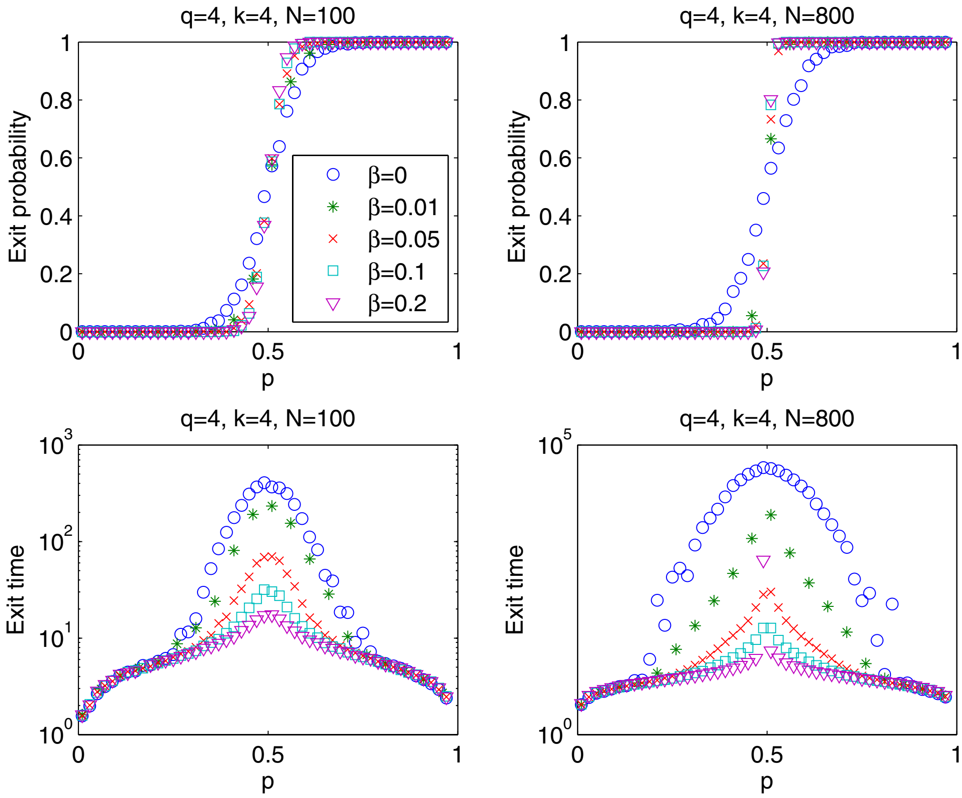

Figure 6. Exit probabilities (upper panels) and exit times (bottom panels) as a function of the initial probability p of spin $+1$ for $q=4$, $k=4$ and several values of β. Right panels correspond to $N=100$ and left panels to $N=800$.

S-shaped function for arbitrary values of parameters q, k and β and can be fitted by an analytical formula (7).

It should be recalled that even for $\beta=0$, $k=1$ and $q=2$ only approximate calculations are available, although surprisingly four independent analytical approaches [7–9, 11] give exactly the same result (1) which is in perfect agreement with Monte Carlo results [5, 10].

In the case of heterogeneous networks, i.e. for $\beta>0$, one could try to apply a powerful technique known as the heterogeneous mean-field (HMF) approach. Recently HMF has been applied to the q-voter model without deadlocks (i.e. for $\epsilon>0$) [18]. It has been argued that for $q=2$ and any network structure, the application of HMF leads to the trivial result as long as $\epsilon=1/2$. In such a case the flipping probability $f(x)=x^q+\epsilon[1-x^q-(1-x)^q]$ is a linear function of the fraction x of neighbors in the opposite state. For values of $q>2$, the application of HMF is hampered by the nonlinearity of evolution equations, even if one considers the system only for the critical value of ϵ, i.e. without the drift term [18]. The case with $\epsilon=0$ is already difficult for $q=2$, as mentioned in the introduction and discussed in [7, 8], not only because of non-linearity but also because of the presence of a drift term in the evolution equation. Therefore, deriving an analytical formula (7) is a challenge for the future.

Another task that could be considered in the future is the exact relation between the finite-size scaling exponent ν and the parameters of the model. Here, it has been found that for $\beta=0$, $q=2$ and some arbitrary value of k the exit probability does not

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depend on the system size. The finite-size effects are also almost invisible for larger values of q, but recent results for very large lattices [10] suggest we be careful with the formulation of final conclusions. Our caution is also dictated by the results for $\beta > 0$, which have shown that for $q = 2$ EP depends much less on the system size than for $q > 2$. Nevertheless, taking into account the results obtained in this paper and earlier papers [5, 10] we may safely say that for $\beta = 0$ and the arbitrary values of q and k the finite-size effects are very weak. This agrees also with recent results for the broad class of Ising-type models. Roy *et al* [13] have investigated the exit probability in several one dimensional Ising-like models with so-called inflow dynamics, such as the Ising–Glauber model at temperature $T = 0$. Using Monte Carlo simulations they have found that a general form for the exit probability is given, as in our case, by equation (7), where the exponent α depends on model details (range of interactions, asymmetry or fluctuations present in a given dynamics). Moreover, they have shown that for the inflow dynamics in one dimension, the exit probability does not depend on the lattice size. However, for $\beta > 0$ the finite-size analysis provides much less trivial results. Using finite-size scaling technique we were able to determine scaling exponents ν for several sets of parameters (q, k, β), though the general dependence is not yet uncovered.

Summarizing, up till now the q-voter model with deadlock has been investigated only on the one-dimensional lattice with nearest neighbors. Here we have examined both the role of the density of the network given by k and its randomness characterized by β. We have been able to show that the exit time decreases with k and β. From a social point of view this means that the consensus is reached faster in societies with larger numbers of links and shorter paths. Moreover, we were able to show that, for any values of parameters, the exit probability can be described by the general relation (7), with the exponent α that depends on model parameters analogously, as in case of Ising-like models with inflow dynamics [13]. Interestingly, it became apparent that, for $\beta = 0$ and arbitrary values of q and k, the EP does not depend on the system size or depends very weakly for $q > 2$. On the contrary, for $\beta > 0$ non-trivial finite-size scaling has been found. Although we leave some problems open, as stressed above, we believe that our paper will contribute to the understanding of non-equilibrium phenomena and opinion spreading on networks. On the other hand we hope that the question posed here will motivate others to study q-voter models with deadlocks more deeply.

Acknowledgments

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Is the Person-Situation Debate Important for Agent-Based Modeling and Vice-Versa?

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Abstract

Background: Agent-based models (ABM) are believed to be a very powerful tool in the social sciences, sometimes even treated as a substitute for social experiments. When building an ABM we have to define the agents and the rules governing the artificial society. Given the complexity and our limited understanding of the human nature, we face the problem of assuming that either personal traits, the situation or both have impact on the social behavior of agents. However, as the long-standing person-situation debate in psychology shows, there is no consensus as to the underlying psychological mechanism and the important question that arises is whether the modeling assumptions we make will have a substantial influence on the simulated behavior of the system as a whole or not.

Methodology/Principal Findings: Studying two variants of the same agent-based model of opinion formation, we show that the decision to choose either personal traits or the situation as the primary factor driving social interactions is of critical importance. Using Monte Carlo simulations (for Barabasi-Albert networks) and analytic calculations (for a complete graph) we provide evidence that assuming a person-specific response to social influence at the microscopic level generally leads to a completely different and less realistic aggregate or macroscopic behavior than an assumption of a situation-specific response; a result that has been reported by social psychologists for a range of experimental setups, but has been downplayed or ignored in the opinion dynamics literature.

Significance: This sensitivity to modeling assumptions has far reaching consequences also beyond opinion dynamics, since agent-based models are becoming a popular tool among economists and policy makers and are often used as substitutes of real social experiments.

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Data Availability: The authors confirm that all data underlying the findings are fully available without restriction. All relevant data are within the paper and its Supporting Information files.

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Introduction

Agent-based models (ABM) are believed to be a very powerful tool in many disciplines [1–8]. Traditionally this kind of approach, that takes into account how individuals interact, was the domain of statistical physics [9]. Physicists were able to describe complex collective phenomena (e.g. phase transitions) as a result of microscopic interactions [10,11]. The models used by statistical physicists were rather simple, usually not because the physical reality was simple but because simple models were much easier to deal with and able to describe universal features. Moreover, within simple models it is possible to assess what microscopic factors are the most important from the macroscopic point of view. Agent-based models that are nowadays used in other disciplines are often more complicated, although the seminal model proposed by Thomas Schelling [12] to describe spatial segregation in societies was as simple as the simplest models in statistical physics can get. It is not the aim of this article to discuss if agent-based models have to be simple or not. It is obvious that a model should be designed to describe the problem at hand and this will determine the level of complexity [2]. However, the fundamental question that arises in all applications is how to introduce interactions.

Even if we consider only the simplest models of opinion formation – in which opinions are represented by binary variables and in which social influence is limited to conformity – we can find in the literature a number of competing and commonly used approaches, including the voter model [9,13], the majority rule [14,15], the Sznajd model [16] and Glauber-type opinion dynamics [17]. There have been also a few attempts at unifying these models. The first one was proposed by Galam and is presently known as the general sequential probabilistic model [18]. In 2009 the q-voter model was introduced [19] and it includes the voter and Sznajd models as special cases. In 2013 yet another generalization of the q-voter model was proposed [20]. Interestingly, the q-voter model belongs to the broad class of so-called

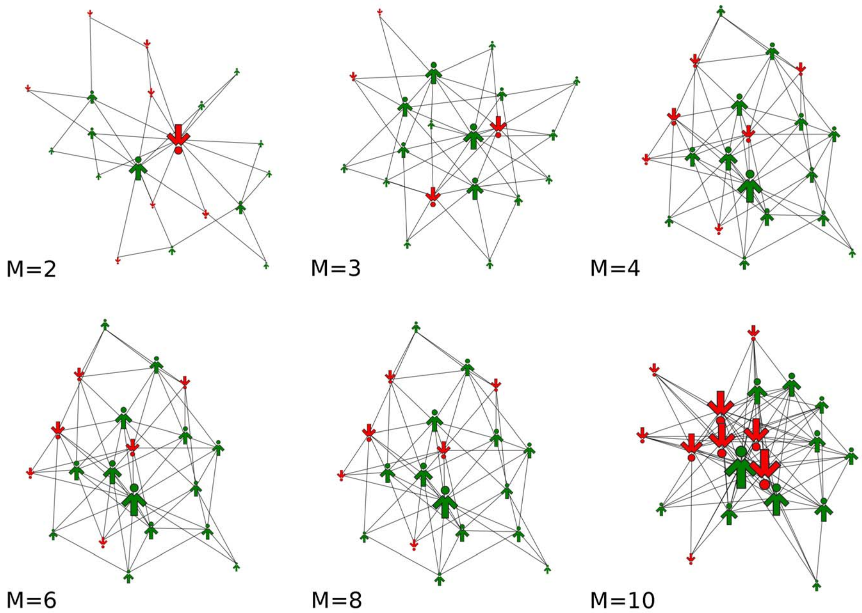

Figure 1. Sample Barabasi-Albert network structures for different densities of links, represented by parameter M. The agents are described by a single binary variable and called *spinsons* to reflect the dyadic nature of the agent (*spin*) and the object of study (*person*). The spinson size is proportional to the number of outgoing links, the color (and simultaneously orientation) represent the binary opinion.
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threshold models [21], which have gained considerable popularity in the social sciences.

Even greater confusion prevails if different types of social influence are considered, such as conformity, nonconformity, anticonformity, etc. An extended discussion of these issues can be found in [20]. Let us only briefly mention that the presence of zealots [22], inflexibles [23], experts [5] or independent agents [20] significantly changes the output of the model and introduces phase transitions [24]. Moreover, in some papers a nonconformist behavior is introduced as an individual trait [22,23], whereas in other as a situational factor [20,24–27]. This raises the question of the role of ABMs. Certainly some of them are just interesting in themselves and can be investigated from the point of view of basic research, in the domain of non-equilibrium statistical physics [9]. However, the ultimate objective of opinion dynamics ABMs is (or at least should be) making them applicable in the social sciences. But if so, the models should be based on realistic assumptions.

These considerations nicely lead us to two questions which have been the motivation for this paper:

1. *Micro level*: What determines human behavior – personal traits or rather the situation?
2. *Macro level*: Do the modeling assumptions we make regarding social interactions (personal traits vs. situation) have substantial impact on the simulated behavior of the system as a whole or not?

Obviously answering the first question lies beyond the domain of physics. In fact, among psychologists there has been a longstanding and vigorous discussion on this topic, known as the *person-situation debate*. The debate started in the late 1960s and recently has been announced to be over (for a review see the special issue of the Journal of Research in Personality [28]). On the other hand, there is still a lot of controversy around the subject. For instance, some psychologists argue that the debate is an academic problem because the concept of situation is not well defined or the questions in the debate are poorly posed. Nevertheless, intuitively the subject of the debate is quite clear and relatively easy to describe with agent-based modeling. It seems that nowadays most psychologists agree that both, the situation and personal traits, influence human behavior and such an approach is also visible in some ABMs [29]. The problems that are still under consideration in the literature are rather related with the question what factors and when are more important [30–32]. It is not the aim of this paper to solve one the most significant debates in the history of psychology. For us, the debate is an excellent excuse for a closer look at some fundamental problems in the area of agent-based modeling. On the other hand, the results obtained within agent-based modeling may shed some light on the debate itself.

Let us now focus on the second question. Certainly the debate is very important for psychologists but it is also important from the macroscopic or societal point of view? Imagine that we have a group of 1000 people and consider two approaches. In the first approach 100 individuals are independent, i.e. act independently of the social influence, and the remaining 900 are conformists, i.e. follow the behavior of other group members. In the second approach each member of a group acts independently with probability 0.1 and conforms with probability 0.9. The expected value of the independent behavior in both approaches is exactly the same. A first guess would most likely be that no differences between the two approaches will be visible on the macroscopic scale. And this indeed is the reply one of us (KSW) was giving when asked on different occasions. Only recently have we realized that this problem – while very difficult, if possible at all, to solve via social experiments – can be easily addressed within a microscopic agent-based model.

Methods

To investigate the above issue we use *q*-voter models, which have been originally proposed as situation-oriented [20] but can

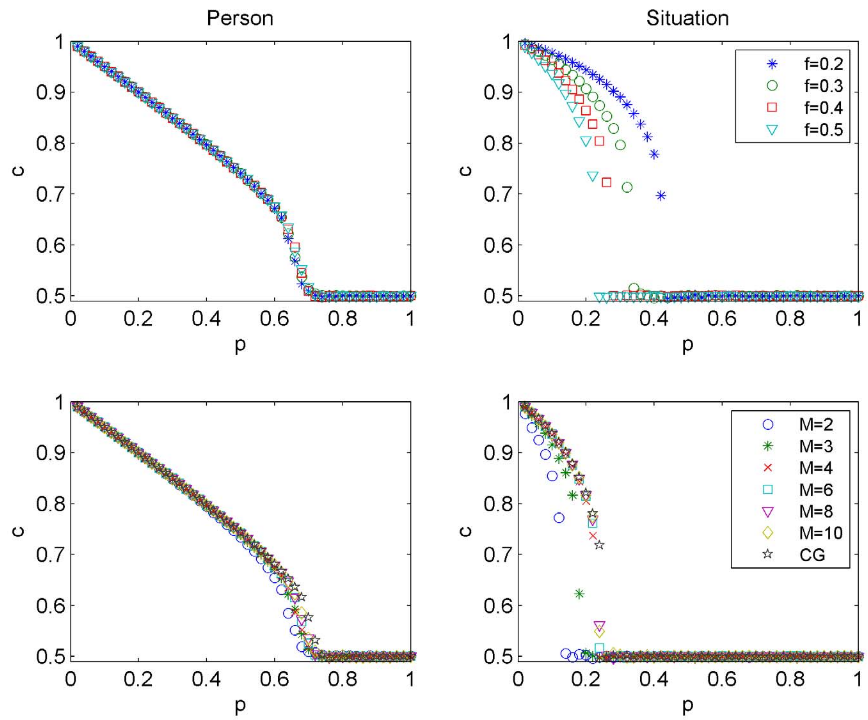

Figure 2. Concentration of adopted *c* in the stationary state as a function of independence *p* for the *person* (left column) and the *situation* (right column) models. Simulation results are averaged over 1000 Monte Carlo runs and concern Barabasi-Albert networks of size $N=10^4$. In the top row the dependence on flexibility f is shown for $M=4$, in the bottom row the dependence on M is shown for $f=0.5$. Note that the results for larger values of M approach the results for the CG, see Fig. 3.
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be easily reformulated to become personality-oriented. We set $q=4$ to reflect the empirically observed fact that a group of four individuals sharing the same opinion has a high chance to 'convince' the fifth, even if no rational arguments are available [33,34]. The agents in our models are described by a single binary variable, which may correspond to 'yes' or 'no' in the field of opinion dynamics or 'adopted' and 'not adopted' when modeling innovation diffusion. For such a simple agent, Nyczka and Sznajd-Weron [20] have recently introduced the name *spinson*, which reflects the dyadic nature of the agent (*spin*) and the object of study (*person*), see Fig. 1. We use this name throughout the paper as it nicely allows to go around gender issues. It should be also emphasized that models like the one discussed here are particularly useful in the field of diffusion of innovation [6,8,35]. Hence, in the remainder of the paper we use the 'innovation diffusion' language.

Interactions between spinsons are very simple, although based on empirical evidence (as reported by social psychologists [34]). Like in [36], in each time step a group of four connected spinsons is chosen and if the group unanimously shares an opinion it will influence one neighbor, which can behave like a conformist (i.e. take the opinion of the group) or act independently. In the case of independent behavior, with probability f the spinson changes its opinion and with $1-f$ stays with the current opinion. Parameter f represents *flexibility*; to calibrate the model to reality it may be set equal to the level of conservatism in the society [37,38]. Such a simple model can be easily formulated as situation-oriented or personality-oriented. In the first case – let us call it *situation* – an agent acts independently with probability p and follows the group with probability $1-p$. As a result, each spinson can sometimes behave independently and sometimes conform with the group. In the second approach or model – dubbed *person* – a fraction p of spinsons in the society are independent. The expected value of independent behavior is exactly the same in both approaches and therefore the first guess could be that both models give the same result on the macroscopic level.

Results

Monte Carlo simulations

We investigate both modeling approaches on Barabasi-Albert networks, as they nicely recover most of the features of a real social network [39]. We build the test network starting from a fully connected graph of M nodes and then preferentially attach M new nodes at each time step until the network achieves the assumed number of nodes N. We then conduct Monte Carlo simulations. In the initial state all spinsons are 'down', which corresponds to the situation prior to introducing the innovation (e.g. a tablet, a new electricity tariff) when none of the agents is 'adopted'. Due to independence some spinsons start to flip and then social influence from a unanimous group of $q=4$ spins may influence a neighboring (and connected) spinson. Eventually the system reaches a stationary state in which concentration of adopted fluctuates around some average value $c=c(p,f)$. As a result of competition between social influence (an ordering force) and independence (which introduces noise and disorders the system), a phase transition appears in both models (see Fig. 2). For level of independence $p<p*$ there is a state in which a majority ($c>0$) coexists with a minority and for $p>p^*$ a status-quo situation is observed ($c=0$). Surprisingly, in the *person* model there is no dependence on parameter f, which describes how often independent spinsons change their opinion. On the other hand, f influences the results significantly in the *situation* model (see the top right panel in Fig. 2).

Not only flexibility is an irrelevant parameter in the *person* model. Also the density of the network, represented by parameter M, does not influence the results (see the bottom left panel in Fig. 2). In the *situation* model the structure of the network is more important, although with increasing M concentration $c(p)$ approaches a limiting case which coincides with the result obtained for a complete graph. One could argue that the differences between the two considered models are visible because of the differences in network structures. However, to our surprise, the results presented in the bottom panels in Fig. 2 suggest that the differences appear even on a complete graph. Indeed, as can be seen in the bottom panels, flexibility f is a redundant parameter in the *person* model (left panel), but not in the *situation* model (right panel).

Analytical calculations for a complete graph

In this section we will perform analytical calculations to answer the intriguing question why the results for the two studied models differ so much even on a complete graph. In a general complex network setup, it is not easy to compute how the number of adopted spinsons changes in time and what is the stationary state. However, in the case of a complete graph this task is exceptionally simple and corresponds to the method known in statistical physics as the *mean field approach* (MFA; see e.g. [9]). On a complete graph each spinson is connected with every other spinson and therefore they are all neighbors. Hence, the system is completely homogeneous in the sense that the local concentration of adopted spinsons is statistically equal to the global concentration $c(t)$. Therefore we can write down the equation that describes the evolution of the system.

Recall that in the *person* approach there are two groups of agents and that the opinion dynamics in each of these groups is different. Let $N_1^{\uparrow}(t)$ and $N_1^{\downarrow}(t)$ denote the number of adopted ($\uparrow$) and unadopted ($\downarrow$) independent spinsons at time t, respectively. Then the total number of independent spinsons is constant in time: $N_1 = N_1^{\uparrow}(t) + N_1^{\downarrow}(t) = pN$. Further, let $N_2^{\uparrow}(t)$ and $N_2^{\downarrow}(t)$ denote the number of adopted ($\uparrow$) and unadopted ($\downarrow$) conformists at time t, respectively. Similarly, the total number of conformists is constant in time: $N_2 = N_2^{\uparrow}(t) + N_2^{\downarrow}(t) = (1-p)N$. Finally, denote by $c_1(t) = N_1^{\uparrow}(t)/N$ the concentration of adopted independent spinsons at time t and by $c_2(t) = N_2^{\uparrow}(t)/N$ the concentration of adopted conformists at time t; both are computed with respect to the whole system size N. Note that the ratio of adopted spinsons $c(t) = c_1(t) + c_2(t) = \{N_1^{\uparrow}(t) + N_2^{\uparrow}(t)\}/N$.

In each elementary time step the number of adopted independent spinsons $N_1^{\uparrow}(t)$ can increase by 1 only if: (i) an independent spinson is drawn from the set of all spinsons (the probability of this event is equal to N_1/N), (ii) this spinson is unadopted (with probability $N_1^{\downarrow}(t)/N_1$), and (iii) the spinson flips (with probability f). Analogously, in each elementary time step $N_1^{\uparrow}(t)$ can decrease by 1 only if: (i) an independent spinson is drawn from the set of all spinsons, (ii) this spinson is adopted (with probability $N_1^{\uparrow}(t)/N_1$), and (iii) the spinson flips. Therefore, we can write the following evolution equation for the number of adopted independent spinsons:

$$N_1^{\uparrow}(t+1) = N_1^{\uparrow}(t) + \frac{N_1}{N}\frac{N_1^{\downarrow}(t)}{N_1}f - \frac{N_1}{N}\frac{N_1^{\uparrow}(t)}{N_1}f = N_1^{\uparrow}(t) + f\left\{\frac{N_1^{\downarrow}(t)}{N} - \frac{N_1^{\uparrow}(t)}{N}\right\}$$

$$= N_1^{\uparrow}(t) + f\left\{\frac{pN - N_1^{\uparrow}(t)}{N} - \frac{N_1^{\uparrow}(t)}{N}\right\} = N_1^{\uparrow}(t) + f\{p - 2c_1(t)\}. \quad (1)$$

Dividing both sides by N we obtain the evolution equation for the concentration of adopted independent spinsons:

$$c_1(t+1) = c_1(t) + \frac{1}{N}f\{p - 2c_1(t)\}. \quad (2)$$

A similar reasoning can be conducted for the number of adopted conformists. In each elementary time step the number of adopted conformists $N_2^{\uparrow}(t)$ can increase by 1 only if: (i) a conformist is drawn from the set of all spinsons (with probability N_2/N), (ii) this spinson is unadopted (with probability $N_2^{\downarrow}(t)/N_2$), and (iii) all four chosen neighbors are adopted (the probability of this event is approximately equal to $c^4(t)$). Let us briefly comment on the later statement. The exact value of the probability in (iii) is equal to:

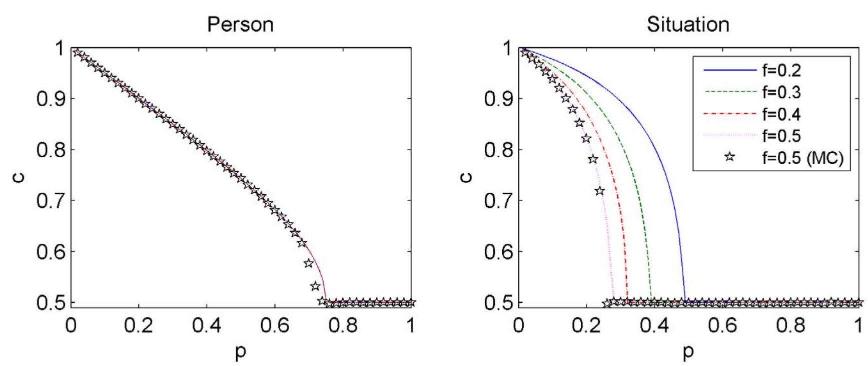

Figure 3. Concentration of adopted c in the stationary state as a function of independence p for the *person* (left) and the *situation* (right) models on a complete graph (CG). Analytic results obtained by iterating formulas (5) and (6) for four values of flexibility f are denoted by lines. For comparison, MC results for $f = 0.5$ (the same as in Fig. 2) are shown as stars. Except for the neighborhood of the critical point, the stars lie on the dotted purple line. This slight discrepancy is caused by the fact that near the critical point very long simulation times are needed to reach the steady state.
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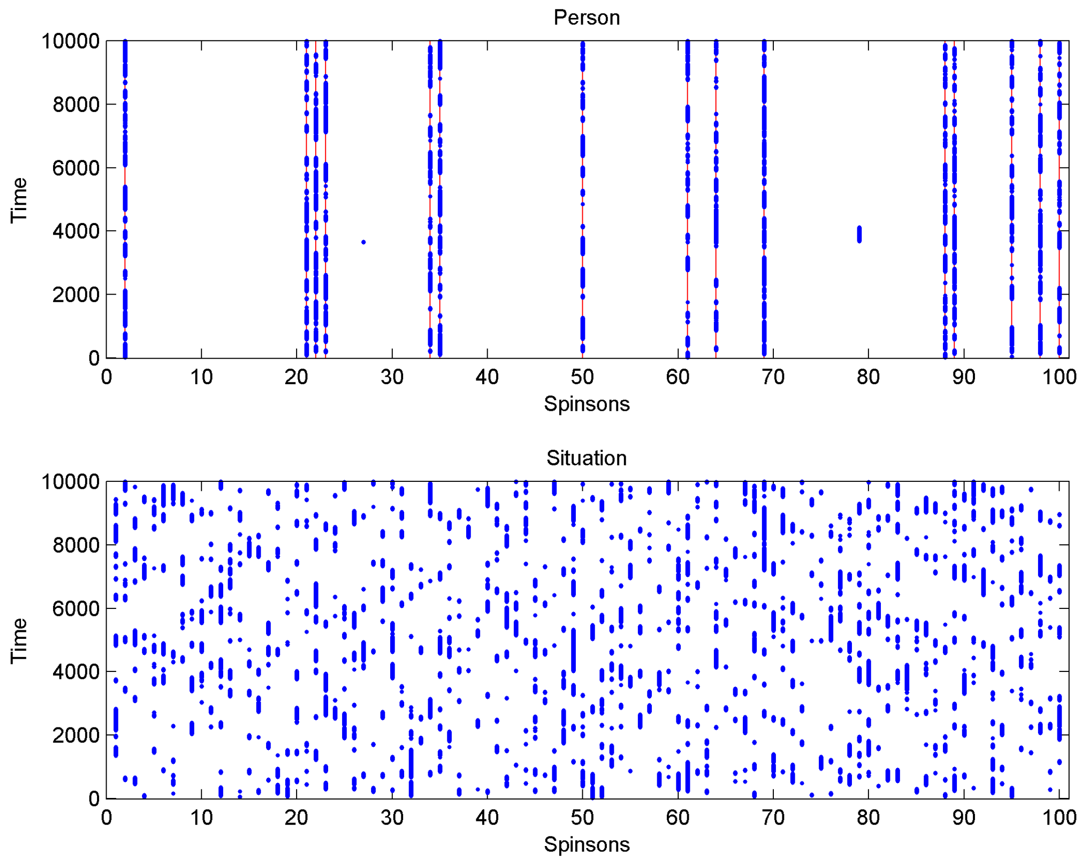

Figure 4. The time evolution of a system of $N=100$ spinsons on a complete graph from an initial state of no adopted. The number of time steps is 10,000, which corresponds to 100 Monte Carlo Steps (MCS). Blue dots denote 'adopted', white spaces denote 'not adopted'. In the *person* model (top panel) the red solid lines denote positions of independent spinsons. Note how the blue dots stay on the red lines – generally only the independent spinsons flip. This is very much unlike the dynamics of the *situation* model (bottom panel).
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$$\frac{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)}{N-1}\cdot\frac{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)-1}{N-2}\cdot\frac{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)-2}{N-3}\cdot\frac{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)-3}{N-4}.$$

However, assuming that for $k<<\{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)\}$ we can approximate $\{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)-k\}$ by $\{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)\}$ and for $l<<N$ we can approximate $(N-l)$ by N, the probability in (iii) can be approximated by

$$\left\{\frac{N_1^{\uparrow}(t)+N_2^{\uparrow}(t)}{N}\right\}^4=c^4(t).$$

Analogously, in each elementary time step $N_2^{\uparrow}(t)$ can decrease by 1 only if: (i) a conformist is drawn from the set of all spinsons, (ii) this spinson is adopted (with probability $N_2^{\uparrow}(t)/N_2$), and (iii) all four chosen neighbors are unadopted (the probability of this event is approximately equal to $\{1-c(t)\}^4$). Therefore, we can write the following evolution equation for the number of adopted conformists:

$$N_2^{\uparrow}(t+1)=N_2^{\uparrow}(t)+\frac{N_2}{N}\frac{N_2^{\downarrow}(t)}{N_2}c^4(t)-\frac{N_2}{N}\frac{N_2^{\uparrow}(t)}{N_2}\{1-c(t)\}^4$$

$$=N_2^{\uparrow}(t)+\{1-p-c_2(t)\}c^4(t)-c_2(t)\{1-c(t)\}^4. \quad (3)$$

Dividing both sides by N we obtain the evolution equation for the concentration of adopted conformists:

$$c_2(t+1)=c_2(t)+\frac{1}{N}\left[\{1-p-c_2(t)\}c^4(t)-c_2(t)\{1-c(t)\}^4\right]. \quad (4)$$

Finally, combining formulas (2) and (4) we obtain the complete set of equations which describe the time evolution of the system in the *person* model:

$$\begin{cases} c_1(t+1) = c_1(t)+\frac{1}{N}f\{p-2c_1(t)\}, \\ c_2(t+1) = c_2(t)+\frac{1}{N}\left[\{1-p-c_2(t)\}c^4(t)-c_2(t)\{1-c(t)\}^4\right], \\ c(t+1) = c_1(t+1)+c_2(t+1) \end{cases} \quad (5)$$

Now, let us briefly comment on the *situation* model. Within this approach all agents are homogeneous and behave independently with probability p and conform with probability $(1-p)$. Following a similar argumentation as in the *person* model, we arrive at the evolution equation for the concentration of adopted spinsons in the *situation* model:

$$c(t+1)=c(t)+\frac{1}{N}pf\{1-2c(t)\}+\frac{1}{N}(1-p)\left[c^4(t)\{1-c(t)\}-\{1-c(t)\}^4c(t)\right]. \tag{6}$$

From formulas (5) and (6) we can obtain the stationary value of the concentration of adopted spinsons, i.e. $c(\infty)$, and it agrees very well with Monte Carlo results (compare Figs. 2 and 3).

But how can we understand the difference more intuitively, without looking at these two figures and the formulas behind them? To do this let us consider again a system in which initially there are no adopted. In the *person* model only independent spinsons can flip. With increasing f they flip more often but this is generally true only for independent spinsons (see the upper panel in Fig. 4). Only if all spinsons in a selected group of q agents are adopted a non-independent neighboring spinson (note that on a complete graph all spinsons are neighbors) may be flipped, which happens quite rarely for smaller values of independence p. On the other hand, in the *situation* model, every spinson can flip with probability pf and therefore with increasing f more and more spinsons flip (see the lower panel in Fig. 4). Therefore the results in this case depend on f.

Conclusions

As suggested by numerous social experiments, the situation can almost completely prevail over personality [34]. On the other hand, personality psychologists argue that *there is considerable agreement that personality attributes exist and that these attributes shape how individuals adapt to the challenges of life* [28]. Although the relative importance of personality versus situational factors is very important from the point of view of psychology, it has been ignored or forgotten in agent-based modeling (at least in the context of binary opinion dynamics models). One of the reasons for such a situation may be the belief that such a detail does not affect the macroscopic behavior of the system.

However, as we have shown, there are significant differences between the personality- and situation-oriented modeling approaches. In the former case, the results on the macroscopic scale do not depend on flexibility (representing the level of conservatism in the society) nor on the network structure, which does not seem to be very realistic. This has far reaching consequences for agent-based modeling in general [7]. Some psychologists argue that the person-situation debate is an academic problem because the concept of situation is not well defined. However, within agent-based modeling this problem can be clearly defined.

Our results indicate that the situation approach may be more relevant when modeling social interactions, which – in a way – validates experimental results [30,33,34]. It may also shed some light on the debate itself. Naturally, we are aware of the limitations of our study, in particular the fact that only one type of model has been analyzed here. Nevertheless we would like to emphasize that when building ABMs we should take into consideration which of the factors – personal traits or situation – determines the behavior in a particular situation.

Supporting Information

Dataset S1 Monte Carlo Simulation results and Matlab codes for Figure 2.
(ZIP)

Author Contributions

Conceived and designed the experiments: KSW JS. Performed the experiments: KSW JS RW. Analyzed the data: KSW JS RW. Wrote the paper: KSW RW.

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