Abstract

The doctoral thesis begins with the discussion of basic physical problems related to Hall systems. These include the dynamics of a single electron in a magnetic field as well as in crossed magnetic and electric fields. The discussion is held in classical and quantum languages. Furthermore, the classical Hall effect is investigated using a simple model of transport, i.e. the Drude model. The dissertation also explains the relation between a perpendicular conductivity and a magnetic field, which is observed in two-dimensional systems and commonly identified with the quantum Hall effect. Technical aspects of transport measurements are not omitted, e.g. the existence of the so-called hot spots, which result in a "leakage" of carriers through voltage-probing electrodes, is explained. Additionally, crucial theories of the quantum Hall effects are presented, i.e. the edge currents model, Laughlin's and Thouless's arguments, and the composite fermion model. It is worth mentioning that the most important problems and conclusions following from the presented theories are emphasized, since they define foundations of the cyclotron subgroup model, i.e. the model developed in the doctoral thesis.

The main part of dissertation describes non-trivial consequences of the quantum indistinguishability of particles, e.g. the quotient structure of configuration space, Ω. Additionally, it defines the braid group, $\pi_1^{QS}(\Omega)$, as well as the relation between its scalar representation with a phase acquired by a wave function after an exchange of arguments. Furthermore, the cyclotron subgroup model – based on the assumption that certain elements of the full braid group are not well-defined in Hall systems – is introduced. The latter is easily justified when elements of $\pi_1^{QS}(\Omega)$ are represented as classical trajectories performed in a classical system, e.g. cyclotron orbits with surfaces proportional to kinetic energy. As a consequence, the cyclotron subgroup model explains unique statistics of composite fermions using modifications of the braid group, and not the Chern-Simons gauge field [Jain, 2007, Lopez and Fradkin, 1991]. It is worth noting that the model has a topological character, because modifications of $\pi_1^{QS}(\Omega)$ are not allowed in three-dimensional spaces; because double-loops from Ω belong to the neutral element of $\pi_1^{QS}(\Omega)$ in three-dimensional spaces. Recall that Thouless revealed the topological nature of the quantization of transverse conductivity [Niu et al., 1985]. The doctoral thesis also defines commensurability conditions of density and magnetic field, i.e. classical crystal and cyclotron orbits. These conditions allow determining filling factors, ν, for which incompressible and statically correlated states can be realized. Furthermore, it demonstrates that the obtained hierarchy is consistent with one evidenced in Hall systems, i.e. established form *plateaus* of transverse conductivity. It is worth noting that the obtained hierarchy contains not only Laughlin's and Jain's, but also enigmatic fillings.

Additionally, potential wave functions of Hall states are presented. These are not antisymmetric with respect to exchanges of arbitrary pairs of arguments. However, they transform according to the one-dimensional unitary representation of the braid group. This change of symmetry indicates the

separation of statistically correlated subsystems. In order to verify the proposed Ψ_N, expectation values of potential energy, density and pair distribution functions are determined and examined. For this purpose, numerical simulations in the disc geometry are carried out. Furthermore, no boundary potential is assumed and the Metropolis Monte Carlo algorithm is used. It is demonstrated that the expectation values are equivalent to ones estimated from exact diagonalizations and agree with predictions of the composite fermion model. As a result, it is demonstrated that the cyclotron subgroup model provides a novel description of composite fermions.

The last part of doctoral thesis introduces an algorithm which allows revealing correlations between particle positions. The latter introduces the well-known simulated annealing technique in order to determine the most probable configuration (i.e. the global maximum of $|\Psi_N|^2$). Furthermore, it introduces a novel measure of proximity of configurations in order to establish the configuration probability density. Introduced algorithm is implemented to reveal and characterize correlations in Hall systems. For example, it allows relating parameters of the cyclotron subgroup model with the robustness of Hall states in experiments; with the energy gain of Hall states. Additionally, it admits used to identify previously unrecognized deformations of composite fermion systems introduced by the projection onto the lowest Landau level.